Kinematics and Dynamics of Mechanisms

Kinematics of mechanisms is concerned with the motion of the parts without considering how the influencing factors (force and mass) affect the motion. Therefore, kinematics deals with the fundamental concepts of space and time and the quantities velocity and acceleration derived there from.

Kinetics deals with action of forces on bodies. This is where the the effects of gravity come into play.

Dynamics is the combination of kinematics and kinetics.

Dynamics of mechanisms concerns the forces that act on the parts -- both balanced and unbalanced forces, taking into account the masses and accelerations of the parts as well as the external forces.

Links, Frames and Kinematic Chains

A link is defined as a rigid body having two or more pairing elements which connect it to other bodies for the purpose of transmitting force or motion (Ham et al. 58).

In every machine, at least one link either occupies a fixed position relative to the earth or carries the machine as a whole along with it during motion. This link is the frame of the machine and is called the fixed link.

The combination of links and pairs without a fixed link is not a mechanism but a kinematic chain Skeleton Outli

For the purpose of kinematic analysis, a mechanism may be represented in an abbreviated, or skeleton, form called the skeleton outline of the mechanism. The skeleton outline gives all the geometrical information necessary for determining the relative motions of the links. In Figure 3-1, the skeleton outline has been drawn for the engine shown in Figure 2-1. This skeleton contains all necessary information to determine the relative motions of the main links, namely, the length AB of the crank; the length BC of the connecting rod; A the location of the axis of the main bearing; and the path AC of point C, which represents the wrist-pin axis.

Pairs, Higher Pairs, Lower Pairs and Linkages

A pair is a joint between the surfaces of two rigid bodies that keeps them in contact and relatively movable. For example, in Figure 3-2, a door jointed to the frame with hinges makes revolute joint (pin joint), allowing the door to be turned around its axis. Figure 3-2b and c show skeletons of a revolute joint. Figure 3-2b is used when both links joined by the pair can turn. Figure 3-2c is used when one of the link jointed by the pair is the frame.

Figure 3-2 Revolute pair

In Figure 3-3a a sash window can be translated relative to the sash. This kind of relative motion is called a prismatic pair. Its skeleton outlines are shown in b, c and d. c and d are used when one of the links is the frame.

Basic Kinematics of Constrained Rigid Bodies

4.1 Degrees of Freedom of a Rigid Body

4.1.1 Degrees of Freedom of a Rigid Body in a Plane

The degrees of freedom (DOF) of a rigid body is defined as the number of independent movements it has. Figure 4-1 shows a rigid body in a plane. To determine the DOF of this body we must consider how many distinct ways the bar can be moved. In a two dimensional plane such as this computer screen, there are 3 DOF. The bar can be translated along the x axis, translated along the y axis, and rotated about its centroid.

Figure 4-1 Degrees of freedom of a rigid body in a plane

4.1.2 Degrees of Freedom of a Rigid Body in Space

An unrestrained rigid body in space has six degrees of freedom: three translating motions along the x, y and z axes and three rotary motions around the x, y and z axes respectively.

Figure 4-2 Degrees of freedom of a rigid body in space

4.2 Kinematic Constraints

Two or more rigid bodies in space are collectively called a rigid body system. We can hinder the motion of these independent rigid bodies with kinematic constraints. Kinematic constraints are constraints between rigid bodies that result in the decrease of the degrees of freedom of rigid body system.

The term kinematic pairs actually refers to kinematic constraints between rigid bodies. The kinematic pairs are divided into lower pairs and higher pairs, depending on how the two bodies are in contact.

4.2.1 Lower Pairs in Planar Mechanisms

There are two kinds of lower pairs in planar mechanisms: revolute pairs and prismatic pairs.

A rigid body in a plane has only three independent motions -- two translational and one rotary -- so introducing either a revolute pair or a prismatic pair between two rigid bodies removes two degrees of freedom.

Figure 4-3 A planar revolute pair (R-pair)

Figure 4-4 A planar prismatic pair (P-pair)

4.2.2 Lower Pairs in Spatial Mechanisms

There are six kinds of lower pairs under the category of spatial mechanisms. The types are: spherical pair, plane pair, cylindrical pair, revolute pair, prismatic pair, and screw pair.

Figure 4-5 A spherical pair (S-pair)

A spherical pair keeps two spherical centers together. Two rigid bodies connected by this constraint will be able to rotate relatively around x, y and z axes, but there will be no relative translation along any of these axes. Therefore, a spherical pair removes three degrees of freedom in spatial mechanism. DOF = 3.

Figure 4-6 A planar pair (E-pair)

A plane pair keeps the surfaces of two rigid bodies together. To visualize this, imagine a book lying on a table where is can move in any direction except off the table. Two rigid bodies connected by this kind of pair will have two independent translational motions in the plane, and a rotary motion around the axis that is perpendicular to the plane. Therefore, a plane pair removes three degrees of freedom in spatial mechanism. In our example, the book would not be able to raise off the table or to rotate into the table. DOF = 3.

Figure 4-7 A cylindrical pair (C-pair)

A cylindrical pair keeps two axes of two rigid bodies aligned. Two rigid bodies that are part of this kind of system will have an independent translational motion along the axis and a relative rotary motion around the axis. Therefore, a cylindrical pair removes four degrees of freedom from spatial mechanism. DOF = 2.

Figure 4-8 A revolute pair (R-pair)

A revolute pair keeps the axes of two rigid bodies together. Two rigid bodies constrained by a revolute pair have an independent rotary motion around their common axis. Therefore, a revolute pair removes five degrees of freedom in spatial mechanism. DOF = 1.

Figure 4-9 A prismatic pair (P-pair)

A prismatic pair keeps two axes of two rigid bodies align and allow no relative rotation. Two rigid bodies constrained by this kind of constraint will be able to have an independent translational motion along the axis. Therefore, a prismatic pair removes five degrees of freedom in spatial mechanism. DOF = 1.

Figure 4-10 A screw pair (H-pair)

The screw pair keeps two axes of two rigid bodies aligned and allows a relative screw motion. Two rigid bodies constrained by a screw pair a motion which is a composition of a translational motion along the axis and a corresponding rotary motion around the axis. Therefore, a screw pair removes five degrees of freedom in spatial mechanism.

4.3 Constrained Rigid Bodies

Rigid bodies and kinematic constraints are the basic components of mechanisms. A constrained rigid body system can be a kinematic chain, a mechanism, a structure, or none of these. The influence of kinematic constraints in the motion of rigid bodies has two intrinsic aspects, which are the geometrical and physical aspects. In other words, we can analyze the motion of the constrained rigid bodies from their geometrical relationships or using Newton's Second Law.

A mechanism is a constrained rigid body system in which one of the bodies is the frame. The degrees of freedom are important when considering a constrained rigid body system that is a mechanism. It is less crucial when the system is a structure or when it does not have definite motion.

Calculating the degrees of freedom of a rigid body system is straight forward. Any unconstrained rigid body has six degrees of freedom in space and three degrees of freedom in a plane. Adding kinematic constraints between rigid bodies will correspondingly decrease the degrees of freedom of the rigid body system. We will discuss more on this topic for planar mechanisms in the next section.

4.4 Degrees of Freedom of Planar Mechanisms

4.4.1 Gruebler's Equation

The definition of the degrees of freedom of a mechanism is the number of independent relative motions among the rigid bodies. For example, Figure 4-11 shows several cases of a rigid body constrained by different kinds of pairs.

In Figure 4-11a, a rigid body is constrained by a revolute pair which allows only rotational movement around an axis. It has one degree of freedom, turning around point A. The two lost degrees of freedom are translational movements along the x and y axes. The only way the rigid body can move is to rotate about the fixed point A.

In Figure 4-11b, a rigid body is constrained by a prismatic pair which allows only translational motion. In two dimensions, it has one degree of freedom, translating along the x axis. In this example, the body has lost the ability to rotate about any axis, and it cannot move along the y axis.

In Figure 4-11c, a rigid body is constrained by a higher pair. It has two degrees of freedom: translating along the curved surface and turning about the instantaneous contact point.

In general, a rigid body in a plane has three degrees of freedom. Kinematic pairs are constraints on rigid bodies that reduce the degrees of freedom of a mechanism. Figure 4-11 shows the three kinds of pairs in planar mechanisms. These pairs reduce the number of the degrees of freedom. If we create a lower pair (Figure 4-11a,b), the degrees of freedom are reduced to 2. Similarly, if we create a higher pair (Figure 4-11c), the degrees of freedom are reduced to 1.

4.4.2 Kutzbach Criterion

The number of degrees of freedom of a mechanism is also called the mobility of the device. The mobility is the number of input parameters (usually pair variables) that must be independently controlled to bring the device into a particular position. The Kutzbach criterion, which is similar to Gruebler's equation, calculates the mobility.

In order to control a mechanism, the number of independent input motions must equal the number of degrees of freedom of the mechanism. For example, the transom in Figure 4-13a has a single degree of freedom, so it needs one independent input motion to open or close the window. That is, you just push or pull rod 3 to operate the window.

To see another example, the mechanism in Figure 4-15a also has 1 degree of freedom. If an independent input is applied to link 1 (e.g., a motor is mounted on joint A to drive link 1), the mechanism will have the a prescribed motion.

Figure 3-3 Prismatic pair

Generally, there are two kinds of pairs in mechanisms, lower pairs and higher pairs. What differentiates them is the type of contact between the two bodies of the pair. Surface-contact pairs are called lower pairs. In planar (2D) mechanisms, there are two subcategories of lower pairs -- revolute pairs and prismatic pairs, as shown in Figures 3-2 and 3-3, respectively. Point-, line-, or curve-contact pairs are called higher pairs. Figure 3-4 shows some examples of higher pairs Mechanisms composed of rigid bodies and lower pairs are called linkages.

Figure 3-4 Higher pairs

3.6 Kinematic Analysis and Synthesis

In kinematic analysis, a particular given mechanism is investigated based on the mechanism geometry plus other known characteristics (such as input angular velocity, angular acceleration, etc.). Kinematic synthesis, on the other hand, is the process of designing a mechanism to accomplish a desired task. Here, both choosing the types as well as the dimensions of the new mechanism can be part of kinematic synthesis

5.1 Introduction

5.1.1 What are Linkage Mechanisms?

Have you ever wondered what kind of mechanism causes the wind shield wiper on the front widow of car to oscillate (Figure 5-1a)? The mechanism, shown in Figure 5-1b, transforms the rotary motion of the motor into an oscillating motion of the windshield wiper.

Figure 5-1 Windshield wiper

Let's make a simple mechanism with similar behavior. Take some cardboard and make four strips as shown in Figure 5-2a.

Take 4 pins and assemble them as shown in Figure 5-2b.

Now, hold the 6in. strip so it can't move and turn the 3in. strip. You will see that the 4in. strip oscillates.

Figure 5-2 Do-it-yourself four bar linkage mechanism

The four bar linkage is the simplest and often times, the most useful mechanism. As we mentioned before, a mechanism composed of rigid bodies and lower pairs is called a linkage (Hunt 78). In planar mechanisms, there are only two kinds of lower pairs --- revolute pairs and prismatic pairs.

The simplest closed-loop linkage is the four bar linkage which has four members, three moving links, one fixed link and four pin joints. A linkage that has at least one fixed link is a mechanism. The following example of a four bar linkage was created in SimDesign in simdesign/fourbar.sim

Figure 5-3 Four bar linkage in SimDesign

This mechanism has three moving links. Two of the links are pinned to the frame which is not shown in this picture. In SimDesign, links can be nailed to the background thereby making them into the frame.

How many DOF does this mechanism have? If we want it to have just one, we can impose one constraint on the linkage and it will have a definite motion. The four bar linkage is the simplest and the most useful mechanism.

Reminder: A mechanism is composed of rigid bodies and lower pairs called linkages (Hunt 78). In planar mechanisms there are only two kinds of lower pairs: turning pairs and prismatic pairs.

5.1.2 Functions of Linkages

The function of a link mechanism is to produce rotating, oscillating, or reciprocating motion from the rotation of a crank or vice versa (Ham et al. 58). Stated more specifically linkages may be used to convert:

Continuous rotation into continuous rotation, with a constant or variable angular velocity ratio.

Continuous rotation into oscillation or reciprocation (or the reverse), with a constant or variable velocity ratio.

Oscillation into oscillation, or reciprocation into reciprocation, with a constant or variable velocity ratio.

Linkages have many different functions, which can be classified according on the primary goal of the mechanism:

Function generation: the relative motion between the links connected to the frame,

Path generation: the path of a tracer point, or

Motion generation: the motion of the coupler link.

5.2 Four Link Mechanisms

One of the simplest examples of a constrained linkage is the four-link mechanism. A variety of useful mechanisms can be formed from a four-link mechanism through slight variations, such as changing the character of the pairs, proportions of links, etc. Furthermore, many complex link mechanisms are combinations of two or more such mechanisms. The majority of four-link mechanisms fall into one of the following two classes:

the four-bar linkage mechanism, and

the slider-crank mechanism.

5.2.1 Examples

Parallelogram Mechanism

In a parallelogram four-bar linkage, the orientation of the coupler does not change during the motion. The figure illustrates a loader. Obvioulsy the behavior of maintaining parallelism is important in a loader. The bucket should not rotate as it is raised and lowered. The corresponding SimDesign file is simdesign/loader.sim.

Figure 5-4 Front loader mechanism

Slider-Crank Mechanism

The four-bar mechanism has some special configurations created by making one or more links infinite in length. The slider-crank (or crank and slider) mechanism shown below is a four-bar linkage with the slider replacing an infinitely long output link. The corresponding SimDesign file is simdesign/slider.crank.sim.

Figure 5-5 Crank and Slider Mechanism

This configuration translates a rotational motion into a translational one. Most mechanisms are driven by motors, and slider-cranks are often used to transform rotary motion into linear motion.

Crank and Piston

You can also use the slider as the input link and the crank as the output link. In this case, the mechanism transfers translational motion into rotary motion. The pistons and crank in an internal combustion engine are an example of this type of mechanism. The corresponding SimDesign file is simdesign/combustion.sim.

Figure 5-6 Crank and Piston

You might wonder why there is another slider and a link on the left. This mechanism has two dead points. The slider and link on the left help the mechanism to overcome these dead points.

Block Feeder

One interesting application of slider-crank is the block feeder. The SimDesign file can be found in simdesign/block-feeder.sim

Figure 5-7 Block Feeder

5.2.2 Definitions

In the range of planar mechanisms, the simplest group of lower pair mechanisms are four bar linkages. A four bar linkage comprises four bar-shaped links and four turning pairs as shown in Figure 5-8.

Figure 5-8 Four bar linkage

The link opposite the frame is called the coupler link, and the links whick are hinged to the frame are called side links. A link which is free to rotate through 360 degree with respect to a second link will be said to revolve relative to the second link (not necessarily a frame). If it is possible for all four bars to become simultaneously aligned, such a state is called a change point.

Some important concepts in link mechanisms are:

Crank: A side link which revolves relative to the frame is called a crank.

Rocker: Any link which does not revolve is called a rocker.

Crank-rocker mechanism: In a four bar linkage, if the shorter side link revolves and the other one rocks (i.e., oscillates), it is called a crank-rocker mechanism.

Double-crank mechanism: In a four bar linkage, if both of the side links revolve, it is called a double-crank mechanism.

Double-rocker mechanism: In a four bar linkage, if both of the side links rock, it is called a double-rocker mechanism.

5.2.3 Classification

Before classifying four-bar linkages, we need to introduce some basic nomenclature.

In a four-bar linkage, we refer to the line segment between hinges on a given link as a bar where:

s = length of shortest bar

l = length of longest bar

p, q = lengths of intermediate bar

Grashof's theorem states that a four-bar mechanism has at least one revolving link if

$$s + l \leq p + q$$

(5-1)

and all three mobile links will rock if

$$s + l > p + q$$

(5-2)

The inequality 5-1 is Grashof's criterion.

All four-bar mechanisms fall into one of the four categories listed in Table 5-1:

Table 5-1 Classification of Four-Bar Mechanisms

Case	l + s vers. p + q	Shortest Bar	Type
1	<	Frame	Double-crank
2	<	Side	Rocker-crank

3	<	Coupler	Doubl rocker
4	=	Any	Change point
5	>	Any	Double-rocker

From Table 5-1 we can see that for a mechanism to have a crank, the sum of the length of its shortest and longest links must be less than or equal to the sum of the length of the other two links. However, this condition is necessary but not sufficient. Mechanisms satisfying this condition fall into the following three categories:

When the shortest link is a side link, the mechanism is a crank-rocker mechanism. The shortest link is the crank in the mechanism.

When the shortest link is the frame of the mechanism, the mechanism is a double-crank mechanism.

When the shortest link is the coupler link, the mechanism is a double-rocker mechanism.

5.2.4 Transmission Angle

In Figure 5-11, if AB is the input link, the force applied to the output link, CD, is transmitted through the coupler link BC. (That is, pushing on the link CD imposes a force on the link AB, which is transmitted through the link BC.) For sufficiently slow motions (negligible inertia forces), the force in the coupler link is pure tension or compression (negligible bending action) and is directed along BC. For a given force in the coupler link, the torque transmitted to the output bar (about point D) is maximum when the angle between coupler bar BC and output bar CD is /2. Therefore, angle BCD is called transmission angle.1 Transmission angle

When the transmission angle deviates significantly from /2, the torque on the output bar decreases and may not be sufficient to overcome the friction in the system. For this reason, the deviation angle =|/2-| should not be too great. In practice, there is no definite upper limit for , because the existence of the inertia forces may eliminate the undesirable force relationships that is present under static conditions. Nevertheless, the following criterion can be followed.

5.2.5 Dead Point

When a side link such as AB in Figure 5-10, becomes aligned with the coupler link BC, it can only be compressed or extended by the coupler. In this configuration, a torque applied to the link on the other side, CD, cannot induce rotation in link AB. This link is therefore said to be at a dead point (sometimes called a toggle point).

Dead point

In Figure 5-11, if AB is a crank, it can become aligned with BC in full extension along the line AB1C1 or in flexion with AB2 folded over B2C2. We denote the angle ADC by and the angle DAB by . We use the subscript 1 to denote the extended state and 2 to denote the flexed state of links AB and BC. In the extended state, link CD cannot rotate clockwise without stretching or compressing the theoretically rigid line AC1. Therefore, link CD cannot move into the forbidden zone below C1D, and must be at one of its two extreme positions; in other words, link CD is at an extremum. A second extremum of link CD occurs with = 1.

Note that the extreme positions of a side link occur simultaneously with the dead points of the opposite link.

In some cases, the dead point can be useful for tasks such as work fixturing (Figure 5-11).

Work fixturing

In other cases, dead point should be and can be overcome with the moment of inertia of links or with the asymmetrical deployment of the mechanism (Figure 5-12).

Overcoming the dead point by asymmetrical deployment (V engine)

5.2.6 Slider-Crank Mechanism

The slider-crank mechanism, which has a well-known application in engines, is a special case of the crank-rocker mechanism. Notice that if rocker 3 in Figure 5-13a is very long, it can be replaced by a block sliding in a curved slot or guide as shown. If the length of the rocker is infinite, the guide and block are no longer curved. Rather, they are apparently straight, as shown in Figure 5-13b, and the linkage takes the form of the ordinary slider-crank mechanism.

Figure 5-13 Slider-Crank mechanism

5.2.7 Inversion of the Slider-Crank Mechanism

Inversion is a term used in kinematics for a reversal or interchange of form or function as applied to kinematic chains and mechanisms. For example, taking a different link as the fixed link, the slider-crank mechanism shown in Figure 5-14a can be inverted into the mechanisms shown in Figure 5-14b, c, and d. Different examples can be found in the application of these mechanisms. For example, the mechanism of the pump device in

Inversions of the crank-slide mechanipump device

Keep in mind that the inversion of a mechanism does not change the motions of its links relative to each other but does change their absolute motions.

Gears are machine elements that transmit motion by means of successively engaging teeth. The gear teeth act like small levers.

7.1 Gear Classification

Gears may be classified according to the relative position of the axes of revolution. The axes may be

parallel,

intersecting,

neither parallel nor intersecting.

Here is a brief list of the common forms. We will discuss each in more detail later.

Gears for connecting parallel shafts

Gears for connecting intersecting shafts

Neither parallel nor intersecting shafts

Gears for connecting parallel shafts

Spur gears

The left pair of gears makes external contact, and the right pair of gears makes internal contact

Parallel helical gears

Herringbone gears (or double-helical gears)

Rack and pinion (The rack is like a gear whose axis is at infinity.)

Gears for connecting intersecting shafts

Straight bevel gears

Spiral bevel gears

Neither parallel nor intersecting shafts

LCrossed-helical gears

Hypoid gears

Worm and wormgear

7.2 Gear-Tooth Action

7.2.1 Fundamental Law of Gear-Tooth Action

Figure 7-2 shows two mating gear teeth, in which

7.2.2 Constant Velocity Ratio

For a constant velocity ratio, the position of P should remain unchanged. In this case, the motion transmission between two gears is equivalent to the motion transmission between two imagined slipless cylinders with radius R1 and R2 or diameter D1 and D2. We can get two circles whose centers are at O1 and O2, and through pitch point P. These two circle are termed pitch circles. The velocity ratio is equal to the inverse ratio of the diameters of pitch circles. This is the fundamental law of gear-tooth action.

The fundamental law of gear-tooth action may now also be stated as follow (for gears with fixed center distance) (Ham 58):

The common normal to the tooth profiles at the point of contact must always pass through a fixed point (the pitch point) on the line of centers (to get a constant velocity ration).

7.2.3 Conjugate Profiles

To obtain the expected velocity ratio of two tooth profiles, the normal line of their profiles must pass through the corresponding pitch point, which is decided by the velocity ratio. The two profiles which satisfy this requirement are called conjugate profiles. Sometimes, we simply termed the tooth profiles which satisfy the fundamental law of gear-tooth action the conjugate profiles.

Although many tooth shapes are possible for which a mating tooth could be designed to satisfy the fundamental law, only two are in general use: the cycloidal and involute profiles. The involute has important advantages -- it is easy to manufacture and the center distance between a pair of involute gears can be varied without changing the velocity ratio. Thus close tolerances between shaft locations are not required when using the involute profile. The most commonly used conjugate tooth curve is the involute curve (Erdman & Sandor 84).

7.3 Involute Curve

The following examples are involute spur gears. We use the word involute because the contour of gear teeth curves inward. Gears have many terminologies, parameters and principles. One of the important concepts is the velocity ratio, which is the ratio of the rotary velocity of the driver gear to that of the driven gears.

The SimDesign file for these gears is simdesign/gear15.30.sim. The number of teeth in these gears are 15 and 30, respectively. If the 15-tooth gear is the driving gear and the 30-teeth gear is the driven gear, their velocity ratio is 2.

Other examples of gears are in simdesign/gear10.30.sim and simdesign/gear20.30.sim

7.3.1 Generation of the Involute Curve

Figure 7-3 Involute curve

The curve most commonly used for gear-tooth profiles is the involute of a circle. This involute curve is the path traced by a point on a line as the line rolls without slipping on the circumference of a circle. It may also be defined as a path traced by the end of a string which is originally wrapped on a circle when the string is unwrapped from the circle. The circle from which the involute is derived is called the base circle.

In Figure 7-3, let line MN roll in the counterclockwise direction on the circumference of a circle without slipping. When the line has reached the position M'N', its original point of tangent A has reached the position K, having traced the involute curve AK during the motion. As the motion continues, the point A will trace the involute curve AKC.

7.3.2 Properties of Involute Curves

The distance BK is equal to the arc AB, because link MN rolls without slipping on the circle.

For any instant, the instantaneous center of the motion of the line is its point of tangent with the circle.

Note: We have not defined the term instantaneous center previously. The instantaneous center or instant center is defined in two ways (Bradford & Guillet 43):

When two bodies have planar relative motion, the instant center is a point on one body about which the other rotates at the instant considered.

When two bodies have planar relative motion, the instant center is the point at which the bodies are relatively at rest at the instant considered.

The normal at any point of an involute is tangent to the base circle. Because of the property (2) of the involute curve, the motion of the point that is tracing the involute is perpendicular to the line at any instant, and hence the curve traced will also be perpendicular to the line at any instant.

There is no involute curve within the base circle.

7.4 Terminology for Spur Gears

Figure 7-4 shows some of the terms for gears.

Figure 7-4 Spur Gear

In the following section, we define many of the terms used in the analysis of spur gears. Some of the terminology has been defined previously but we include them here for completeness. (See (Ham 58) for more details.)

Pitch surface : The surface of the imaginary rolling cylinder (cone, etc.) that the toothed gear may be considered to replace.

Pitch circle: A right section of the pitch surface.

Addendum circle: A circle bounding the ends of the teeth, in a right section of the gear.

Root (or dedendum) circle: The circle bounding the spaces between the teeth, in a right section of the gear.

Addendum: The radial distance between the pitch circle and the addendum circle.

Dedendum: The radial distance between the pitch circle and the root circle.

Clearance: The difference between the dedendum of one gear and the addendum of the mating gear.

Face of a tooth: That part of the tooth surface lying outside the pitch surface.

Flank of a tooth: The part of the tooth surface lying inside the pitch surface.

Circular thickness (also called the tooth thickness) : The thickness of the tooth measured on the pitch circle. It is the length of an arc and not the length of a straight line.

Tooth space: The distance between adjacent teeth measured on the pitch circle.

Backlash: The difference between the circle thickness of one gear and the tooth space of the mating gear.

Circular pitch p: The width of a tooth and a space, measured on the pitch circle.

Diametral pitch P: The number of teeth of a gear per inch of its pitch diameter. A toothed gear must have an integral number of teeth. The circular pitch, therefore, equals the pitch circumference divided by the number of teeth. The diametral pitch is, by definition, the number of teeth divided by the pitch diameter. That is,

p = circular pitch

P = diametral pitch

N = number of teeth

D = pitch diameter

That is, the product of the diametral pitch and the circular pitch equals .

Module m: Pitch diameter divided by number of teeth. The pitch diameter is usually specified in inches or millimeters; in the former case the module is the inverse of diametral pitch.

Fillet : The small radius that connects the profile of a tooth to the root circle.

Pinion: The smaller of any pair of mating gears. The larger of the pair is called simply the gear.

Velocity ratio: The ratio of the number of revolutions of the driving (or input) gear to the number of revolutions of the driven (or output) gear, in a unit of time.

Pitch point: The point of tangency of the pitch circles of a pair of mating gears.

Common tangent: The line tangent to the pitch circle at the pitch point.

Line of action: A line normal to a pair of mating tooth profiles at their point of contact.

Path of contact: The path traced by the contact point of a pair of tooth profiles.

Pressure angle : The angle between the common normal at the point of tooth contact and the common tangent to the pitch circles. It is also the angle between the line of action and the common tangent.

Base circle :An imaginary circle used in involute gearing to generate the involutes that form the tooth profiles

Ordinary Gear Trains

Gear trains consist of two or more gears for the purpose of transmitting motion from one axis to another. Ordinary gear trains have axes, relative to the frame, for all gears comprising the train. Figure 7-6a shows a simple ordinary train in which there is only one gear for each axis. In Figure 7-6b a compound ordinary train is seen to be one in which two or more gears may rotate about a single axis.

Figure 7-6 Ordinary gear trains

7.6.1 Velocity Ratio

We know that the velocity ratio of a pair of gears is the inverse proportion of the diameters of their pitch circle, and the diameter of the pitch circle equals to the number of teeth divided by the diametral pitch. Also, we know that it is necessary for the to mating gears to have the same diametral pitch so that to satisfy the condition of correct meshing. Thus, we infer that the velocity ratio of a pair of gears is the inverse ratio of their number of teeth.

The tooth number in the numerator are those of the driven gears, and the tooth numbers in the denominator belong to the driver gears.

Gear 2 and 3 both drive and are, in turn, driven. Thus, they are called idler gears. Since their tooth numbers cancel, idler gears do not affect the magnitude of the input-output ratio, but they do change the directions of rotation. Note the directional arrows in the figure. Idler gears can also constitute a saving of space and money (If gear 1 and 4 meshes directly across a long center distance, their pitch circle will be much larger.)

There are two ways to determine the direction of the rotary direction. The first way is to label arrows for each gear as in Figure 7-6. The second way is to multiple mth power of "-1" to the general velocity ratio. Where m is the number of pairs of external contact gears (internal contact gear pairs do not change the rotary direction). However, the second method cannot be applied to the spatial gear trains.

Thus, it is not difficult to get the velocity ratio of the gear train in Figure 7-6b:

(7-16)

7.7 Planetary gear trains

Planetary gear trains, also referred to as epicyclic gear trains, are those in which one or more gears orbit about

the central axis of the train. Thus, they differ from an ordinary train by having a moving axis or axes. Figure 7-8 shows a basic arrangement that is functional by itself or when used as a part of a more complex system. Gear 1 is called a sun gear , gear 2 is a planet, link H is an arm, or planet carrier.

Figure 7-8 Planetary gear trains

Figure 7-7 Planetary gears modeled using SimDesign

The SimDesign file is simdesign/gear.planet.sim. Since the sun gear (the largest gear) is fixed, the DOF of the above mechanism is one. When you pull the arm or the planet, the mechanism has a definite motion. If the sun gear isn't frozen, the relative motion is difficult to control.

7.7.1 Velocity Ratio

To determine the velocity ratio of the planetary gear trains is slightly more complex an analysis than that required for ordinary gear trains. We will follow the procedure:

Invert the planetary gear train mechanism by imagining the application a rotary motion with an angular velocity of H to the mechanism. Let's analyse the motion before and after the inversion with Table 7-3:

Table 7-3 Inversion of planetary gear trains.

Note: H is the rotary velocity of gear i in the imagined mechanism.

Notice that in the imagined mechanism, the arm H is stationary and functions as a frame. No axis of gear moves any more. Hence, the imagined mechanism is an ordinary gear train.

Apply the equation of velocity ratio of the ordinary gear trains to the imagined mechanism. We get

(7-17)

or

(7-18)

7.7.2 Example

Take the planetary gearing train in Figure 7-8 as an example. Suppose N1 = 36, N2 = 18, 1 = 0, 2 = 30. What is the value of N?

Module3

Static and dynamic force

Mechanisms are designed to carry out certain desired work, by producing the

specified motion of certain output member. It is usually required to find the force or

torque to be applied on an input member, when one or more forces act on certain output member(s). The external force may be constant or varying through the whote cycle of motion. Calculation of input force or torque over the complete cycle will be needed to determine the power requirement. When the masses and moments of inertia of the members are negligible, static force analysis may be carried out. Otherwise, particularly at high speeds, significant forces or torques will be required to produce linear or angular accelerations of the various members. The same will have to be considered in the analysis. It is also required to find the forces at the joints for proper design. These also vary depending upon the position/configuration in the cycle.

Static analysis is carried out by the usual methods of collinearity of forces, equilibrium of forces / moments. Input is determined as that force or moment to bring the system to equilibrium. In the case of dynamic systems, linear acceleration of each link (CC) and the angular accelerations of the members are evaluated. The corresponding forces and moments are calculated (product of acceleration and inertia).

D'Alembert's principle is a method of applying fictitious forces / torques called inertia force / torque, equal and opposite to the force or torque required to produce acceleration in each member, so as to produce a static condition which is called dynamic equilibrium. Then the system can be treated as static, which permits application of techniques of static force analysis.Dynamic force analysis is the evaluation of input forces or torques and joint forcesconsideiing motion of members. Evaluation of the inertia force /torque are exPlainedfirst. Methods of static force analysis are explained.

[redacted]

The rate of change of velocity with respect to time is known as acceleration and acts in the direction of the change in velocity. A change in the velocity requires any of the following conditions to be fulfilled.

(i) A change in the magnitude only

(ii) A change in direction only

(iii) A change in both magnitude and direction.

<u>Acceleration</u>: Let a link OA, of length r , rotate in a circular path in the clockwise direction as shown in figure . It has instantaneous angular velocity ω and an angular acceleration α in the same direction , i.e the angular velocity increases in the clock wise direction. Tangential velocity of A , $v_a = \omega r$. In short interval of time δt , let OA assume the new position OA′ by rotating through a small angle $\delta\theta$.

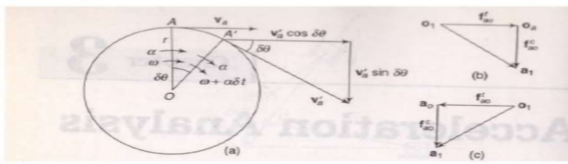


Figure - 30

Angular velocity of OA′ , $\omega_a' = \omega + \alpha \cdot \delta t$.

Tangential velocity of A′, $V_a' = (\omega + \alpha \cdot \delta t) r$.

The tangential velocity of A′ may be considered to have two components ; one perpendicular to OA and the other parallel to OA .

<u>Change of velocity perpendicular to OA:</u>

Velocity of A $\perp$ to OA = V_a

Velocity of A′ $\perp$ to OA = $V_a' \cos \delta\theta$

Change of velocity = $V_a' \cos \delta\theta - V_a$

Acceleration of A $\perp$ to OA $= \dfrac{(\omega + \alpha \cdot \delta t) r \cos \delta\theta - \omega r}{\delta t}$

In the limit, as $\delta t \to 0$, $\cos \delta\theta \to 1$

$\therefore$ Acceleration of A $\perp$ to OA $= \dfrac{\omega r + \alpha r \delta t - \omega r}{\delta t} = \alpha r = \dfrac{d\omega}{dt} r = \dfrac{d(\omega r)}{dt} = \dfrac{dv}{dt}$

This represents the rate of change of velocity in the tangential direction of the motion of A relative to O and thus is known as the tangential acceleration.

<u>Change of velocity parallel to OA</u>:

Velocity of A parallel to OA = 0

Velocity of A′ parallel to OA = $V_a' \sin \delta\theta$

Change of Velocity = $V_a' \sin \delta\theta - 0$

Acceleration of A parallel to OA $= \omega r \dfrac{d\theta}{dt} = \omega r \cdot \omega = \omega^2 r = \dfrac{V^2}{r}$

This represents the rate of change of velocity along OA, the direction being from A towords O or towards the center of rotation. It is known as centripetal acceleration and denoted by f_{ao}^{c} .

In this way f_{fa} can be found.

Acceleration of slider–crank mechanism :

Writing the acceleration equation,

Acc. of B rel. to O = Acc. Of B rel. to A + Acc. Of A rel. to O

$$\vec{f}_{bo} = \vec{f}_{ba} + \vec{f}_{ao} \,;$$

$$\vec{f}_{bg} = \vec{f}_{ao} + \vec{f}_{ba} = \vec{f}_{ao} + \vec{f}^{\,c}_{ba} + \vec{f}^{\,t}_{ba}$$

$$\vec{g_1 b_1} = \vec{o_1 a_1} + \vec{a_1 b_a} + \vec{b_a b_1}$$

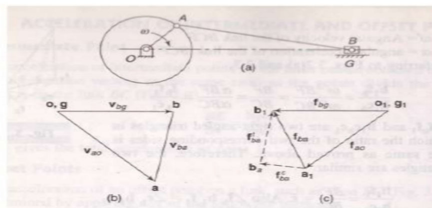


Figure - 32

The crank OA rotates at a uniform velocity, therefore, the acceleration of A relative tyo O has only the centripetal component. Similarly, the slider moves in a linear direction and thus has no centripetal component.

Setting the vector table:

S.N.	Vector	Magnitude	Direction	Sense
1.	$\vec{f}_{ao}$ or $\vec{o_1 a_1}$	$\frac{(oa)^2}{OA}$	∥ OA	→ O
2.	$\vec{f}^{\,c}_{ba}$ or $\vec{a_1 b_a}$	$\frac{(ab)^2}{AB}$	∥ AB	→ A
3.	$\vec{f}^{\,t}_{ba}$ or $\vec{b_a b_1}$	—	⊥ AB	—
4.	$\vec{f}_{bg}$ or $\vec{g_1 b_1}$	—	∥ to line of motion of B	—

Construct the acceleration diagram as follows:

1. Take the first vector $\vec{f}_{ao}$.
2. Add the second vector to the 1st.
3. For the third vector, draw as line ⊥ to AB through the head b_a of the second vector.
4. For the fourth vector, draw a line through g_1 parallel to the line of motion of the slider.

This completes the velocity diagram.
Acceleration of the slider B = g_1b_1
Total acceleration of B relative to A = a_1b_1.

Note that if the direction of the acceleration of slider is opposite to that of the velocity, then the slider decelerates.

Coriolis Acceleration Component:

Coriolis component exists only if there are two coincident points which have linear relative velocity of sliding and angular motion about fixed finite centres of rotation.

[redacted]

$$= -\omega^2 \sin\theta \left[\frac{n^2 - 1}{\left(n^2 - \sin^2\theta\right)^{3/2}} \right]$$

The negative sign indicates that the sense of angular acceleration of the rod is such that it tends to reduce the angle β.

Engine Force Analysis:

An engine is acted upon by various forces such as weight of reciprocating masses and connecting rod, gas forces, forces due to friction and inertia forces due to acceleration and retardation of engine elements, the last being dynamic in nature. The analysis is made of the forces neglecting the effect of the weight and the inertia effect of the connecting rod.

(i) Piston Effort (effective driving force):

Piston effort is termed as the net or effective force applied on the piston. In reciprocating engines, the reciprocating masses accelerate during the first half of the stroke and the inertia force tends to resist the same. Thus the net force on the piston is decreased. During the later half of the stroke, the reciprocating masses decelerate and the inertia force opposes this deceleration or acts in the direction of the applied gas pressure and thus, increases the effective force on the piston.

In vertical engine, the weight of the reciprocating masses assists the piston during the down stroke, thus, increases the piston effort by an amount equal the weight of the piston. During the upstroke, piston effort is decreased by the same amount.

Let A_1 = area of the cover end

A_2 = area of the piston rod end

p_1 = pressure on the cover end

p_2 = pressure on the rods end

m = mass of the reciprocating parts

Force on the piston due to the gas pressure, $F_p = p_1A_1 - p_2A_2$

Inertia force, $F_b = mf = mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$. It is opposite direction to that of the acceleration of the piston.

Resistant force = F_f

Net effective force on t5he piston, $F = F_p - F_b - F_f$

In case vertical engines, the weight of the piston or reciprocating parts also acts.

Force on the piston, $F = F_p + mg - F_b - F_f$

(ii) Force (thrust) along rthe Connecting rod:

Let F_c = Force in the3 connecting rod

Then equating the horizontal components of vforces,

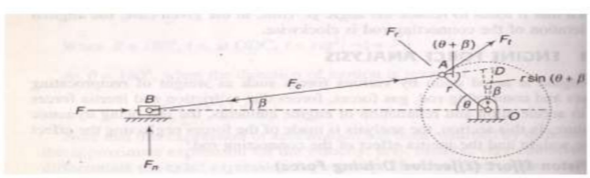


Figure - 38

$$F_c \times \cos\beta = F \text{ or } F_c = \frac{F}{\cos\beta}$$

(iii) Thrust on the Sides of Cylinder

It is the normal reaction on the cylinder walls.

$$F_n = F_c \sin\beta = F \tan\beta$$

(iv) Crank Effort

As $\quad F_t \times r = F_c \times r \times sin(\theta+\beta)$

$$= \frac{F}{cos\,\beta} sin(\theta+\beta)$$

(v) Thrust on the Bearings

The component of F_c along the crank (in the radial direction) produces a thrust on the crankshaft bearings.

$$F_r = F_c\, cos(\theta+\beta) = \frac{F}{cos\,\beta} cos(\theta+\beta)$$

Turning moment on Crankshaft:

$$T = F_t \times r$$

$$= \frac{F}{cos\,\beta} sin(\theta+\beta) \times r = \frac{Fr}{cos\,\beta}(sin\,\theta\, cos\,\beta + cos\,\theta\, sin\,\beta)$$

$$= Fr\left(sin\,\theta + cos\,\theta\, sin\,\beta \frac{1}{cos\,\beta}\right)$$

$$= Fr\left(sin\,\theta + cos\,\theta \frac{sin\,\theta}{n} \frac{1}{\frac{1}{n}\sqrt{n^2 - sin^2\theta}}\right)$$

$$= Fr\left(sin\,\theta + \frac{2\,sin\,\theta\, cos\,\theta}{2\sqrt{n^2 - sin^2\theta}}\right)$$

$$= Fr\left(sin\,\theta + \frac{sin\,2\theta}{2\sqrt{n^2 - sin^2\theta}}\right)$$

Also as $r\, sin(\theta+\beta) = OD\, cos\,\beta$

$$T = F_t \times r$$

$$= \frac{F}{cos\,\beta} r\, sin(\theta+\beta)$$

$$= \frac{F}{cos\,\beta}(OD\, cos\,\beta)$$

$$= F \times OD$$

Dynamically Equivalent System:

As neither the mass of the connecting rod uis uniformly distributed nor the motion is linear, its inertia cannot be found as general manner.

Usually, the inertia of the connecting rod is taken into account by considering a dynamically – equivalent system.

Dynamically–equivalent system means that the rigid link is replaced by a link w2ith tw2o point masses in such a way that it has the same motion as the rigid link when subjected to the same force i.e. the center of mass of the equivalent link has the same linear acceleration and the link has the same angular acceleration.

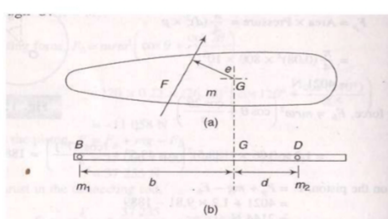


Figure - 39

Figure 39 (a) shows a rigid bnody of mass 'm' with center of mass at G. Let a

MODULE 4

Friction effect

2.12 SCREW JACK

With Square Threads

A screw jack with its spindle, having square threads, is shown in Figure 2.7. The theory discussed till now is directly applicable to this case.

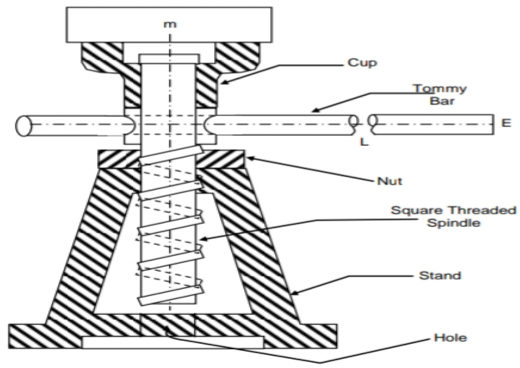


Figure 2.7 : Screw Jack

Let m = Mass on the jack,

P = Force applied at the screw tangentially in a horizontal plane,

P_e = Horizontal force applied tangentially at the end E of a tommy bar in a horizontal plane, and

L = Horizontal distance between central axis of the screw and the end E of the bar as shown.

In this screw jack nut is stationary and the screw is rotated with the help of tommy bar.

$$P_e \times L = P \times r$$

$$P_e = \frac{P \times r}{L}$$

$$P = mg \tan (\alpha + \phi) = \frac{mg (\tan \alpha + \tan \phi)}{1 - \tan \alpha \tan \phi}$$

But $\quad \tan \alpha = \frac{p}{\pi d_m}$ and $\tan \phi = \mu$

Substituting for $\tan \alpha$ and $\tan \phi$

$$\Rightarrow \qquad P = \frac{mg \left(\frac{p}{\pi d_m} + \mu \right)}{1 - \frac{p}{\pi d_m} \times \mu} = \frac{mg (p + \mu \pi d_m)}{(\pi d_m - p \mu)}$$

Hence, $\quad \boldsymbol{P = \frac{mg (p + \mu \pi d_m)}{(\pi d_m - p \mu)}}$

$$P_e = \frac{\frac{mg \times r}{L} (p + \mu \pi d_m)}{(\pi d_m - p \mu)}$$

Mechanical advantage of the jack with tommy bar

$$= \frac{\text{Load lifted}}{\text{Force applied}}$$

$$= \frac{mg}{P_e} = \frac{L}{r} \times \frac{(\pi d_m - p \mu)}{(p + \mu \pi d_m)}$$

Velocity ratio with tommy bar :

$$= \frac{\text{Distance covered by } P_e}{\text{Distance covered by load in one revolution}}$$

$$= \frac{2 \pi L}{2 \pi r_m \times \tan \alpha} = \frac{L}{r_m \tan \alpha}$$

$$\boldsymbol{V.R. = \frac{2 \pi L}{p}}$$

With *V*-threads

The square threads, by their nature, take the axial load mg perpendicular to them where as in *V*-threads, the axial load does not act perpendicular to the surface of the threads as shown in Figure 2.8. The normal reaction R_N between the threads and the screw should be such that its axial component is equal and opposite to the axial load mg.

$$P_0 = mg \tan \alpha \frac{30}{2 \times 1000} = 20 \times 0.091 \times 15 = 27.3$$

(b) Efficiency $e_{up} = \frac{P_0}{P} = \frac{27.3}{73} \times 100 = 37.4\%$

2.13 PIVOT AND COLLAR FRICTION

The shafts of ships, steam and water turbines are subjected to *axial thrust*. In order to take up the axial thrust, they are provided with one or more bearing surfaces at right angle to the axis of the shaft. A bearing surface provided at the end of a shaft is known as a *pivot* and that provided at any place along with the length of the shaft with bearing surface of revolution is known as a *callar*. Pivots are of two forms : flat and conical. The bearing surface provided at the foot of a vertical shaft is called *foot step bearing*.

Due to the axial thrust which is conveyed to the bearings by the rotating shaft, rubbing takes place between the contacting surfaces. This produces friction as well as wearing of the bearing. Thus, power is lost in over-coming the friction, which is ultimately to be determined in this unit.

Obviously, the rate of wearing depends upon the intensity of thrust (pressure) and relative velocity of rotation. Since velocity is proportional to the radius, therefore,

$\therefore$ Rate of wear $\propto pr$.

Assumptions Taken

(a) *Firstly, the intensity of pressure is uniform over the bearing surface*. This assumption only holds good with newly fitted bearings where fit between the two contacting surfaces is assumed to be perfect. After the shaft has run for quite sometime the pressure distribution will not remain uniform due to varying wear at different radii.

(b) *Secondly, the rate of wear is uniform*. As given previously, the rate of wear is proportional to pr which means that the pressure will go on increasing radially inward and at the centre where $r = 0$, the pressure will be infinite which is not true in practical sense. *However, this assumption of uniform wear gives better practical results* when bearing has become older.

The various types of bearings mentioned above will be dealt with separately for each assumption.

2.13.1 Flat Pivot

A flat pivot is shown in Figure 2.9.

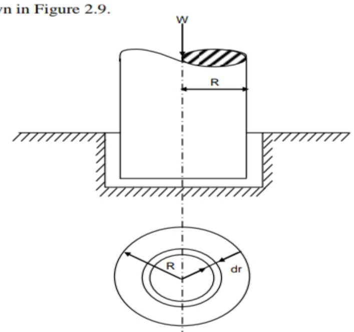


Figure 2.9 : Flat Pivot

Let W = Axial thrust or load on the bearing,

R = External radius of the pivot,

p = Intensity of pressure, and

μ = Coefficient of friction between the contacting surfaces.

Consider an elementary ring of the bearing surfaces, at a radius r and of thickness dr as shown in Figure 2.9.

Axial load on the ring

$$dW = p \times 2\pi r \times dr$$

Total load $$W = \int_0^R p \times 2\pi r \times dr \quad \ldots (2.8)$$

Frictional force on the ring

$$dF = \mu \times dW = \mu \times p \times 2\pi r \times dr$$

Frictional moment about the axis of rotation

$$dM = dF \times r = \mu \times p \times 2\pi r^2 \times dr$$

Total frictional moment

$$M = \int_0^R dM = \int_0^R \mu \times p \times 2\pi r^2 \times dr \quad \ldots (2.9)$$

Uniform Pressure

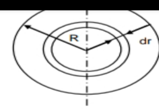


Figure 2.9 : Flat Pivot

Let W = Axial thrust or load on the bearing,

R = External radius of the pivot,

p = Intensity of pressure, and

μ = Coefficient of friction between the contacting surfaces.

Consider an elementary ring of the bearing surfaces, at a radius r and of thickness dr as shown in Figure 2.9.

Axial load on the ring

$$dW = p \times 2\pi r \times dr$$

Total load $$W = \int_0^R p \times 2\pi r \times dr \qquad \ldots (2.8)$$

Frictional force on the ring

$$dF = \mu \times dW = \mu \times p \times 2\pi r \times dr$$

Frictional moment about the axis of rotation

$$dM = dF \times r = \mu \times p \times 2\pi r^2 \times dr$$

Total frictional moment

$$M = \int_0^R dM = \int_0^R \mu \times p \times 2\pi r^2 \times dr \qquad \ldots (2.9)$$

Uniform Pressure

If the intensity of pressure p is assumed to be uniform and hence constant.

From Eq. (2.8)

$$W = p \times 2\pi \int_0^R r \times dr = p \times 2\pi \left[\frac{r^2}{2}\right]_0^R = p \times 2\pi \times \frac{R^2}{2}$$

$$\Rightarrow \qquad \boldsymbol{W = p \times \pi R^2}$$

And from Eq. (2.9)

$$M = \mu \times p \times 2\pi \int_0^R r^2\, dr$$

$$M = \mu\, p \times 2\pi \times \left[\frac{r^3}{3}\right]_0^R = \frac{2}{3} \mu \times p \times \pi \times R^3$$

But $\qquad p\, \pi\, R^2 = W$

$$\Rightarrow \qquad \boldsymbol{M = \frac{2}{3} \mu\, WR = \mu W \times \frac{2}{3} R}$$

The friction force μW can be considered to be acting at a radius of $\frac{2}{3} R$.

Uniform Rate of Wear

By Eq. (2.8),

$$W = \int_0^R p \times 2\pi r \times dr$$

As the rate of wear is taken as constant and proportional to $pr = a$ constant say c. Substituting for $pr = c$ in the above equation.

$$W = \int_0^R 2\pi \times c \times d\,r = 2\pi R \times c$$

$$\Rightarrow \qquad c = \frac{W}{2\pi R} \qquad \ldots (2.10)$$

By Eq. (2.9), total frictional moment

$$M = \int_0^R \mu \times p \times 2\pi r^2 \times dr = \int_0^R \mu \times 2\pi \times c \times r \times dr$$

$$= \mu \times 2\pi \times c \times \frac{R^2}{2} = \mu \times 2\pi \times \frac{W}{2\pi R} \times \frac{R^2}{2}$$

$$\therefore \qquad \boldsymbol{M = \mu W \times \frac{R}{2}} \qquad \ldots (2.11)$$

Thus, the frictional force : μW acts at a distance $\frac{R}{2}$ from the axis.

2.14 CLUTCH

It is a mechanical device which is widely used in automobiles for the purpose of engaging driving and the driven shaft, at the will of the driver or the operator. The driving shaft is the engine crankshaft and the driven shaft is the gear-box driving shaft. This means that the clutch is situated between the engine flywheel mounted on the crankshaft and the gear box.

In automobile, gears are required to be changed for obtaining different speeds. It is possible only if the driving shaft of the gear box is stopped for a while without stopping the engine. These two objects are achieved with the help of a clutch.

Broadly speaking, a clutch consists of two members; one fixed to the crankshaft or the flywheel of the engine and the other mounted on a splined shaft, of the gear box so that this could be engaged or disengaged as the case may be with the member fixed to the engine crankshaft.

2.15 TYPES OF CLUTCHES

Clutches can be classified into two types as follows :

(a) conical clutch, and

(b) the place or disc clutches can be of single plate or of multiple plates.

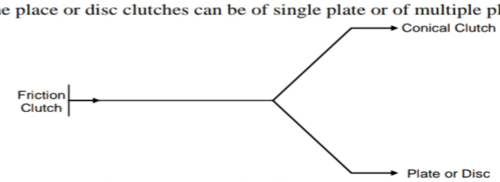


Figure 2.13 : The Single Cone Clutch

2.15.1 Conical Clutch

A conical clutch is shown in Figure 2.14. It consists of a cone *A* mounted on engine crankshaft. Cone *B* has internal splines in its boss which fit into the corresponding splines provided in the gear box shaft. Cone *B* could rotate the gear box shaft as well as may slide along with it. The outer surface of cone *B* is lined with friction material. In the normal or released position of the clutch pedal *P*, cone *B* fits into the inner conical surface of cone *A* and by means of the friction between the contacting surfaces, power is transmitted from crankshaft to the gear box shaft. When the clutch pedal is pressed, pivot *D* being the fulcrum provided in it, the collar *E* is pressed towards the right side, thus disengaging cone *B* from cone *A* and keeping the compression spring *S* compressed. On releasing the pedal, by the force of the spring, the cone *B* is thrusted back to engage cone *A* for power transmission.

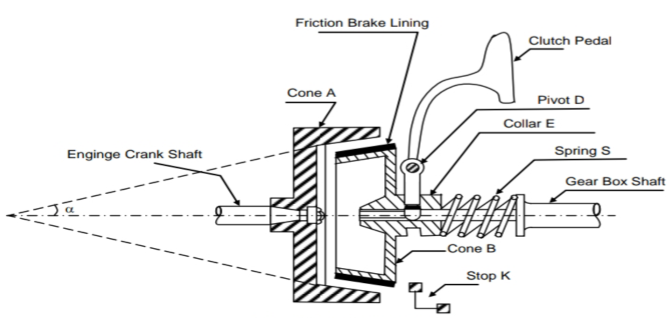


Figure 2.14 : Conical Clutch

For calculating frictional moments or torque transmitted on account of friction in clutches, unless otherwise specifically stated uniform rate of wear is assumed. For torque transmitted formulae of conical pivot can be used.

2.15.2 Single Plate Clutch

A single plate clutch is known as *single disc clutch*. It is shown in Figure 2.15. It has two sides which are driving and the driven side. The driving side comprises of the driving shaft or engine crankshaft *A*. A boss *B* is keyed to it to which flywheel *C* is bolted as

[redacted]

Figure 2.14 : Conical Clutch

For calculating frictional moments or torque transmitted on account of friction in clutches, unless otherwise specifically stated uniform rate of wear is assumed. For torque transmitted formulae of conical pivot can be used.

2.15.2 Single Plate Clutch

A single plate clutch is known as *single disc clutch*. It is shown in Figure 2.15. It has two sides which are driving and the driven side. The driving side comprises of the driving shaft or engine crankshaft *A*. A boss *B* is keyed to it to which flywheel *C* is bolted as shown. On the driven side, there is a driven shaft *D*. It carries a boss *E* which can freely slide axially along with the driven shaft through splines *F*. The clutch plate is mounted on the boss *E*. It is provided with rings of friction material – known as *friction linings*, on the both sides indicated *H*. One friction lining is pressed on the flywheel face and the other on the pressure plate *I*. A small spigot, bearing *J*, is provided in the end of the driving shaft for proper alignment.

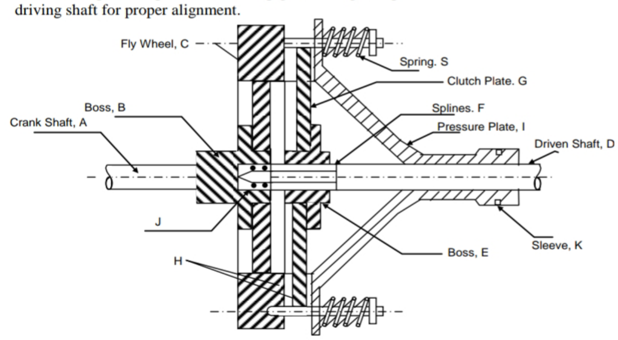


Figure 2.15 : Single Plate Clutch

The pressure plate provides axial thrust or pressure between clutch plate *G* and the flywheel *C* and the pressure plate *I* through the linings on its either side, by means of the springs, *S*.

The pressure plate remains engaged and as such clutch remains in operational position. Power from the driving shaft is transferred to the driven shaft from flywheel to the clutch

plate through the friction lining between them. From pressure plate the power is transmitted to clutch plate through friction linings. Both sides of the clutch plate are effective. When the clutch is to be disengaged the sleeve *K* is moved towards right hand side by means of clutch pedal mechanism (it is not shown in the figure). By doing this, there is no pressure between the pressure plate, flywheel and the clutch plate and no power is transmitted. In medium size and heavy vehicles, like truck, single plate clutch is used.

2.15.3 Multi Plate Clutch

As already explained in a plate clutch, the torque is transmitted by friction between one or more pairs of co-axial annular faces kept in contact by an axial thrust provided by springs. In a single plate clutch, both sides of the plate are effective so that it has two pairs of surfaces in contact or $n = 2$.

Obviously, in a single plate clutch limited amount of torque can be transmitted. When large amount of torque is to be transmitted, more pair of contact surfaces are needed and it is precisely what is obtained by a multi-plate clutch.

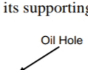
SAQ 5

(a) Clutches are used for which purpose?

(b) Where do we use single plate clutch. Name the vehicles?

2.16 JOURNAL BEARING

The portion of a shaft, which revolves in the bearing and subjected to load at right angle to the axis of the shaft, is known as *journal* as clearly indicated in Figure 2.16. The whole unit consisting of the journal and its supporting part (or bearing) is known as *journal bearing*.



Flexible Mechanical Elements

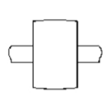
Belts, ropes, chains, and other similar elastic, or flexible machine elements are used in conveying systems and in the transmission of power over comparatively long distance.

Type of belts

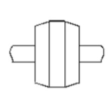
Belt Type	Figure	Joint	Size Range	Center Distance
Flat		Yes	$t = 0.75 - 5mm$	No upper limit
Round		Yes	$d = 3 - 20mm$	No upper limit
V		None	$b = 8 - 19mm$	Limited
Timing		None	$p = \geq 2mm$	Limited

- Crowned pulleys are used for flat belts.
- Grooved pulleys or sheaves are used for round or V belts.
- Toothed wheels or sprockets are used for timing belts.

Crowned pulley



FULL RADIUS



ANGLED

Flat and round-belt drives

The efficiency of a V-belt drive ranges from about 70 to 96 percent. Flat-belt drives produce very little noise and absorb more torsional vibration from the system than either V-belt or gear drives. When an open-belt drive is used, the contact angles are found to be

$$\theta_d = \pi - 2\sin^{-1}\frac{D-d}{2C}$$

$$\theta_D = \pi + 2\sin^{-1}\frac{D-d}{2C}$$

Where D= diameter of large pulley
d= diameter of small pulley
C= center distance
$\theta =$ angle of contact

The Length of the belt

$$L = [4C^2 - (D-d)^2]^{1/2} + 1/2(D\theta_D + d\theta_d)$$

The angle of wrap is the same for both pulleys and is

$$\theta = \pi + 2\sin{-1}\frac{D+d}{2C}$$

The belt length for crossed belts is

$$L = [4C^2 - (D+d)^2]^{1/2} + \frac{\theta}{2}(D+d)$$

$$V = \frac{\pi dn}{1000} m/\sec, d = mm, n = r/s$$

$$\omega = \frac{\gamma bt}{10^6},$$

$$Fc = \frac{\omega}{g}(V)^2 = \frac{\omega v^2}{9.81} N$$

$$F_1 = F_i + F_c + \Delta F' = F_i + F_c + T/D$$

$$F_2 = F_i + F_c - \Delta F' = F_i + F_c - T/D$$

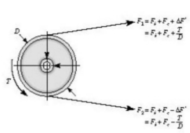
$$\frac{F_1 - mr^2\omega^2}{F_2 - mr^2\omega^2} = \frac{F_1 - F_c}{F_2 - F_c} = \exp(f\phi)$$

$$F_c = mr^2\omega^2$$

$$F_1 - F_2 = (F_1 - F_c)\frac{\exp(f\phi) - 1}{\exp(f\phi)}$$

Where
F_i= initial tension
F_c=hoop tension due to centrifugal force
$\Delta F'$= tension due to the transmitted torque T
D= diameter of the pulley

Forces and torques on a pulley



$$F_1 - F_2 = \frac{2T}{D} = \frac{T}{D/2}$$

$$F_i = \frac{F_1 + F_2}{2} - F_c$$

$$F_i = \frac{T}{D}\frac{\exp(f\phi) + 1}{\exp(f\phi) - 1}$$

$$F_1 = F_c + F_i\frac{2\exp(f\phi)}{\exp(f\phi) + 1}$$

$$F_2 = F_c + F_i\frac{2}{\exp(f\phi) + 1}$$

MODULE 5 BRAKE AND DYNAMO METERS

A dynamometer is a device used for measuring the torque and brake power required to operate a driven machine.

Dynamometers can be broadly classified into two types. They are:

Power Absorption Dynamometers: Power Absorption dynamometers measure and absorb the power output of the engine to which they are coupled. The power absorbed is usually dissipated as heat by some means.

Examples of power absorption dynamometers are Prony brake dynamometer, Rope brake dynamometer, Eddy current dynamometer, Hydraulic dynamometer, etc.

Power Transmission Dynamometers: In power transmission dynamometers the power is transmitted to the load coupled to the engine after it is indicated on some scale. These are also called torque meters.

Some of the different types of dynamometers are discussed below in brief.

Prony Brake Dynamometer:

Pony Brake is one of the simplest dynamometers for measuring power output (brake power). It is to attempt to stop the engine using a brake on the flywheel and measure the weight which an arm attached to the brake will support, as it tries to rotate with the flywheel.

Prony Brake

Figure 1: Prony Brake

The Prony brake shown in the above consists of a wooden block, frame, rope, brake shoes and flywheel. It works on the principle of converting power into heat by dry friction. Spring-loaded bolts are provided to increase the friction by tightening the wooden block.

The whole of the power absorbed is converted into heat and hence this type of dynamometer must the cooled.

The brake power is given by the formula

Brake Power (bp) = 2π NT

Where T = Weight applied (W) × distance (l)

Rope Brake Dynamometer:

The rope brake as shown in below figure is another device for measuring brake power of an engine. It consists of some turns of rope wound around the rotating drum attached to the output shaft. One side of the rope is connected to a spring balance and the other side to a loading device. The power is absorbed in friction between the rope and the drum. Therefore drum in rope brake requires cooling.

Rope brake

Figure 2: Rope Brake

Rope brake dynamometers are cheap and can be constructed quickly but brake power can't be measured accurately because of change in the friction coefficient of the rope with a change in temperature.

The brake power is given by the formula

Brake Power (bp) = π DN (W − S)

where D is the brake drum diameter,

W is the weight of the load and

S is the spring balance reading.

BRAKE AND TYPES:

Today we will discuss about types of brakes. The brake is one of the most important controlling component of vehicle. We have heard about drum brake and disk brake. Drum brake is widely used in the automobile. The brakes are required for stop the vehicle within the smallest possible distance or to slow down the vehicle when we needed. Without the brakes we cannot control the vehicle speed so it is the most important system in automobiles.All brakes are working on same principle by converting the kinetic energy of the vehicle into the heat energy which is dissipated into the automobile. There are two most important requirement of brakes as follow.

1. The brake must be enough strong to stop the vehicle within a minimum distance in an emergency with safety. The driver should have total control over vehicle during emergency

braking and the vehicle must not skid.

2. With prolonged application of brake their effectiveness should not be decreases. These characteristics called antifade characteristics.

Types of brakes:

Brakes is one of the most important element of automobile. There are many types of brakes available in automobile industries. Theses are primary brake, Secondary brakes, vacuum brake, air brake, disk brake, drum brake etc.The classification of brakes are as follow.

According to the purpose:

1. Primary or service brake:

This brake is used when the vehicle is in running condition to stop or slow down the vehicle. This is the main braking system, which is situated in both rear and front wheels of the vehicle.

2. Secondary brakes:

Secondary brakes, which is also known as parking brake or emergency brake , are used to keep the vehicle stationary. It is generally operated by hand, so also known as hand brake. The main function of this brake, is to keep the vehicle stationary when it is parked.

According to the construction:

1. Drum brake:

In this type of brakes a drum is attached to the axle hub whereas on the axle casing is mounted a back plate. The back plate is made of pressed steal sheet. It provide support for the expander, anchor and brake shoes. It also protect the drum and shoe assembly from mud and dust. It also known as the torque plate because it absorbed the complete torque reaction of the shoe. Two brake shoe is mounted on back plate with friction linnings. On or two retractor spring are used to seprate brake shoe from drum when the brakes are not applied. The brake shoe are anchored at one end, whereas on the other ends force is applied by means of some brake actuating mechanism which forces the brake shoe against the revolving drum, so the friction force is generated between drum and the shoe and brake applied.

An adjuster is also provided to compensate for wear of friction lining with use. This brakes are widely used in motorcycle and the cars.

2. Disk brake:

Disk brake consists of a cast iron disc bolted to the wheel hub and a stationary housing called caliper. The caliper is connected to some stationary part of the vehicle and it cast in two parts, each part containing a piston. In between each piston and the disc there is a friction pad held in position by retaining pins, springs plates etc. There are arrangements in the caliper for the fluid to enter or leave each housing. There passages are also connected to another one for bleeding. Each cylinder contains a rubber sealing ring between the cylinder and the piston.

When the brakes are applied, hydraulically actuated piston move the friction pads into contact with the disc, applying equal and opposite forces on the later. On releasing the brakes, the rubber sealing rings act as return springs and retract the pistons and the friction pads away from the disc.

According to the actuation:

1. Mechanical brakes:

In this brakes the brake force is applied mechanically used where we needed small force to braking. This brakes are used in the small vehicle such as in scooters, bikes etc where small braking force is needed.

2. Hydraulic brakes:

In the hydraulic brakes, brake force is applied by the hydraulic oil. It is one of the most useful and reliable braking system. This brakes is used in most of passenger cars.