

Lecture 3

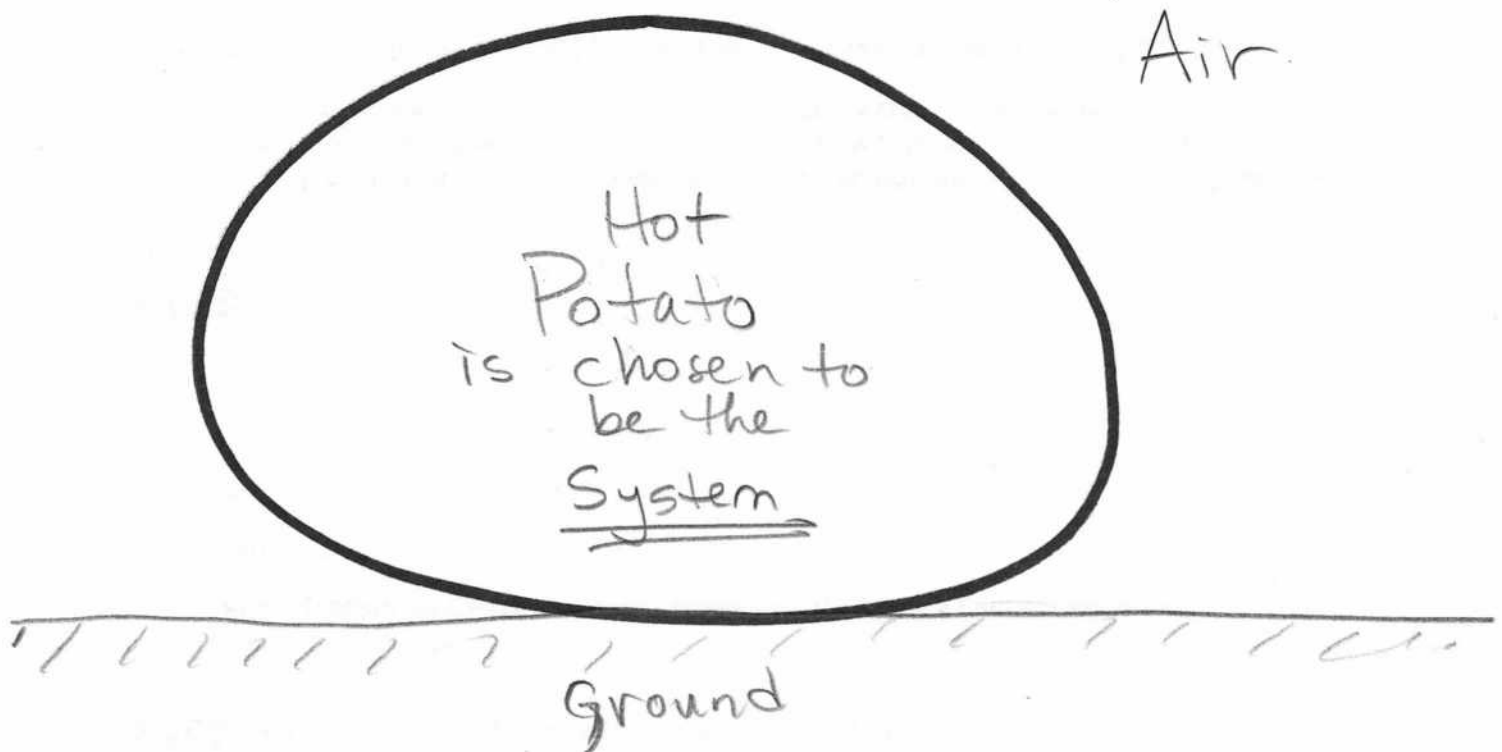
Fundamental Concepts for Thermodynamic Analysis

①

The next few lectures are going to cover some very basic definitions and concepts that we will need to analyze and design devices in Engineering. (Use your Glossary!)

System

A "System is any quantity of matter or a region of space chosen for study.



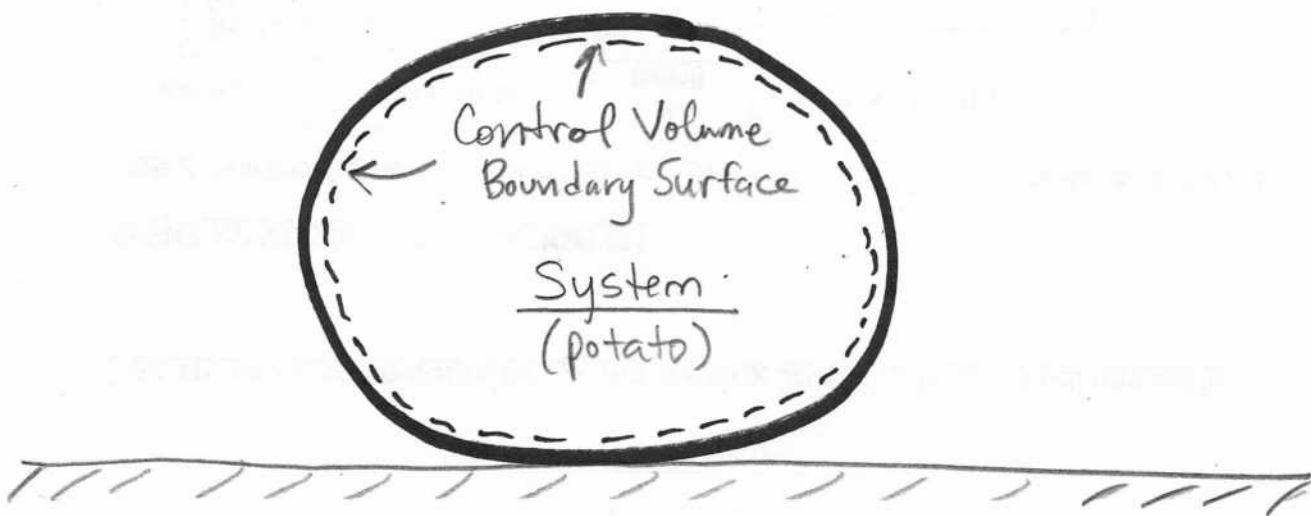
CONTROL VOLUME/SURFACE BOUNDARY

2

A **Control Volume (CV)** is an imaginary surface that separates the System from its surroundings. A Control Volume can change position, orientation, shape, and/or volume with time while mass either does or doesn't cross the boundary. Also, Energy and Entropy/Exergy can be transferred across the CV boundary. Within the System CV, Entropy can be generated (or Exergy destroyed), species concentrations, and phases of matter can change with time.

Mathematically speaking, the System CV boundary surface has zero thickness and it is infinitesimally close to any real System surfaces.

The purpose of a CV is to help you keep track (i.e., "control") of all the processes that are affecting the System under study. A CV is ESSENTIAL for rational analysis, even at the professional level. A System CV can change its position, orientation, shape, and/or volume with time.



According to the textbook:

1. **Control Mass** = when mass *does not* cross this boundary, the System is called "Closed"
2. **Control Volume** = when mass *does* cross this boundary, the System is called "Open".

I think the textbook (and most other texts') distinction between "Control Mass and Control Volume" is superfluous and confusing. Why? Because the only difference in how these cases are analyzed is with respect to the mathematical description of the balance of mass, Energy, and Entropy/Exergy across the bounding surface. We will deviate from the textbook's terminology in this class.

In this ME 315/515 class, we will define the System using the terminology "Control Volume" only. All you need consider is whether mass crosses the Control Boundary Surface or not:

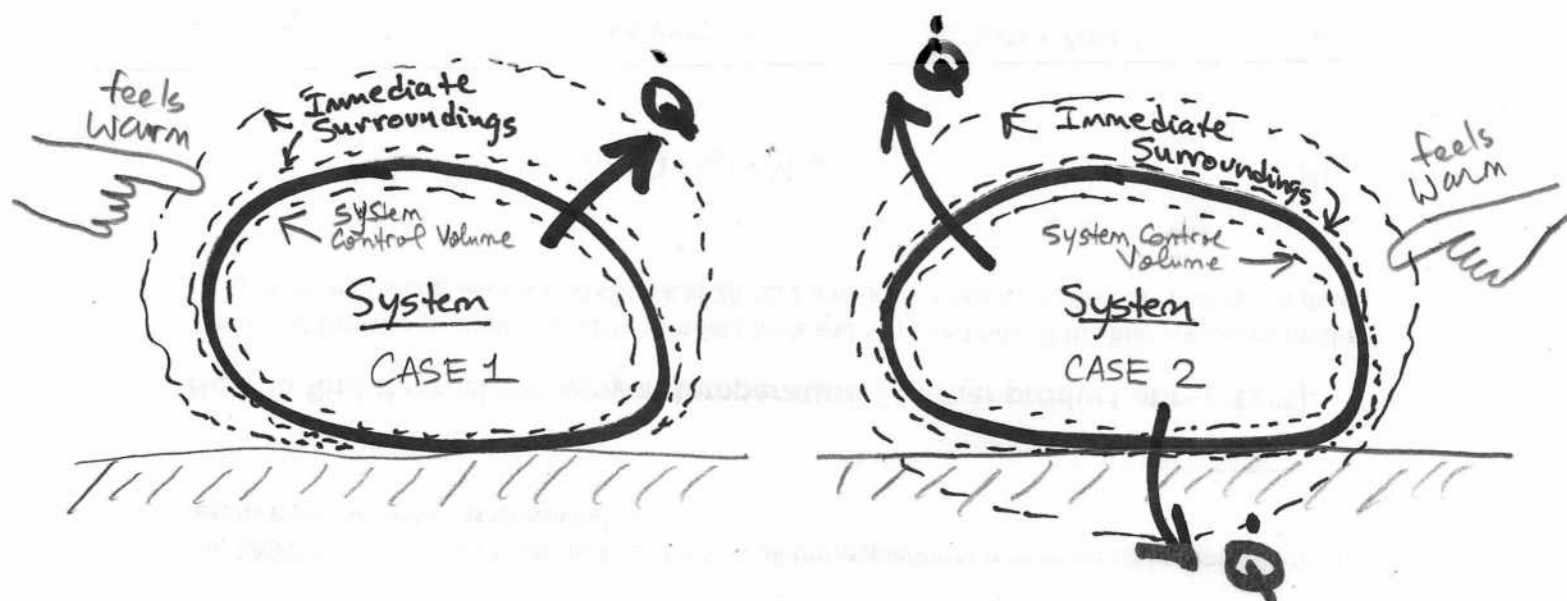
1. If mass does not cross the boundary, then it is called a Closed System.
2. If mass does cross the boundary, then it is called an Open System.

IMMEDIATE SURROUNDINGS

The **Immediate Surroundings** is the mass or region of space in "close proximity" to the System under study.

We will encounter situations in which significant mass, Energy, and Entropy/Exergy transfers are taking place across the System Control Volume Boundary Surface.

Say that I just placed a potato with a temperature of $100\text{ }^{\circ}\text{C}$ on the ground that is at $20\text{ }^{\circ}\text{C}$. Initially, the air is also at $20\text{ }^{\circ}\text{C}$. Within a minute or so, if you place your hand near the potato (i.e., within the "Immediate Surroundings") you may feel that the air is slightly warmer than the original $20\text{ }^{\circ}\text{C}$ because Thermal Energy has been transferred out of the potato into the air (this is "Heat" Transfer).



However, your choice of what you call the "Immediate Surroundings" will depend upon what constitutes a good engineering approximation for the particular problem at hand.

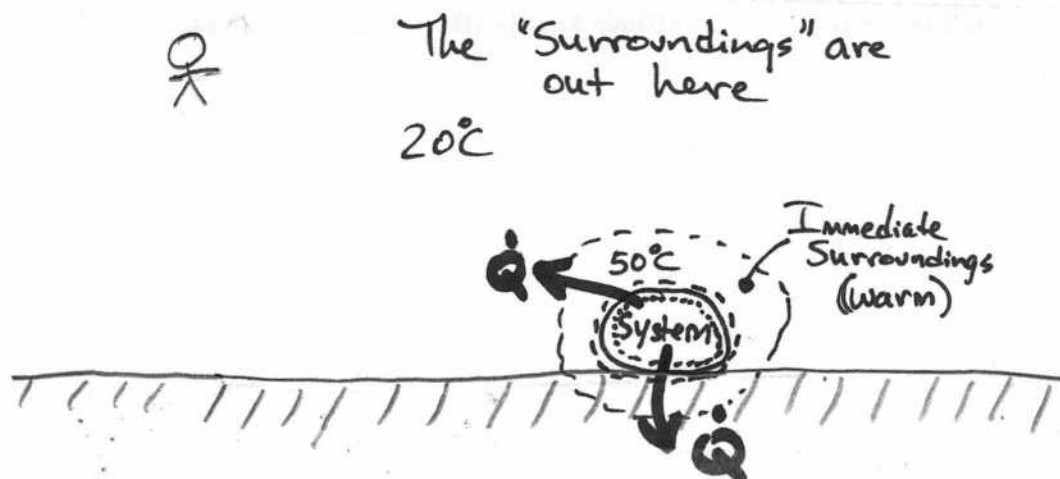
For example, in one figure I have placed the Immediate Surroundings Surface Boundary around the entire potato in order to include heat interactions between the System, air, and the ground. However, the engineer may have prior knowledge that heat interactions between the System and the ground are much smaller in magnitude than those with the air. In this case, the engineer may place the Immediate Surroundings control boundary around only the potato-air interface, as shown in the other figure. (case 1)

This illustrates that just like the proper identification of the System Control Volume, a proper (or easier) analysis will depend upon a good choice of where you place the Immediate Surroundings surface boundary.

To use the Immediate Surroundings Boundary Surface in an analysis, generally you should focus your attention on that part of the Immediate Surroundings boundary surface nearest the System CV surface. Why? Because this is usually where "the action is" i.e., this is where significant mass, Energy (thermal and work), and Entropy/Exergy interactions will be taking place with the System.

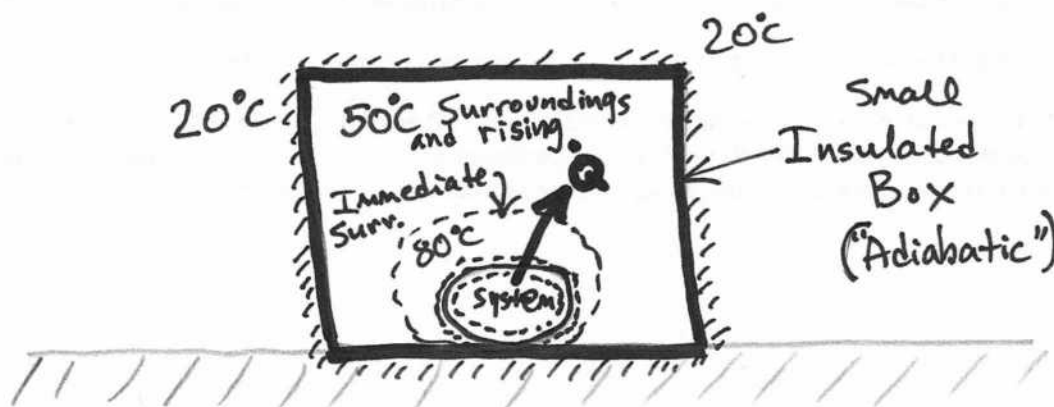
SURROUNDINGS

The **Surroundings** is the mass or region of space far enough away from both the System and Immediate Surroundings that it is, for practical engineering purposes, not affected by interactions between the System and the Immediate Surroundings.



If you stand back about 20 meters from the 100 °C potato you probably will not feel any change in the air temperature, even after the potato has cooled down and its temperature is not changing with time. This region where you stand would then be called "The Surroundings".

The temperature rise in The Surroundings is not perceptible to your senses because of the enormous number of air and ground molecules. During the cooling of the potato each air and ground molecule received, on average, very little of the original internal thermal energy of the hot potato.

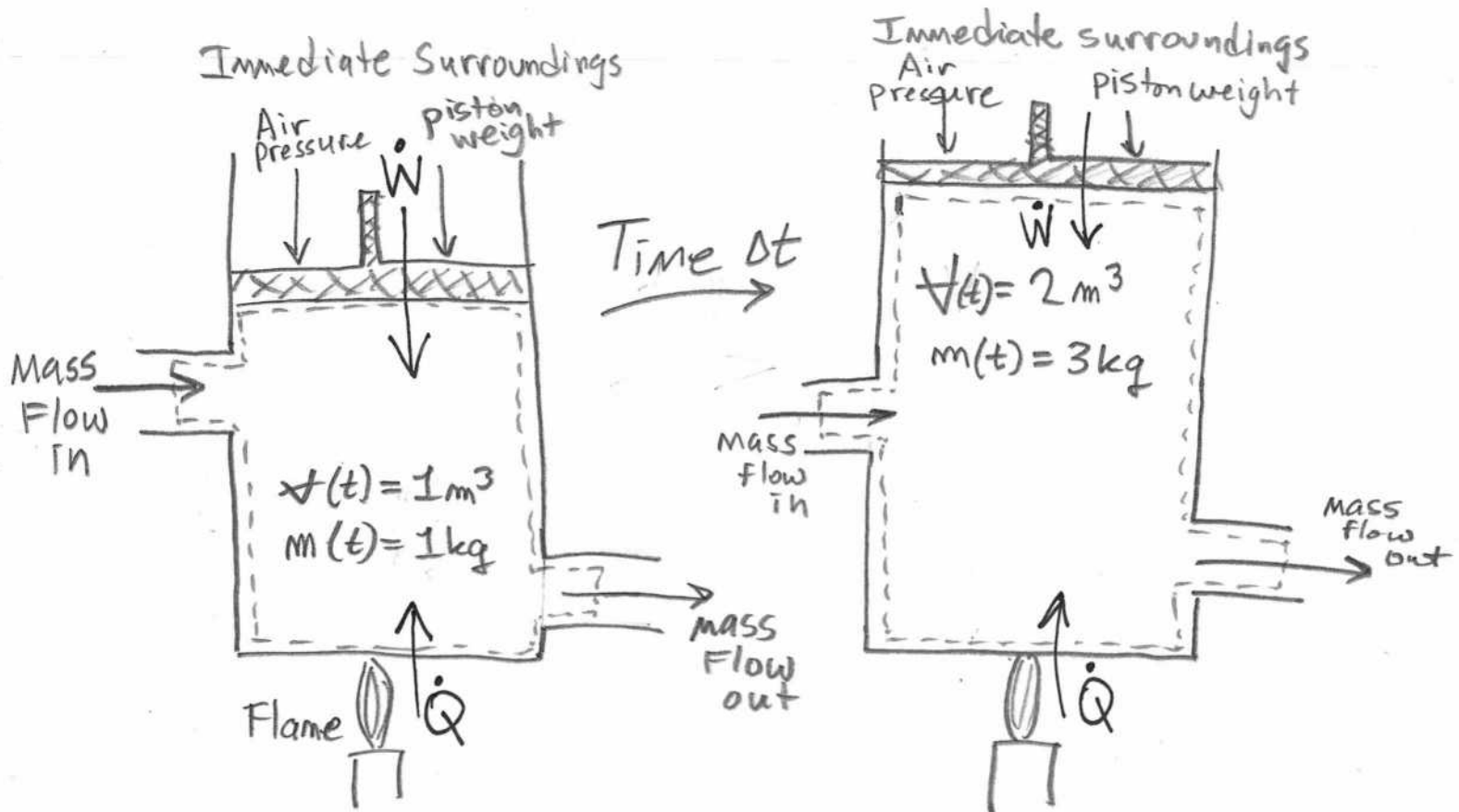


On the other hand, if the potato were to be placed in a small box (i.e., the box volume now constitutes the Surroundings), then there is a good chance that you would notice a temperature change in the entire box.

OPEN, CLOSED, and ISOLATED SYSTEMS

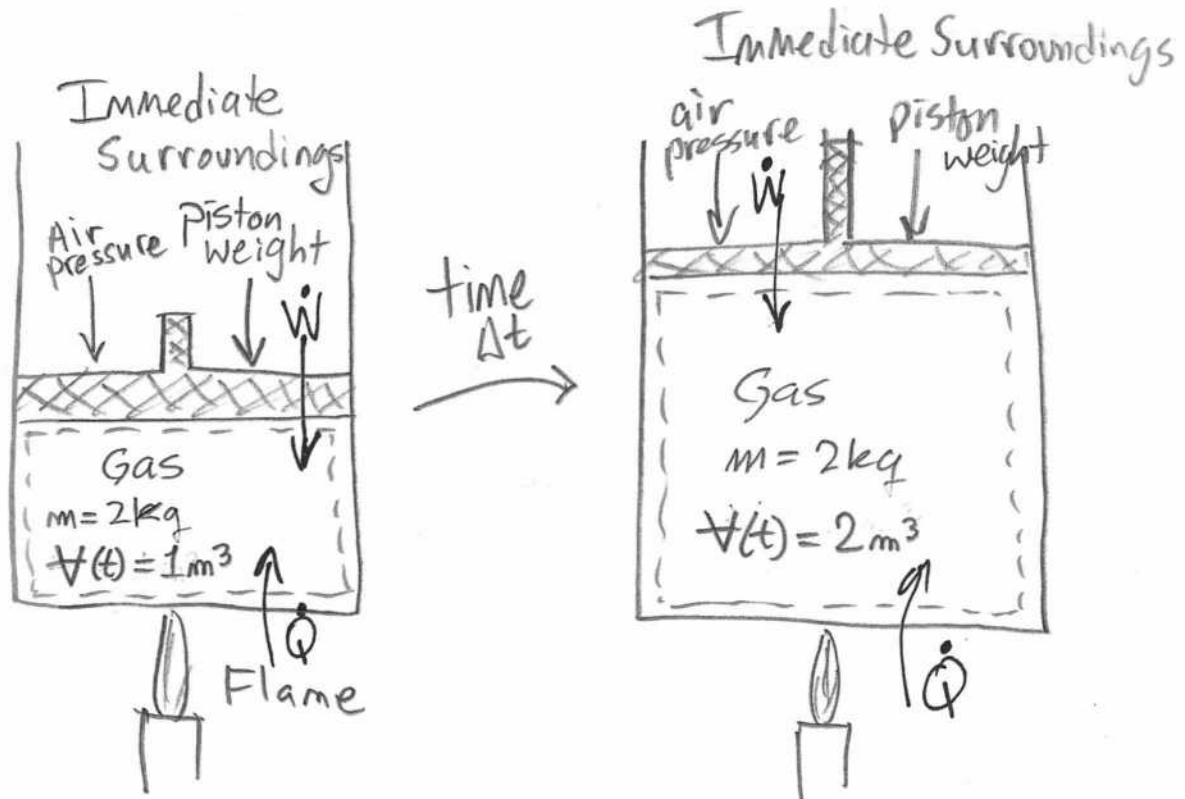
- An Open System is one in which mass crosses the CV. Energy and Entropy/Exergy can also cross the CV boundary.
- A Closed System is one in which mass does not cross the CV. Energy and Entropy/Exergy can also cross the CV boundary.
- An Isolated System is an idealized Closed System in which *neither Mass, Energy, or Entropy/Exergy may cross the CV Boundary*. It is important to understand that this is **FICTIONAL**, and only serves as an engineering approximation under the appropriate circumstances in order to make the analysis easier.

Example 1



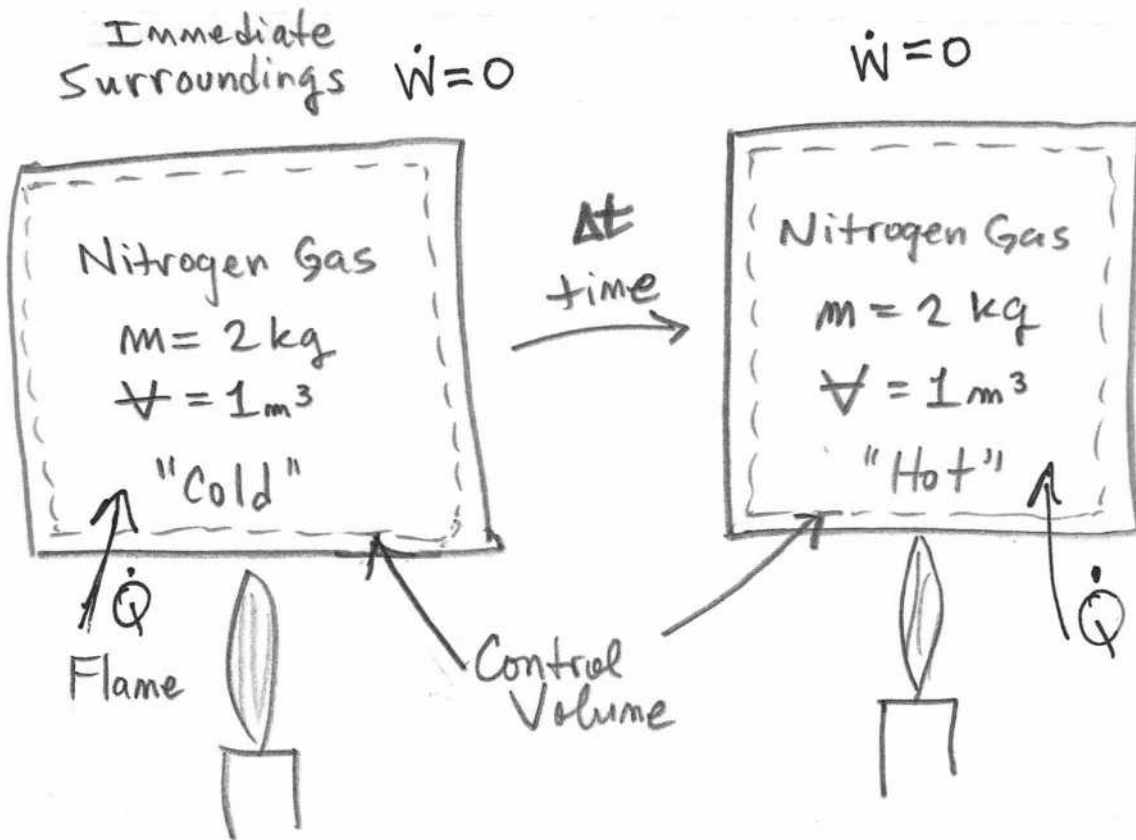
- Open System
- Moving Boundary

Example 2

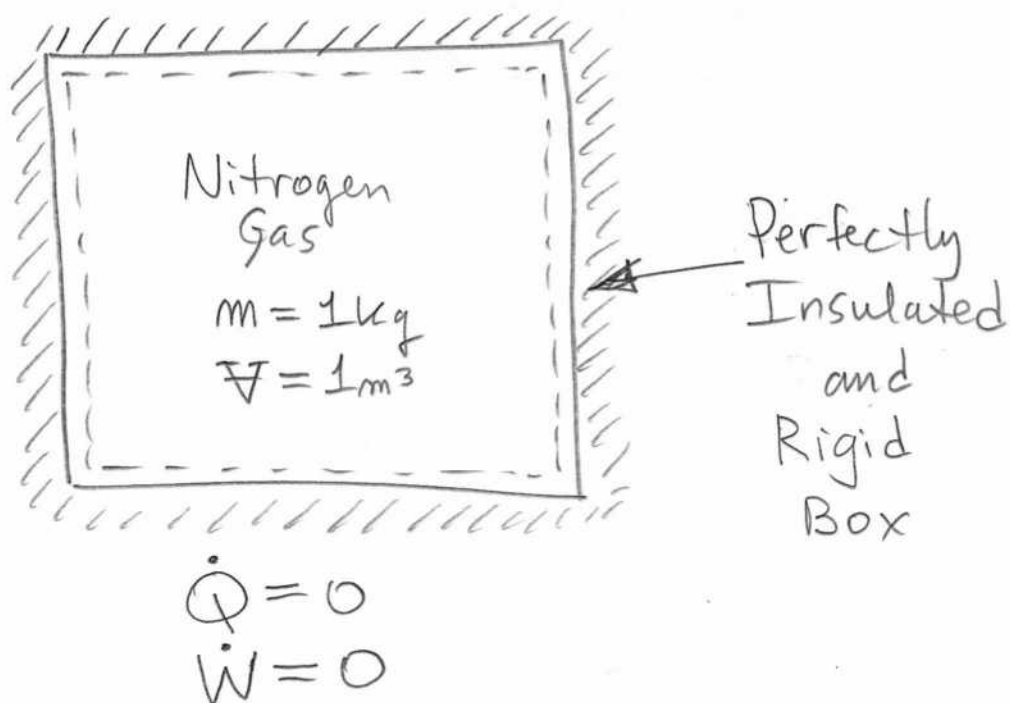


- . Closed System
- . Moving Boundary

Example 3



- Closed System
- Stationary (or "Fixed") Control Volume

Example 4Isolated System

- The System is assumed to be completely isolated from the outside universe
 - \Rightarrow • No Thermal energy interactions/transfers
 - No work interactions/transfers
 - No change in elevation (potential energy)
 - No change in velocity

Lecture 4 Properties, Equilibrium, States, Processes, and Cycles

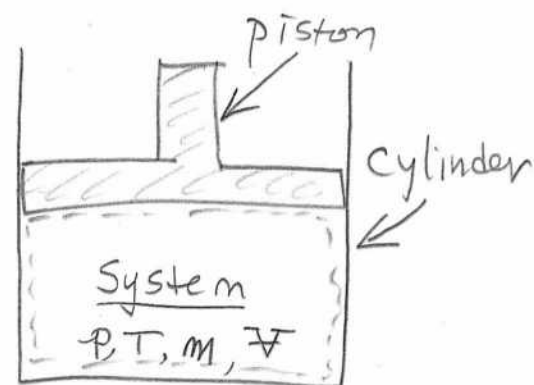
(1)

Properties of a System

Any characteristic of a System is called a "Property"

Examples

- A particular human being is designated as a System. Some Properties are
 - Blue Eyes
 - Black hair
 - Height
 - Weight
- Some Properties of a thermodynamic System may be
 - pressure, P
 - temperature, T
 - mass, m
 - Volume, V
 - Internal Energy, U
 - thermal conductivity, k
 - mass density, ρ
 - specific volume, $v \equiv 1/\rho$
 - speed, V
 - Elevation, z
 - Shape

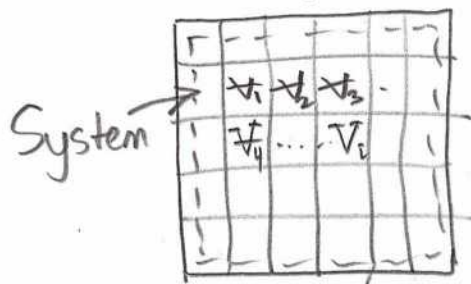


Extensive and Intensive Properties

- In thermodynamics, we need to distinguish between properties of a System that are dependent or independent of the SIZE of the System.
- These two cases will be used in Chapter 3 when we study properties of matter.

(1) Extensive Properties

- Dependent upon the Size or "Extent" of the System
- Examples: mass, m_{system} (kg)
Volume, V_{system} (ft³)
Energy, E_{system} (Btu)
- All Extensive properties are additive:
For example, consider the box below, divided into N partitions, each of volume V_i :

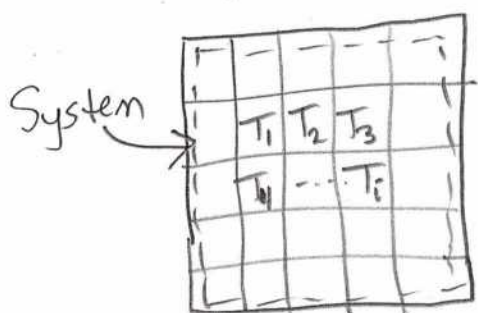


then

$$V_{\text{System}} = \sum_{i=1}^N V_i$$

(2) Intensive Properties

- Independent of the size* of the System
- Intensive properties of a System are not additive.
- Examples: temperature, T_{system} (K)
pressure, P_{system} (psia)
mass density, ρ_{system} (kg/m^3)
- If you divide up the closed system shown below into N partitions, then the properties of each individual partition do not add up to the total value for the System:



then

$$T_{\text{system}} \neq \sum_{i=1}^N T_i$$

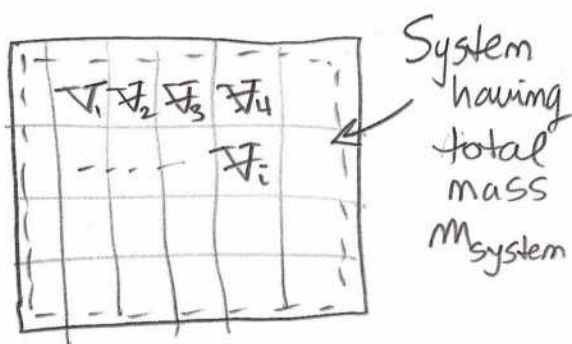
$$P_{\text{system}} \neq \sum_{i=1}^N P_i$$

$$\rho_{\text{system}} \neq \sum_{i=1}^N \rho_i$$

* Our textbook says it is independent of the mass of the system. I am using the definition given by Moran & Shapiro because it is easier to understand, although both are technically equivalent.

(3) Special case: Turning an Extensive property into an Intensive property by dividing by mass

- If we take an Extensive Property and divide it by the System's mass (kg or lb_m), then that property becomes independent upon the size of the system and hence, becomes Intensive:



In this case we always call the property with the prefix "specific"

Examples:

$$\frac{V_{\text{sys}}}{m_{\text{sys}}} \equiv v_{\text{sys}} = \frac{1}{\rho_{\text{sys}}} \equiv \text{"Specific Volume"}$$

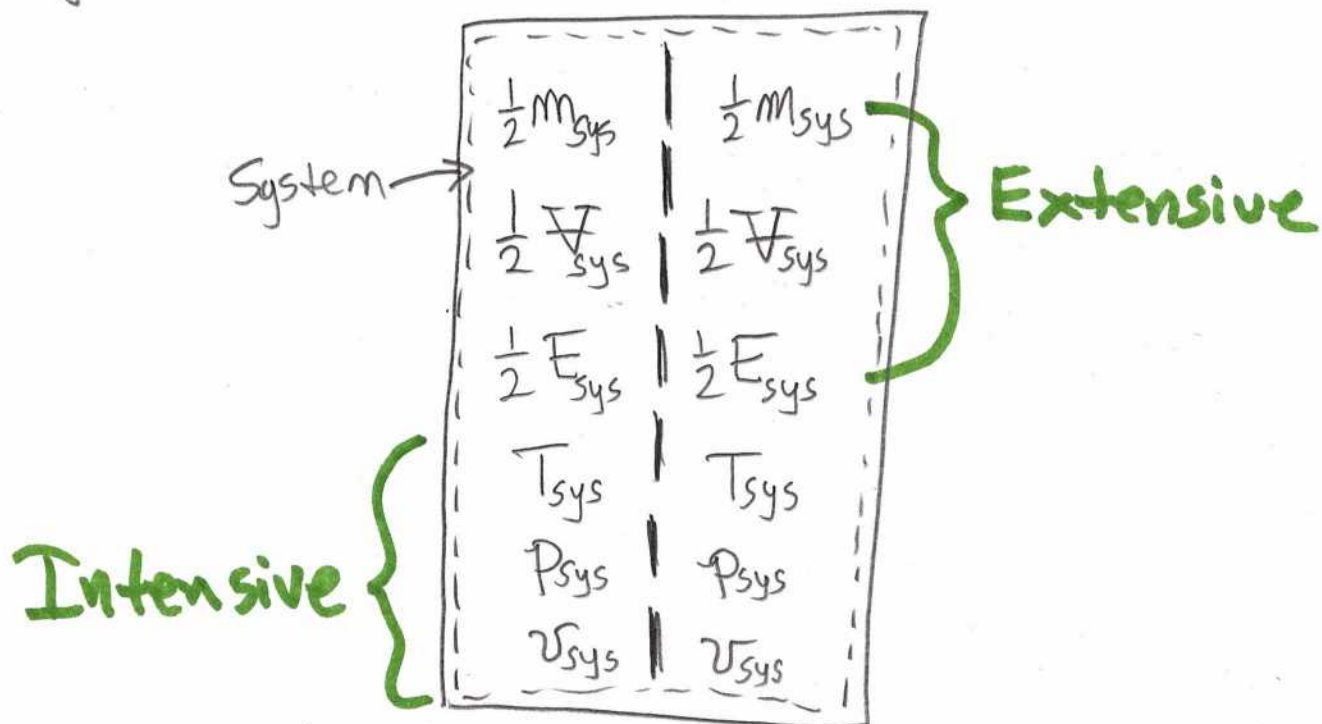
$$e_{\text{sys}} \equiv \frac{E_{\text{sys}}}{m_{\text{sys}}} = \text{"Specific Energy"}$$

$$\frac{\rho}{\rho_{\text{H}_2\text{O}}} \equiv \text{"Specific gravity", or "SG"}$$

$$\text{so, } v_{\text{sys}} \neq \sum_{i=1}^N v_i \quad \text{and} \quad e_{\text{sys}} \neq \sum_{i=1}^N e_i$$

How to determine if a Property is Intensive or Extensive

- First, note that we are dealing only with a System in Thermal Equilibrium (next Lecture) which has spatially uniform distributions of Properties
- Divide the system into two halves using an imaginary partition.
- If the property is Extensive, Then each part will be $\frac{1}{2}$ of the total amount for the System:



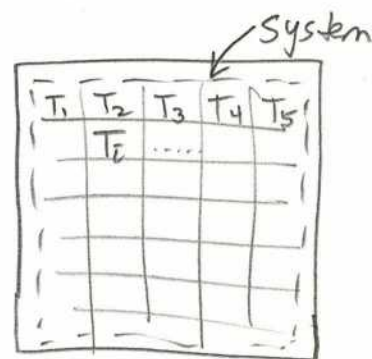
Definition of Thermodynamic Equilibrium

- There are 4 conditions that must be simultaneously satisfied for any System to be in Thermodynamic Equilibrium.

(1) Thermal Equilibrium

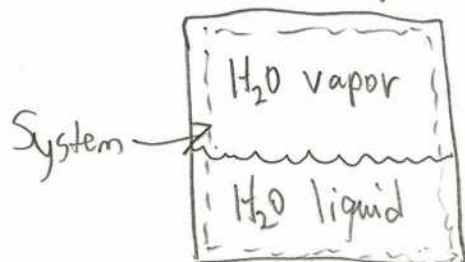
- The temperature within the System is completely uniform and does not change with time:
- The temperatures in each sub-system, or partition, are all equal:

$$T_1 = T_2 = T_3 = T_i \text{ etc.}$$



(2) Phase Equilibrium

- There are 3 common phases of matter:
 - a) gas
 - b) liquid
 - c) solid
- A system is in Phase Equilibrium when there is no change of any phases of matter with time:



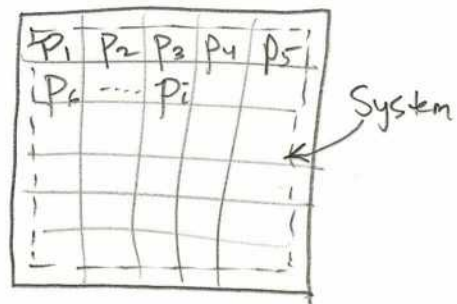
No change in mass of either phase with time \Rightarrow phase equilibrium

(3) Mechanical Equilibrium

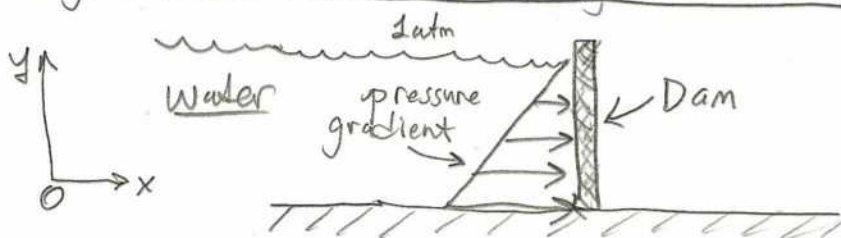
(a) The pressure within the System is completely uniform and does not change with time

- The pressures in each sub-system, or, partition, are all equal:

$$p_1 = p_2 = p_3 = p_i, \text{ etc.}$$



- In the special case of pressure variations within the System due to elevation (gravity), such as hydrostatic pressure, there is no imbalance of forces on the System so the pressure gradient does not change with time.



(b) The System may be moving with respect to an inertial reference frame Oxy , but only under the conditions

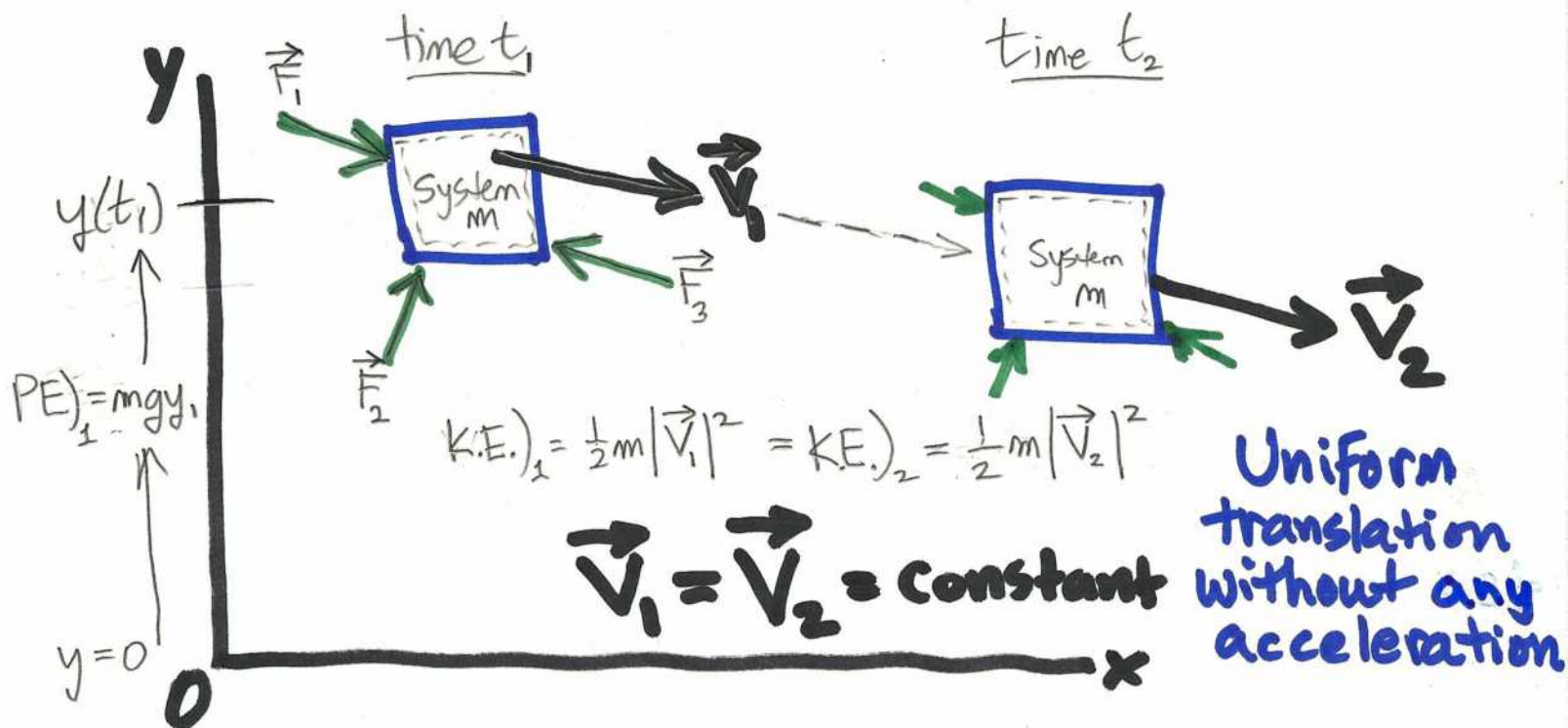
(i) $|\vec{V}_{\text{sys}}| = \text{Constant} \quad (\sum \vec{F}_{\text{forces}} = 0)$

(ii) No net Torque on System $(\sum \vec{M} = 0)$

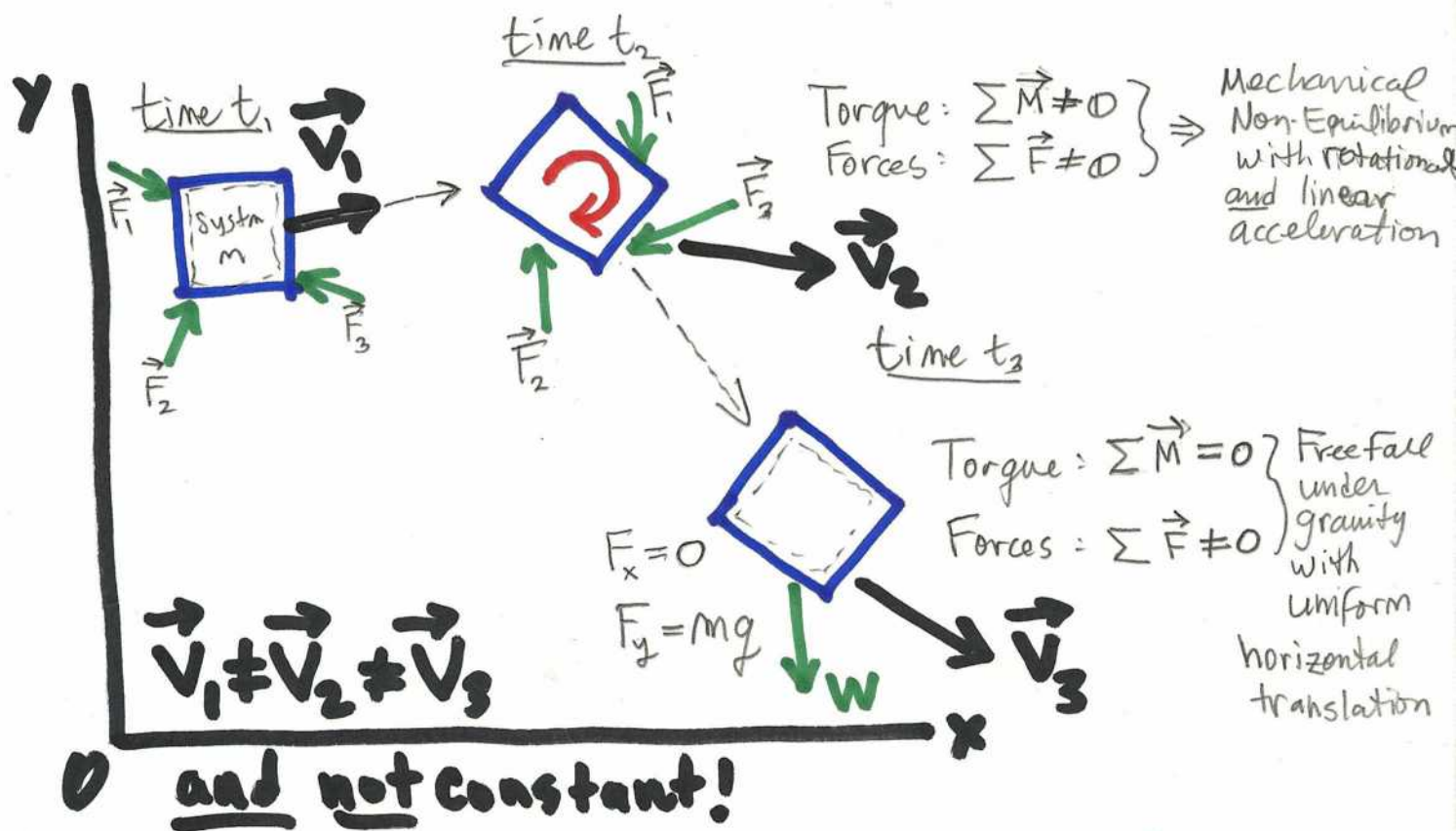
(see Wikipedia for "Mechanical Equilibrium")

Let us look at a specific Examples of Mechanical equilibrium vs. mechanical non-equilibrium according to part (b):

• Mechanical Equilibrium due to motion of the System:



• Mechanical Non-Equilibrium due to motion of the System:

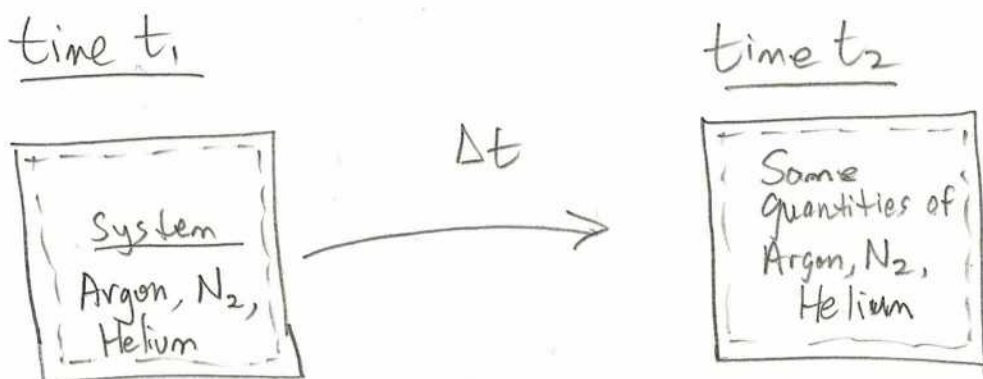


(4) Chemical Equilibrium

(9)

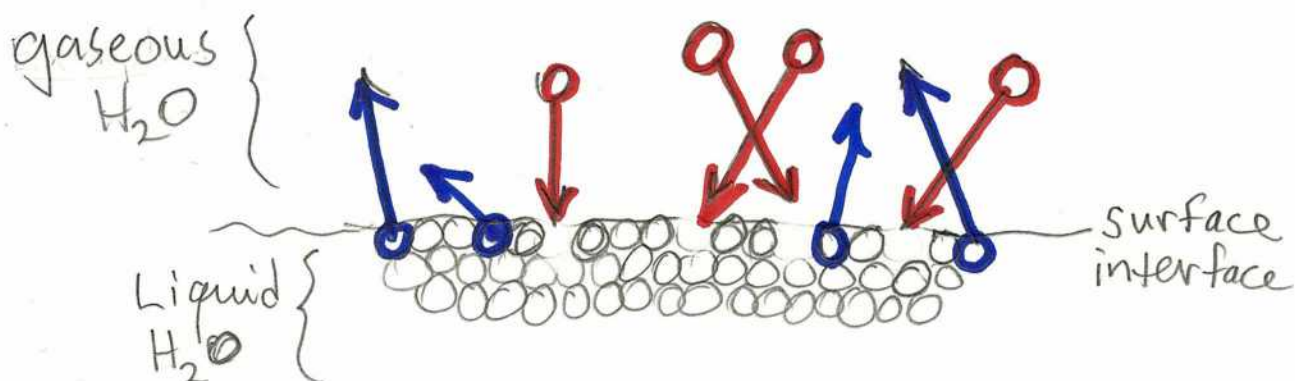
(a) No chemical reactions occur within the System:

- No combustion occurs ($A+B \rightarrow AB$)
- No change in the number of species particles (atoms or molecules) over time



(b) Dynamic Chemical Equilibrium is allowed to occur

- On a molecular/Atomic scale, the chemical reaction's forward and backward reaction rates are equal:



State of a System

"State" is the condition of a System when it is at Thermodynamic Equilibrium

- Let us think back in the notes, now.

State \Rightarrow Equilibrium \Rightarrow Spatially Uniform Properties
Not changing with time

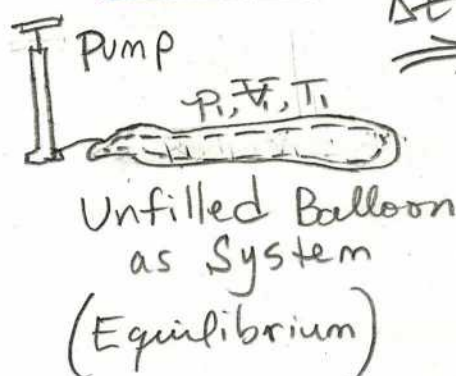
- So "State" refers to the condition of a System when its properties are uniform and unchanging in time.
- A system is said to be at "Steady state" when none of its properties change with time.
- There is only one unique State associated with a given set of properties.
For example, if I assign a system the properties (p, T, ρ, E, \dots etc) then this is a unique State.
- Important: The converse is true: A System cannot have properties unless it is in an Equilibrium State!

Process

Any change that a System undergoes from one Equilibrium State to another.

Example 1

State 1

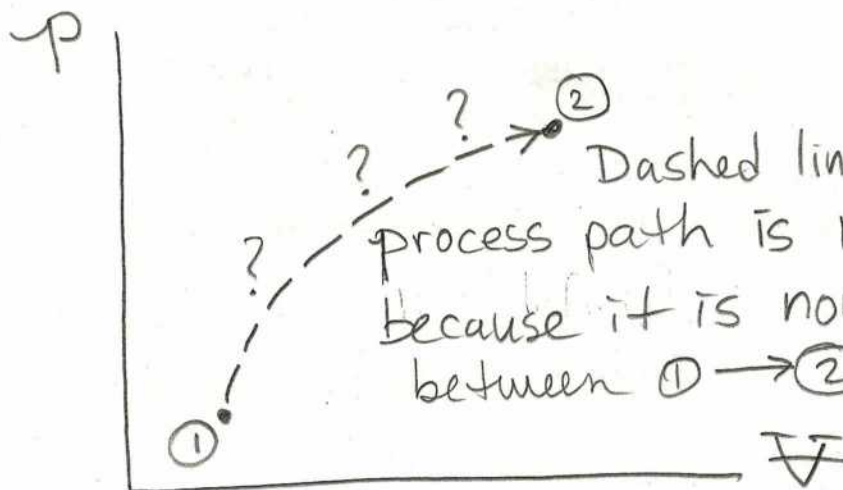
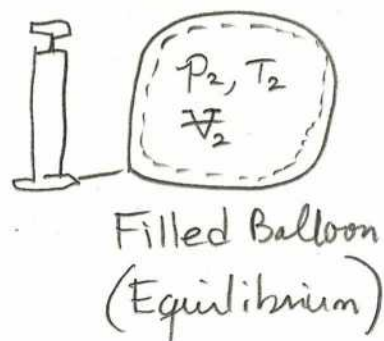


Intermediate non uniform, condition



properties cannot be defined because it is in a nonequilibrium condition \Rightarrow State cannot be defined!

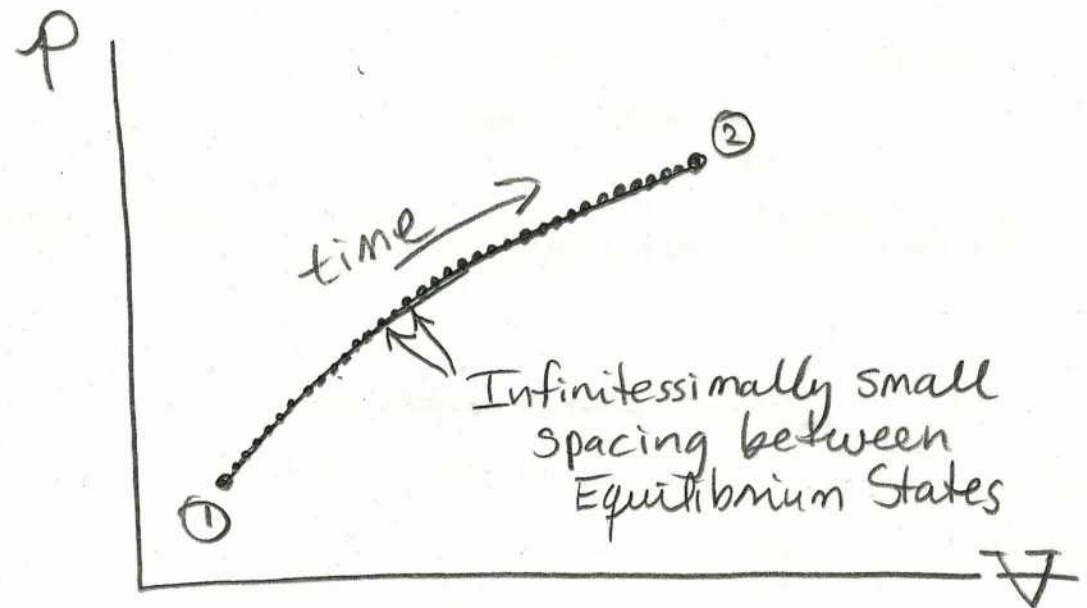
State 2



Example 2 So how do we get from State ① to State ② if any change leads to non-equilibrium conditions in between?!

Answer: We assume that the process path proceeds in such a manner that the System remains infinitesimally close to an equilibrium state at all times:

***Quasi-Equilibrium Process**



- Now, with this "trick", we can say that at each point along the process path there exists an Equilibrium state and therefore it makes sense to talk of a unique set of properties along the path.

- The "price" we pay for using a quasi-equilibrium Process assumption lies with the timescale it takes for the System to self-adjust so that properties remain spatially uniform (approximately) along the path.
- Each System will have it's own timescale ^{for self adjustment} so the quasiequilibrium process from ① → ② might take 1 week, 1 year, 1 microsecond, whatever.
- A quasiequilibrium process is an idealized process and is not a true representation of an actual process.
- But, many actual processes closely approximate quasiequilibrium conditions with negligible error so it is very useful.
- Engineers use a quasiequilibrium Process assumption because
 - (1) Easy to Analyze
 - (2) Result in the highest device efficiencies possible and serve as a benchmark for comparison to the actual designed device

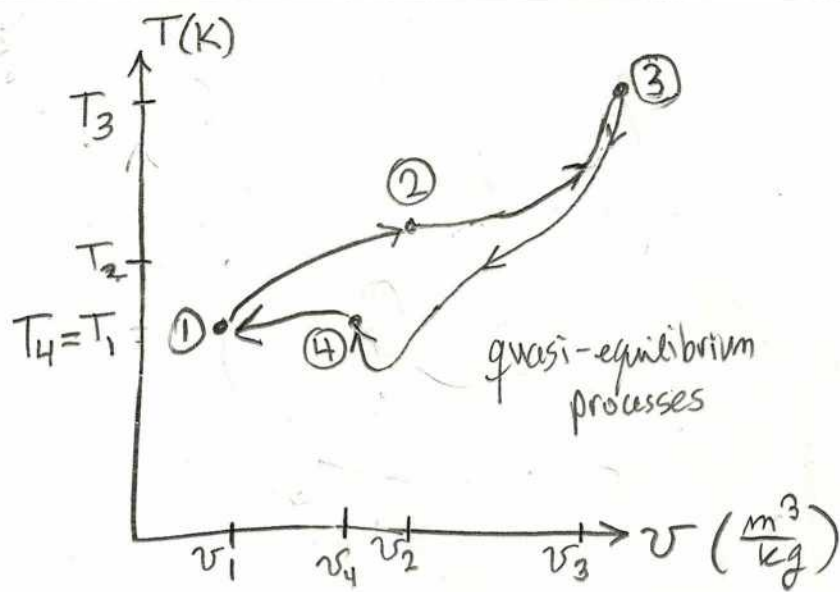
Thermodynamic Cycles

A thermodynamic Cycle is a process, or series of processes that allows a System to undergo State changes and returns to the initial State at the end of the process.

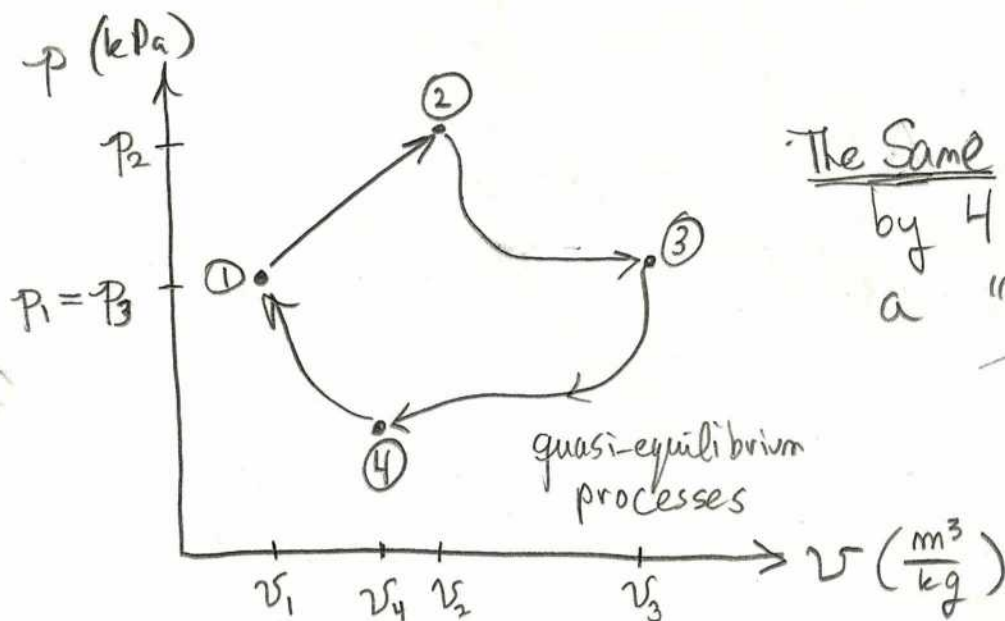
⇒ The Initial and Final States are Identical

Example: Consider a System in which both work and heat interactions are accompanied by changes in the States such that it undergoes a "cycle".

To represent the cycle process paths in terms of System Properties, we can plot the cycle on a 2-D graph. There are many combinations of Properties which we can plot, but two of the important ones for this class are Temperature, pressure, and specific volume:



A Cycle represented by 4 paths in a "T-v" diagram



The Same cycle represented by 4 paths within a "p-v" diagram

Note that each state ①, ②, ③, ④ is the same system equilibrium states except they are represented in different graphs using a different set of system properties.

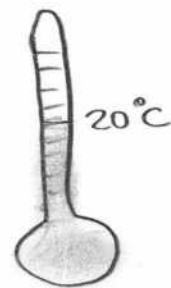
Lecture 5: Temperature, The 0th Law of Thermo, and Pressure

The Concept of "Temperature"

- On a Macroscopic Scale, "temperature" is associated with our human sense of how hot or cold an object or environment feels to us.
- This definition is qualitative and also subjective, in other words it is qualitative because there are no numbers associated with "hotness" and subjective because it depends on each individual person's perception (subjective).
- The qualitative and subjective nature of "hotness" can be eliminated by taking advantage of the fact that several Properties of Materials change with "hotness" in a repeatable and predictable way.

These are two fundamental bases for the definition of "a Science" !!

- One common material is Mercury (Hg), which volumetrically expands in a repeatable and predictable manner when heated



- Say that there is one particular system that everyone in the world agrees upon as a "standard for hotness"

⇒ then the mercury expansion can be used to objectively compare the "hotness" of an unknown system to this defined standard.

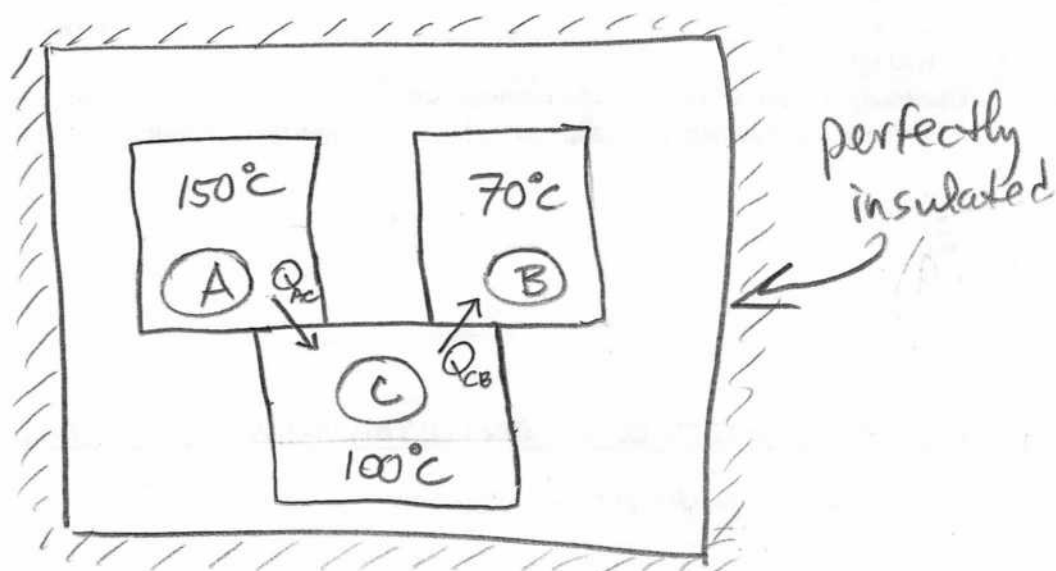
0th Law of Thermodynamics

IF a system (C) is in thermal equilibrium with two other systems (A) & (B),
Then (A) and (B) are also in thermal equilibrium with each other.

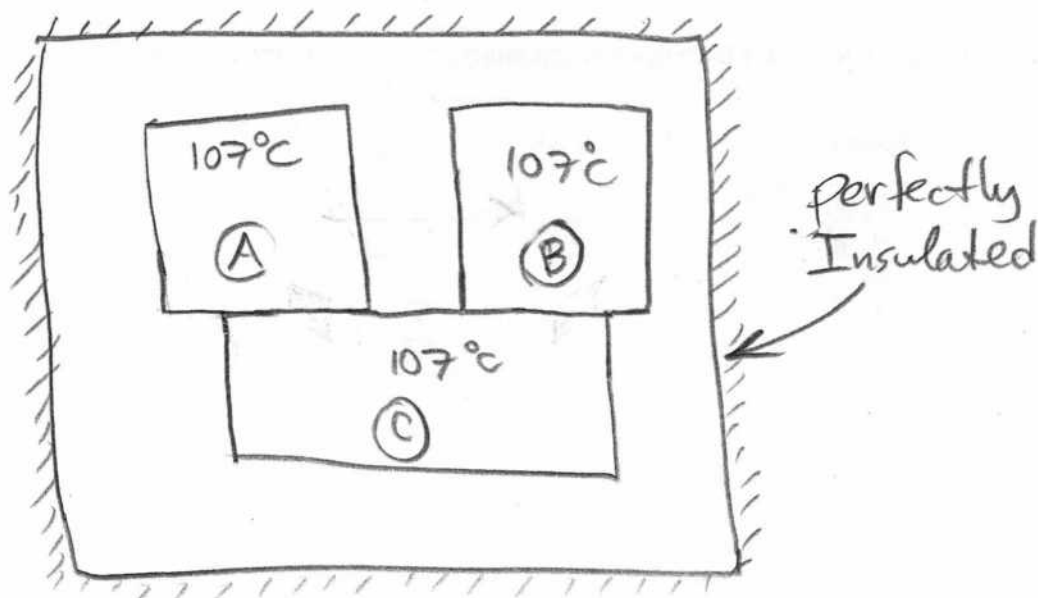
Note: the 0th Law was stated by R.H. Fowler in 1931, long after the 1st, 2nd, and 3rd Laws were conceived. It is so basic that it was named "0th" because it should have been conceived first!

Example 1 0th Law of Thermodynamics

- Consider three systems, (A), (B), and (C) that are not initially in thermal equilibrium.
- Let us place the three systems into a perfectly insulating Box (Isolated) and allow System (C) to interact with both (A) and (B):



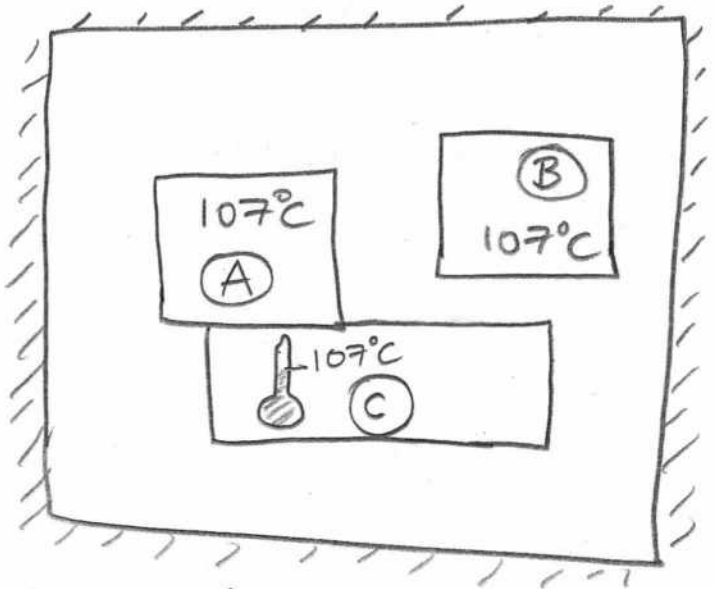
- Experiment Shows that after a time Δt , All three Systems are in mutual Thermal Equilibrium with each other:



Example 2] A Thermometer

- Let System (C) Be a mercury-type thermometer.
- Let Systems (C), (A), and (B) attain mutual thermal equilibrium according to the 0th Law.
- The three Systems are again inside an isolated box.

- Now Disconnect (B) from (C) \Rightarrow



- Experiments show that thermal equilibrium is not affected by disconnecting (or insulating) the three bodies as long as there is no other outside environmental interactions.

\Rightarrow The reading on thermometer (C) will not change if it were in contact with (A) only, or, (B) only.

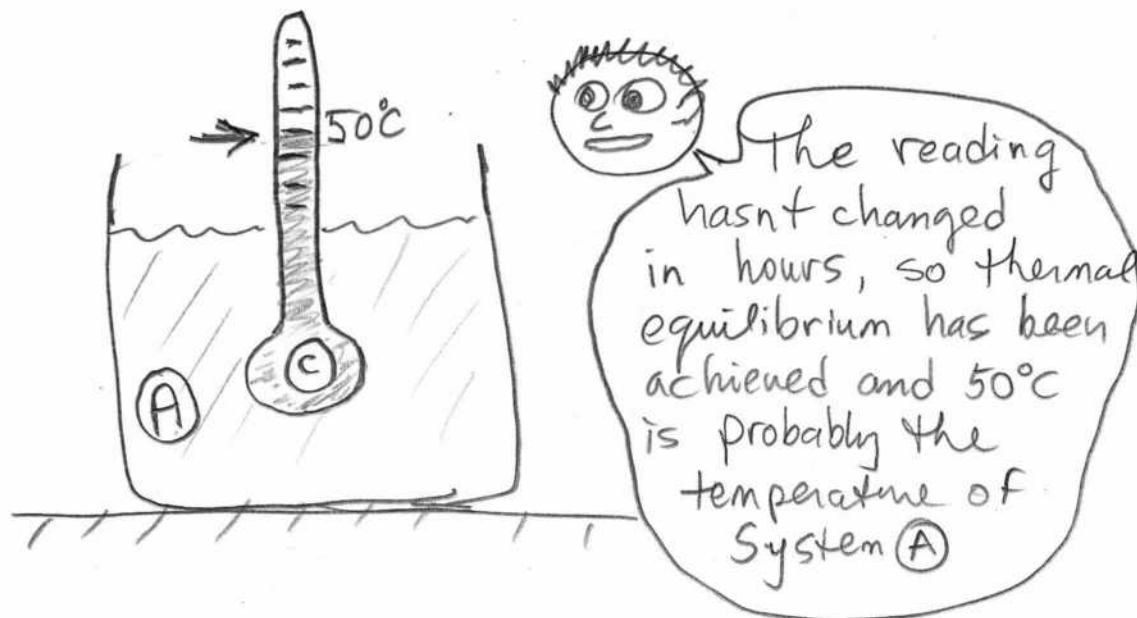
\Rightarrow The two Systems (A) and (C) above are in thermal equilibrium if and only if they have the same temperature.

Note: This last experimentally determined fact is what makes a thermometer useful:

⇒ A thermometer actually measures its own temperature!

⇒ However, when a thermometer is in thermal equilibrium with another body, the temperatures must be equal.

⇒ If a thermometer is built properly, then the readout is an accurate representation of the temperature of the measured body when the two systems are in thermal equilibrium



Temperature Scales

- All temperature scales are based upon some easily reproducible states of matter, such as the freezing and boiling points of H_2O

⇒ This allows anyone in the Universe to construct an identical thermometer scale so that a person can calibrate their thermometer instrument, conduct an experiment, and easily compare data with other scientists and engineers.

Ⓐ "Two-Point" Scales (these are physically arbitrary scales)

(1) Celcius Scale

- Named after the Swedish Astronomer A. Celcius (1702-1744)

- H_2O is chosen as the material "Standard"

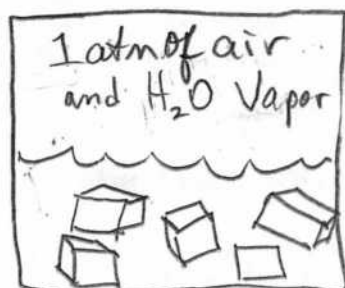
- Two States of H_2O are used to "fix" the Celcius scale:

$$\text{@ } P = 1 \text{ atm } \begin{cases} (1) \text{ Ice point of } H_2O \equiv 0^\circ C \\ (2) \text{ Steam point of } H_2O = 99.975^\circ C \end{cases}$$

Q: What are "Ice Points" and "Steam Points"?

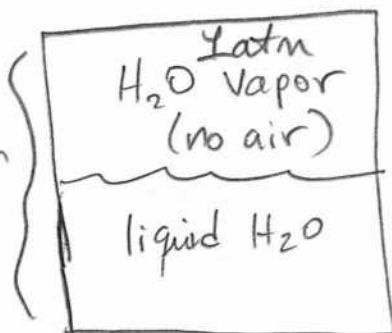
A: At 1 atm (Sealevel pressure)

Thermal &
Phase
Equilibrium
Mixture
of
Ice,
liquid,
and Saturated
air



Ice point, 1 atm

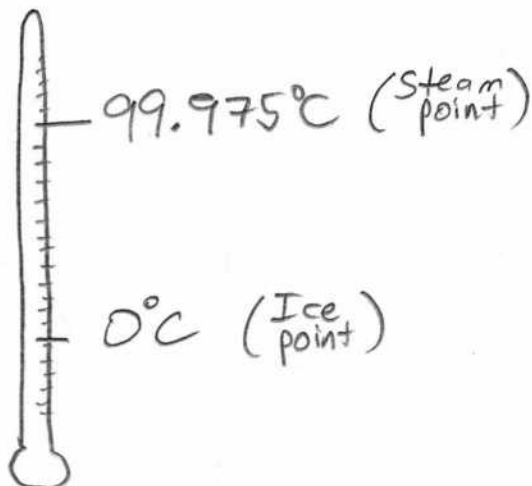
Thermal &
Phase
Equilibrium
mixture
of liquid
and vapor
 H_2O



Steam Point, 1 atm

Celsius
Scale

International
Temperature Scale
of 1990, or
ITS-90



(2) Fahrenheit Scale

- Named after German instrument maker G. Fahrenheit (1686-1736)
- Again, H_2O was chosen as the material "Standard"
- Again, uses two H_2O states to fix the Fahrenheit Scale:

$$@ p = 1 \text{ atm}, \begin{cases} (1) \text{ Ice Point of } H_2O \equiv 32.02^\circ F \\ (2) \text{ Steam Point of } H_2O \equiv 211.95^\circ F \end{cases}$$

(According to the ITS-90 standard)

(B) Thermodynamic Temperature Scale (single point scale)

- In thermodynamics, it is very important to have a temperature scale that is independent of any substance or substances
- This type of temperature scale is actually based upon the 2nd Law of Thermodynamics and a particular process cycle called the Carnot cycle.
- These scales are also known as "Absolute" temperature scales and have physical meaning about Energy & Entropy

(9)

(1) Kelvin Temperature Scale

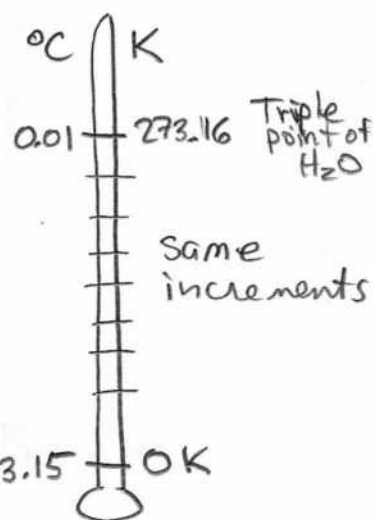
- Named after William Thomson (a.k.a. Lord Kelvin) (1824-1907)
- The only state that fixes this scale is the triple point of water \Rightarrow 273.16 K at 0.006 atm

phase
Equilibrium
between
ice, liquid,
and vapor



- The lowest point on the scale is \equiv 0K (Absolute Zero)

$$\Delta T = 1^{\circ}\text{C} \equiv 1\text{K}$$



(2) Rankine Temperature Scale

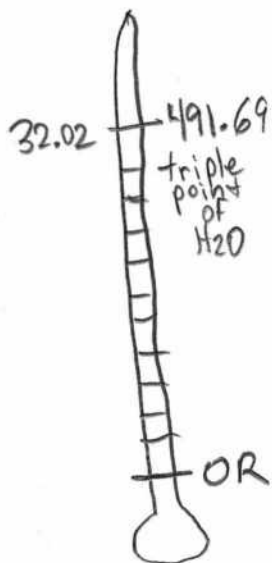
- Named after William Rankine (1820-1872)

- Used with the English System

- Triple point of H₂O is 491.69 R @ 0.006 atm on this scale

- The lowest point on the scale is 0R

$$\Delta T = 1^{\circ}\text{F} \equiv 1\text{R} \quad (\text{same increments})$$



Conversion Factors (see back of textbook cover)

(10)

$T(^{\circ}\text{F}) = 1.8 T(^{\circ}\text{C}) + 32$ $T(^{\circ}\text{C}) = \frac{T(^{\circ}\text{F}) - 32}{1.8}$
$T(\text{K}) = T(^{\circ}\text{C}) + 273.15$ $T(\text{R}) = T(^{\circ}\text{F}) + 459.67$
$T(\text{R}) = 1.8 T(\text{K})$

$$^{\circ}\text{C} \Leftrightarrow ^{\circ}\text{F}$$

$$\begin{array}{l} \text{From } ^{\circ}\text{C} \rightarrow \text{K} \\ ^{\circ}\text{F} \rightarrow \text{R} \end{array}$$

$$\text{R} \Leftrightarrow \text{K}$$

Microscopic View of Temperature (An Intensive Property)

- Temperature is the level of molecular/atomic "agitation" or "activity"

⇒ Temperature is NOT Energy

⇒ Temperature can be related to this average molecular/atomic Energy

⇒ A thermometer measures the Intensity of "warmth". NOT the Quantity of "warmth"

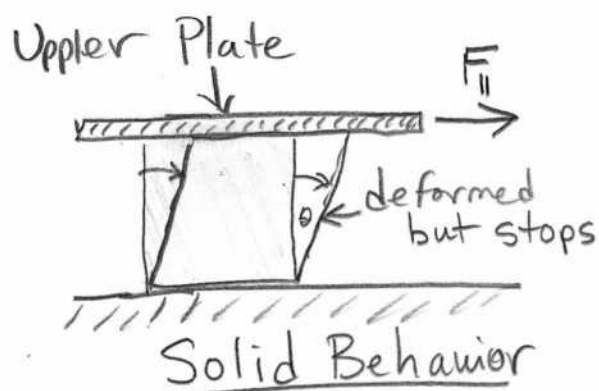
(Quantity of Warmth is Energy ⇒ Extensive!)

The Concept of Pressure

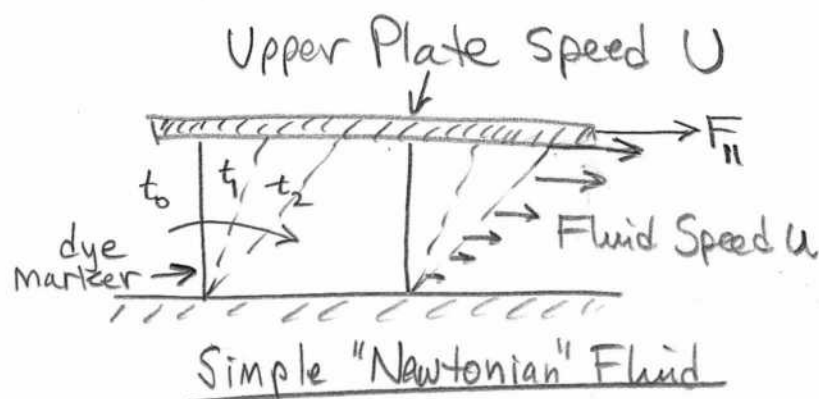
- First, we need to distinguish a "Fluid" (as used in Mechanics and Thermodynamics) from a solid.

FLUID

- A "fluid" is a substance that deforms continuously under the application of a shear (tangential) stress no matter how small the shear stress may be
- Matter comprised by the liquid or gas/vapor phases can be considered as fluid's



shear stress $\propto \frac{F_{\parallel}}{A}$



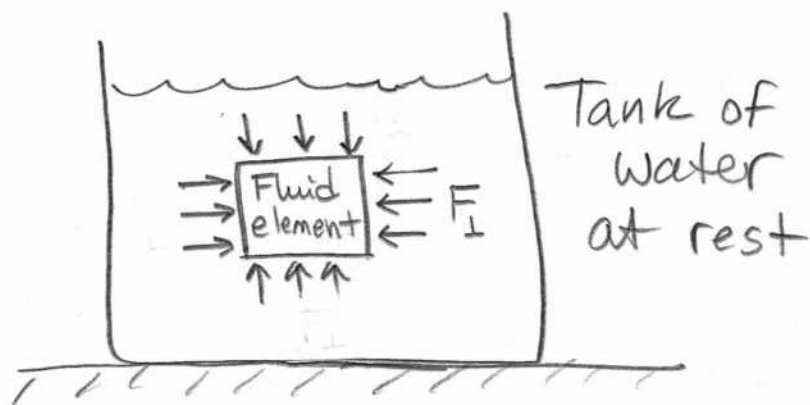
shear stress $\propto \frac{du}{dy}$

Note: A Fluid can be defined as a substance that cannot sustain a shear stress when it is at rest.

Pressure

Pressure is a Normal force exerted by or on a Fluid per-unit-area

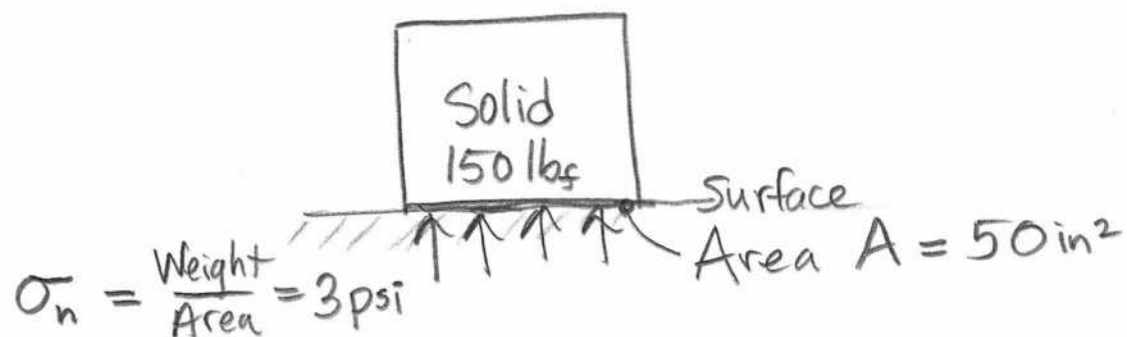
- "Pressure" deals with fluids only — gases, vapors, or liquids



- Pressure is measured in Kilo-Pascals (kPa) or $1000 \frac{\text{N}}{\text{m}^2}$ in this class

[English units: pound force, lb_f/in^2 or psi]

- Solids \Rightarrow the term "normal stress", σ_n , is used, but the units are the same:



"Absolute" vs. "Gage" pressures

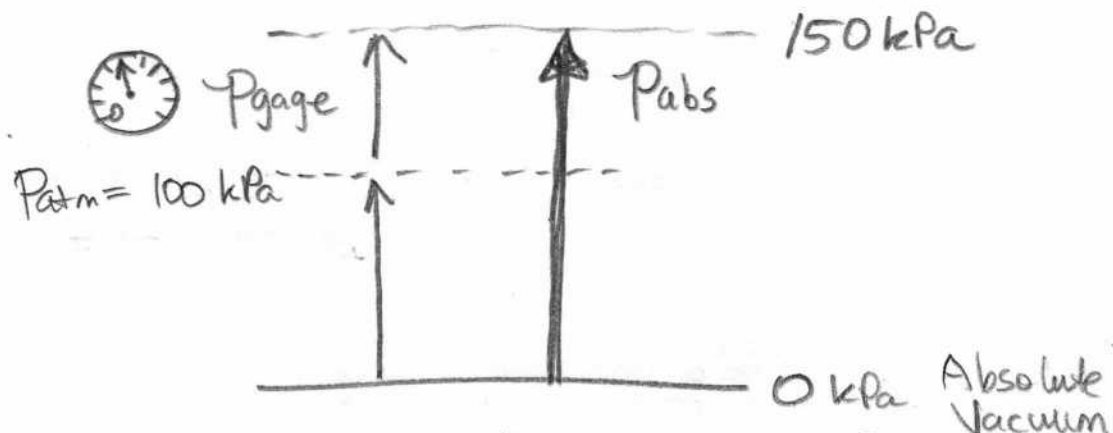
Absolute Pressure

The pressure of a fluid measured with respect to a vacuum (0 kPa)

Gage Pressure

The pressure of a fluid measured with respect to atmospheric pressure

Example



- $P_{abs} = 150 \text{ kPa}$ (compared to vacuum 0 kPa)
- $P_{atm} = 100 \text{ kPa}$
- $P_{gage} = 150 - 100 \text{ kPa} = 50 \text{ kPa}$



$$P_{gage} = P_{abs} - P_{atm}$$

Standard Atmospheric Pressure

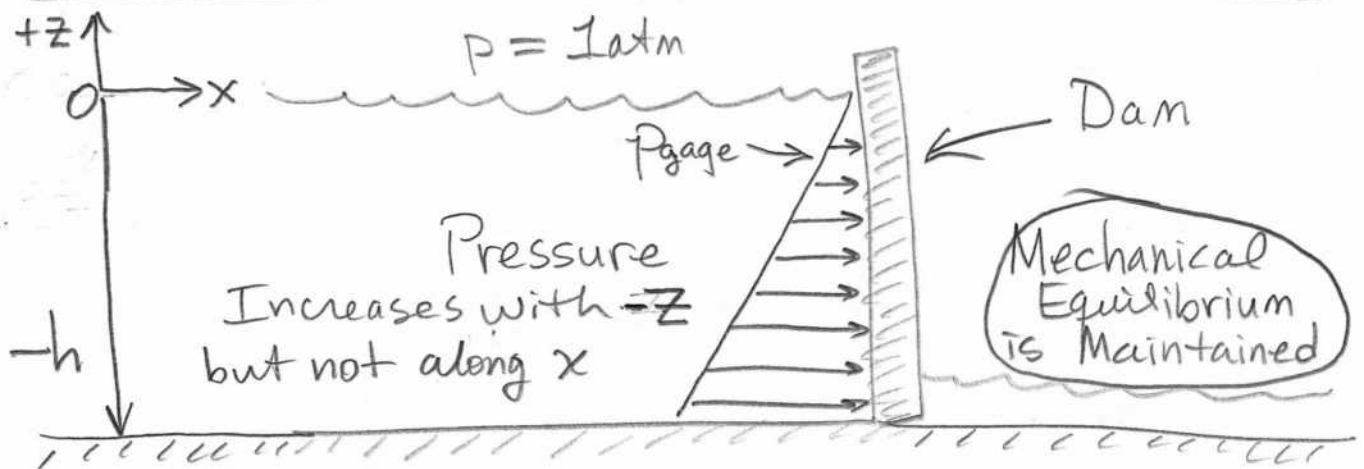
14

This is an Internationally defined Standard based on sea-level averages

$$1 \text{ atm} = 101.325 \text{ kPa}$$

Note: $101.325 \text{ kPa} \approx 100 \text{ kPa}$ (good way to remember it)

Variation of Pressure in a Fluid with Depth



- Pressure in a fluid increases with Depth because more fluid rests on deeper layers, and the effect of this "extra weight" on a deeper layer is balanced by an increasing pressure.

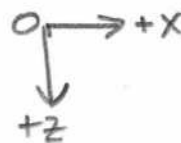
- Fluid pressure varies with depth as

(Pg 24)

$$\frac{dp}{dz} = -\rho g$$

$\rho = \rho(z)$ fluid density at z
 $g = \text{constant}$ gravitational constant

For the Dam picture above, we integrate assuming: (1) Coordinate System is



(2) $\rho = \text{constant}$

So

$$dp = -\rho g dz$$

$$\int_0^{-h} dp = - \int_0^{-h} \rho g dz$$

$$\int_0^{-h} dp = \int_{-h}^0 \rho g dz = \rho g \int_{-h}^0 dz$$

$$p(-h) - p(0) = \rho g (0 - (-h))$$

$$p - p_{\text{atm}} = \rho g h$$

$$p = p_{\text{atm}} + \rho g h$$

↑
absolute
pressure

gauge pressure

Read pgs. 21 - 32 and understand
the Examples Given

Lecture 6: Total Energy, Mechanical Energy, and Internal Energy

ENERGY

Energy is an ability to cause a change in the organization of the Universe.

The word "Energy" was first used in the modern sense by Thomas Young in 1807. Prior to this, there was only an abstract notion of "*vis viva*" which means "living force". Young was most noted for showing that light has wave properties (recall the famous "double slit" experiment) and for "Young's Modulus" which is proportionality constant that relates stress to strain in a simple elastic body.

TOTAL SYSTEM ENERGY

The textbook says that types of Energy are "...in numerous forms such as thermal, mechanical, kinetic, potential, electric, magnetic, chemical, and nuclear, and their sum constitutes the Total Energy of the System."

This is true from an overall classification standpoint, and we are going to be using a few of these forms on a regular basis in this class.

However, I want to give you a different perspective which may help you better understand what "Mechanical" and "Internal" Energies of the System actual mean.

MECHANICAL "BULK" AND INTERNAL (MICROSCOPIC) ENERGIES

In Thermodynamics, the **Total Energy**, E_{tot} , of the System is divided into two parts; Macroscopic "Bulk" Mechanical Energy and Microscopic "Internal" Energy:

$$E_{tot} \equiv \begin{bmatrix} \text{Mechanical} \\ \text{"Bulk"} \\ \text{Energy} \\ E_{\text{Mech}} \end{bmatrix} + \begin{bmatrix} \text{Internal} \\ \text{"Microscopic"} \\ \text{Energy} \\ U \end{bmatrix} \quad (1)$$

Both E_{Mech} and U can be described in terms of *Kinetic* and *Potential* Energies.

KINETIC AND POTENTIAL ENERGIES

In your basic physics courses, you have learned about two types of Energies:

1. **Kinetic Energy (K.E.)** is due solely to a bodies' motion through space:
(Gottfried Leibniz called this concept "*vis viva*", sometime between 1676-1689, but it was Gustave Coriolis, the discoverer of the "Coriolis Force", defined the modern terminology in 1829).

- $$\text{K.E.} = \frac{1}{2} m |\vec{V}|^2$$

2. **Potential Energy (P.E.)** due to a number of different physical sources such as:
(William Rankine defined this terminology in 1853)

- Gravitational P.E. = mgy
- Electrostatic P.E. (Volts) = $V(\vec{r}) = \frac{q}{4\pi\epsilon_0 |\vec{r}|}$

We can use these concepts of KE and PE on either a macroscopic "bulk" scale or on a microscopic particle scale to "build-up" the textbooks' broader definitions of "thermal", "mechanical", "electric", "magnetic", "chemical", and "nuclear" energies.

MECHANICAL ENERGY, E_{Mech} OF A SYSTEM

A System's Mechanical Energy (E_{Mech}) is comprised of:

- A. $\text{K.E.}_{\text{Linear}} = \frac{1}{2} m |\vec{V}|^2$
 B. $\text{P.E.}_{\text{Grav}} = mgy_p$

as shown below in Figure 1. Note that I am talking about the potato in the fixed inertial Oxy reference frame.

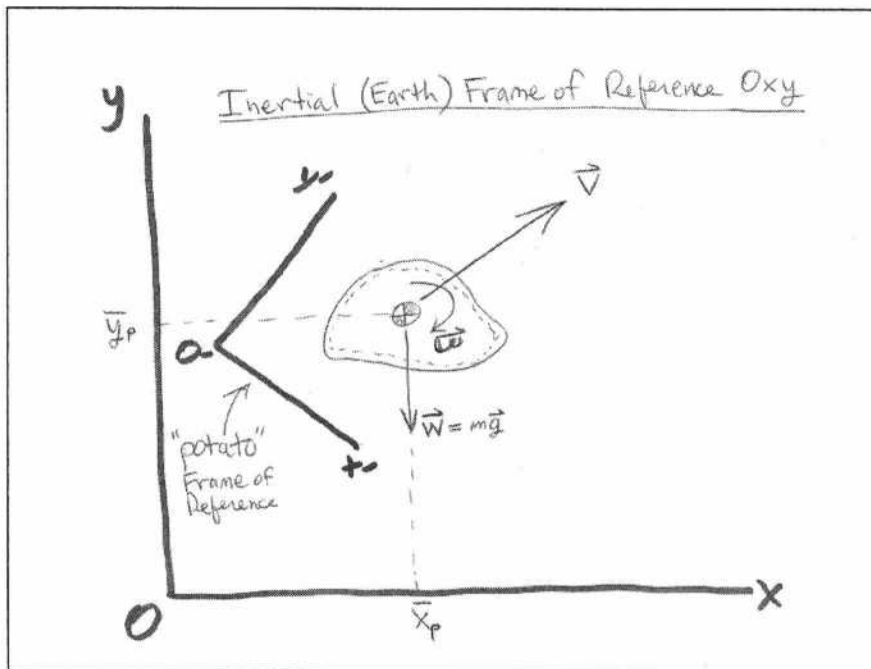


Figure 1. The inertial reference frame Oxy used to describe the Systems' Mechanical Energy. Also shown is an arbitrarily defined non-inertial $O'x'y'$ reference frame in which the potato is completely stationary.

INTERNAL ENERGY, U , OF A SYSTEM

Pretend that you are now moving with the flying, whirling, hot potato in the $O'x'y'$ reference frame shown in Figure 1. From your point of view, the potato is not translating or rotating. It is "fixed".


The Internal Energy, U , of the potato is related to the organization, mutual interactions, and "degree of activity" of the fundamental particles (molecules, atoms, electrons, nuclei, protons, neutrons, etc.) with respect to each other.

System Internal Energy U (kJ)


Thermal

Sensible

Associated with Kinetic energies of the molecule

(1) Molecular Translation 

(2) Molecular Rotation 

(3) Molecular Vibration 

(4) Electron translation



(5) Electron "Spin"

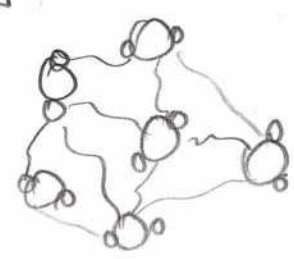


(6) Nuclear "Spin"



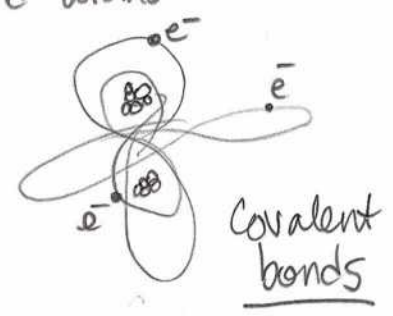
Latent

Associated with the potential energies between molecules and consequently, the phase of the substance



Chemical

Associated with Potential & Kinetic energies of electrons & nucleus within the molecules and atoms



Nuclear

Associated with VERY Strong potential and Kinetic energies within the atomic nucleus

The textbook states on pg. 53 that contrary to the macroscopic Mechanical Energy, the microscopic Internal Energy is independent of the outside reference frame Oxy. This is true to a very good approximation because the mass of the individual microscopic particles is small *and* the rotation rate of the non-inertial reference frame is generally small for the types of problems we will be dealing with in this class (read the first couple of paragraphs of the definition of "inertial reference frame" on Wikipedia and you will easily see what I mean).

Now let's break down the Internal Energy, U , into both KE and PE and see what each part means to us in thermodynamics:

- A. **Internal K.E.** is due to the motion of the molecules, atoms, and subatomic particles. Internal KE is the sum of the microscopic particles' translational, vibrational, rotational, and electronic spin Energies (see page 55 of the text). The degree of molecular motion on the microscopic scale is proportional to what we call "Temperature". The KE due to the *random motion* of the molecules/atoms is what we call "Thermal Energy".
- B. **Internal P.E.** is related to the position of the molecules, atoms, and subatomic particles with respect to each other and leads to attractive and repulsive forces between them (see page 55 of the text). For this class, we will consider only two parts of the Internal P.E.; "Latent Energy" and "Chemical Energy".
 - i. **Latent Energy** is associated with weak attractive forces *in between* the molecules. When sufficient energy is transferred into/out of the System, these bonds will weaken/strengthen and the System will change its phase (solid, liquid, or gas) but the chemical composition does not change (e.g., H_2O from ice \rightarrow water \rightarrow water vapor....it's still H_2O).
 - ii. **Chemical Energy** is associated with the relatively strong atomic bonds *within* the molecules (ionic & covalent, which are much stronger than the attractive forces between the molecules). During a chemical reaction, some chemical bonds are broken while others are formed. This results in a re-arrangement of the interacting atoms into a new species and as such, the Internal Energy changes.

THE BOTTOM LINE

So, the bottom line is that the Total Energy of a System in this class is

$$\begin{aligned} E_{\text{tot}} &\equiv E_{\text{Mech}} + U \\ &= \frac{1}{2}mV^2 + mgy + U \end{aligned} \quad (2)$$

Note that $V^2 = |\vec{V}|^2$ in order to simplify the notation.

THE SCOPE OF CLASSICAL ENGINEERING THERMODYNAMICS

As mentioned above, the science of Thermodynamics is limited in the amount of information it can provide regarding the details of the System as it interacts with its Surroundings.

- Thermodynamics provides no information about the absolute quantity of the Energy that a System possesses.
- Thermodynamics only deals with *changes* of Energy.
- **Energy**, in general, is defined to be zero at an arbitrary, but convenient reference point, called a "datum".

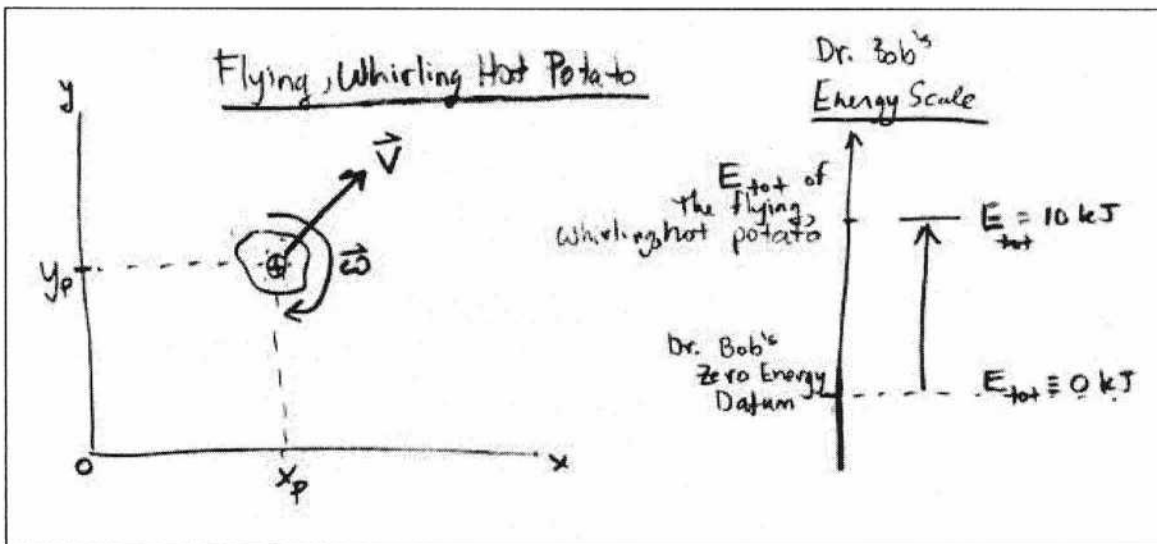


Figure 2. Thermodynamics only gives information about the *change* of a System's Energy.

To understand this, first consider the Mechanical Energy of the "Flying, whirling, hot potato" System shown in Figure 2. An observer in the Oxy reference frame will say that the potato's Mechanical Energy is non-zero ($E_{\text{Mech}} \neq 0 \text{ kJ}$). But an observer in a reference frame fixed to the potato (an $O'x'y'$ frame in which the potato is stationary) will say that $E_{\text{Mech}} = 0 \text{ kJ}$.

Now consider Internal Energy, U . The textbook doesn't mention this, but through *Statistical Thermodynamics*, it is possible, at least in principle, to determine an absolute quantity of the System's *Internal Energy*. However, this is beyond the scope of the class. So the Classical Thermodynamics that we are studying in this class does not describe the physics of the fundamental particle interactions in enough detail, and so we are again limited in only being able to compute the changes in a System's Internal Energy.

Thus, the Mechanical and Internal Energies comprising the Total System Energy, E_{tot} , can only be known to within an arbitrary constant. In Figure 2 I have drawn an energy scale that sets $E_{\text{tot}} = 0 \text{ kJ}$ at an arbitrary value, or "datum".

Lecture 7

Equation of State for a Pure, Simple, Compressible Substance

Pure Substance

A substance that has either

(1) the same chemical composition throughout the system (with same or different phases)

or

(2) A mixture of various chemical compounds as long as the chemical composition is uniform (homogeneous) throughout the system

Examples:

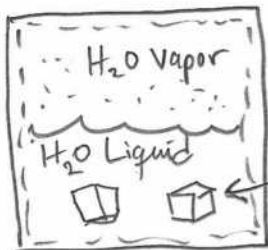
A Pure Substance

(a)



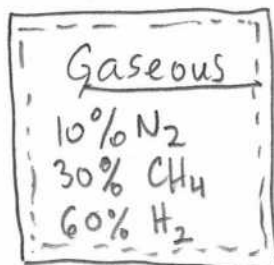
Same chemical substance throughout

(b)



Same chemical substance throughout

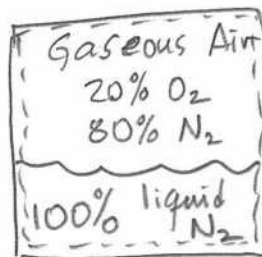
(c)



(Different chemicals, But uniform)

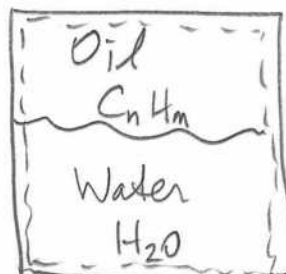
NOT A Pure Substance

(a)



(Different chemicals, Not uniform)

(b)



(Different chemicals, not uniform)

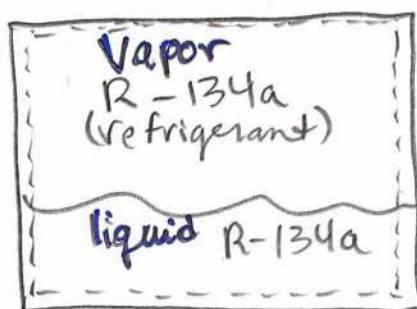
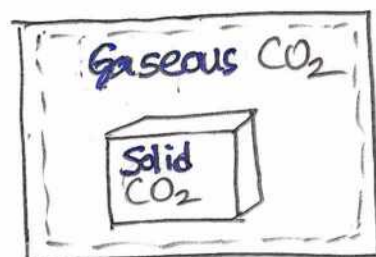
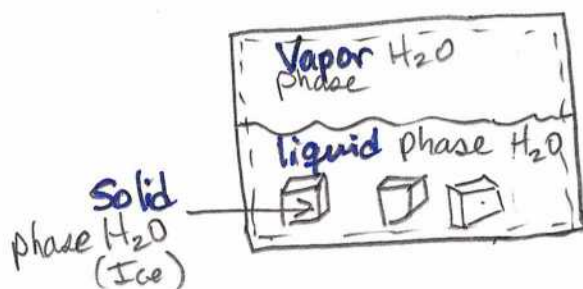
Phase of a Pure Substance

The Phase of a pure substance is identified as having a distinct molecular or atomic arrangement that is homogenous throughout a region of the system and is separated from others by easily identifiable boundary surfaces

The 3 Common phases of matter are

- Solid
- liquid
- gas (or "vapor")

Examples



Simple Compressible System

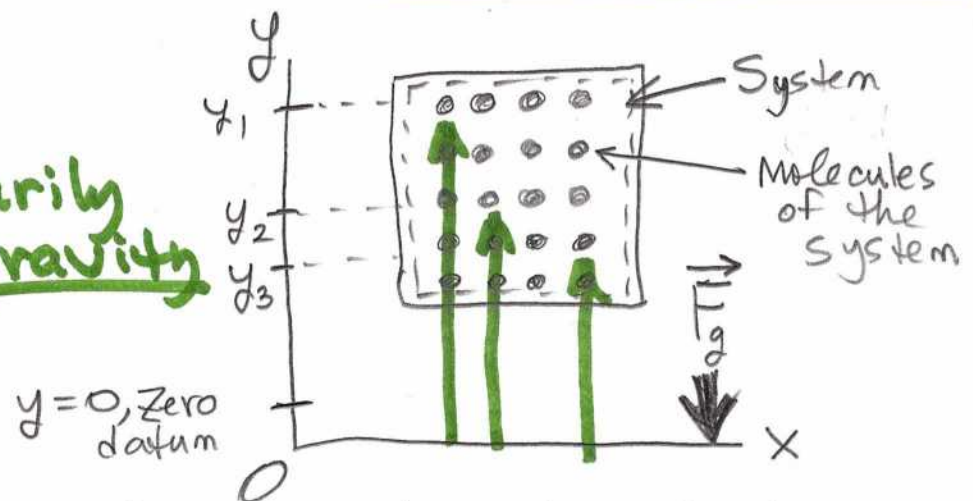
If the individual molecules and atoms within the System are not effected (or the effect(s) are negligible) by external influences such as

- Electric fields
- Magnetic Fields
- Gravitational fields
- Nonuniform accelerations of individual molecules
- Surface tension
- etc.

then the System is a "Simple, Compressible, System"
(See page 15)

Examples :

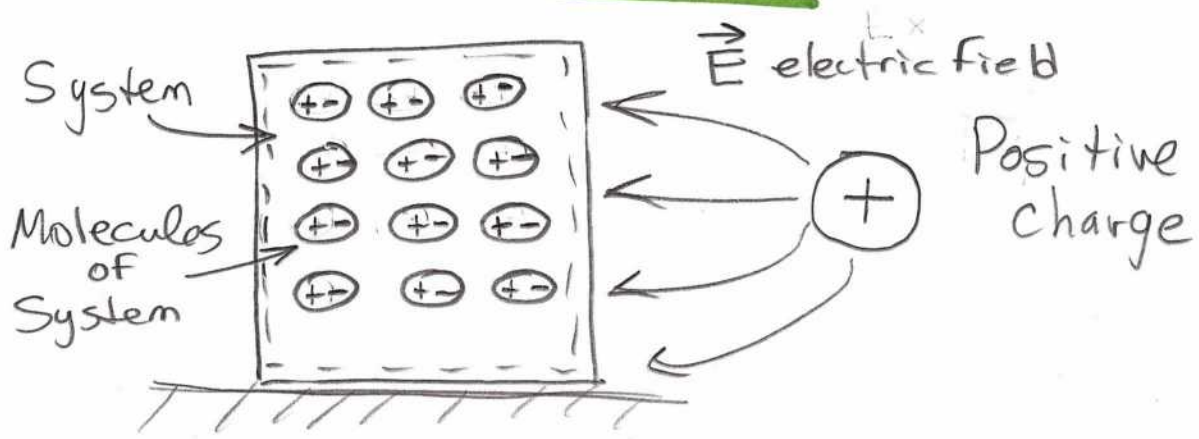
(i) Extrordinarily Strong gravity field



The Field is so strong that each molecule experiences a different gravitational "constant" (g varies from y_1 to y_2 to y_3 , etc.)

⇒ **Not a Simple Compressible System**

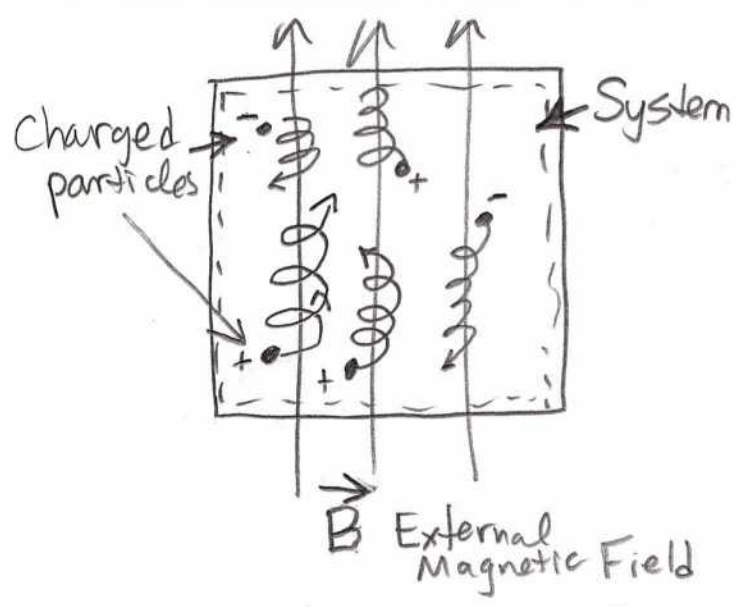
(2) External Electric Fields



The external field and/or the molecules of the system are sufficiently susceptible to the External Electric field that the charges become aligned

⇒ **Not a simple compressible system**

(3) Non-Uniform Accelerations of Molecules in System due to External Magnetic Field



The External Magnetic field is strong enough make charge particles accelerate (circular motion around field lines) in crazy ways!

⇒ **Not a Simple Compressible System**

(the Earth's magnetic field effects is negligible in ME 315/515)

The State Postulate

(see pages 15 and 123)

The State of a Simple Compressible System is completely specified by Two (and only two) Independent Intensive Properties of the System:

$$Z = f(x, y)$$

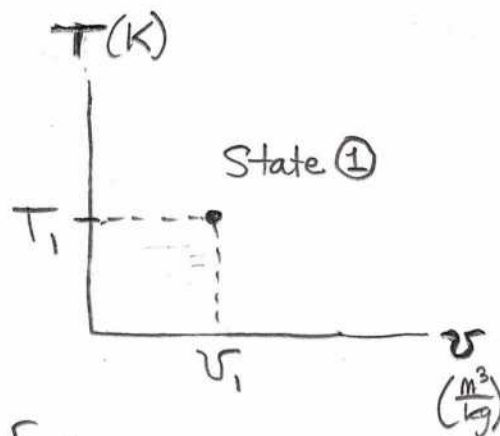
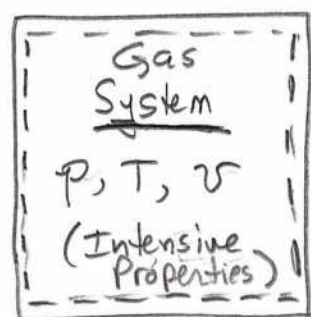
Example :

Ideal Gas

For an "Ideal gas" which we will discuss soon,

$$P = f(T, v)$$

[you may already know that $P = \frac{R \cdot T}{v \cdot m}$]



So given the properties T_1 and v_1 , we can compute p_1 , but the other important point is that STATE ① is now completely specified so that we can also compute the other properties such as Internal energy per-unit-mass (specific internal Energy), $u \left(\frac{kJ}{kg} \right)$.

Equation of State (E.O.S.)

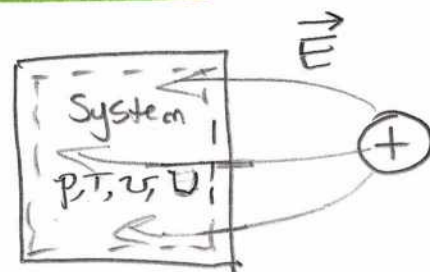
Any Equation (or Table of values) that relates independent, intensive properties of a system, and completely define the state of that system, is called an "Equation of State"

Examples of E.O.S.

- (1) If a system is influenced by an external Electric field, then that field $\vec{E}(x,y,z)$ must be included in the System's E.O.S. :

$$p = f(T, v, \vec{E})$$

⇒ This is NOT a simple compressible system and is not studied in ME 315/515.

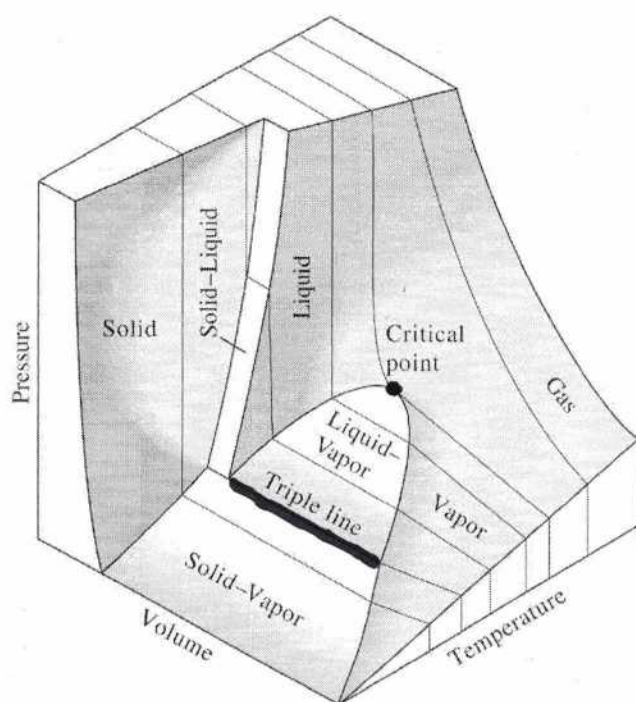


- (2) For a Simple Compressible System, the State Postulate holds; $z = f(x, y)$, so that
- "Phase Diagrams" for p - v - T behavior,
 - Property Tables such as A-4 through A-8 for H_2O
 - Explicit Analytic functions like $p v = R T$

Phase Diagram E.O.S.

⑦

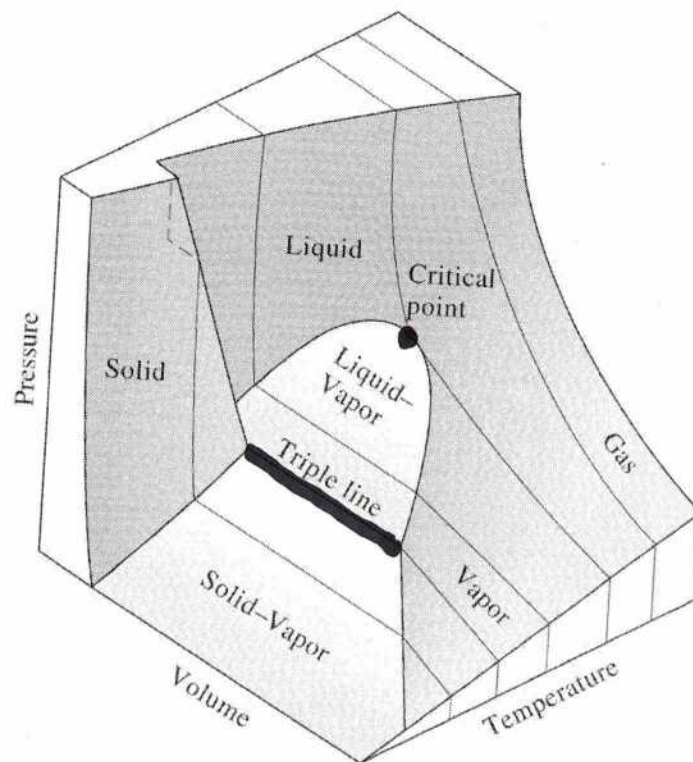
Since the State of a Simple Compressible Substance is fixed by any two independent Intensive properties, we can plot these properties and designate regions where the phases of the substance are homogeneous:



This 3-D plot,
 $p = p(T, v)$,
is called
"p-v-T"
Surface

Simple Compressible Substance that
Contracts upon freezing (CO_2 for e.g.)

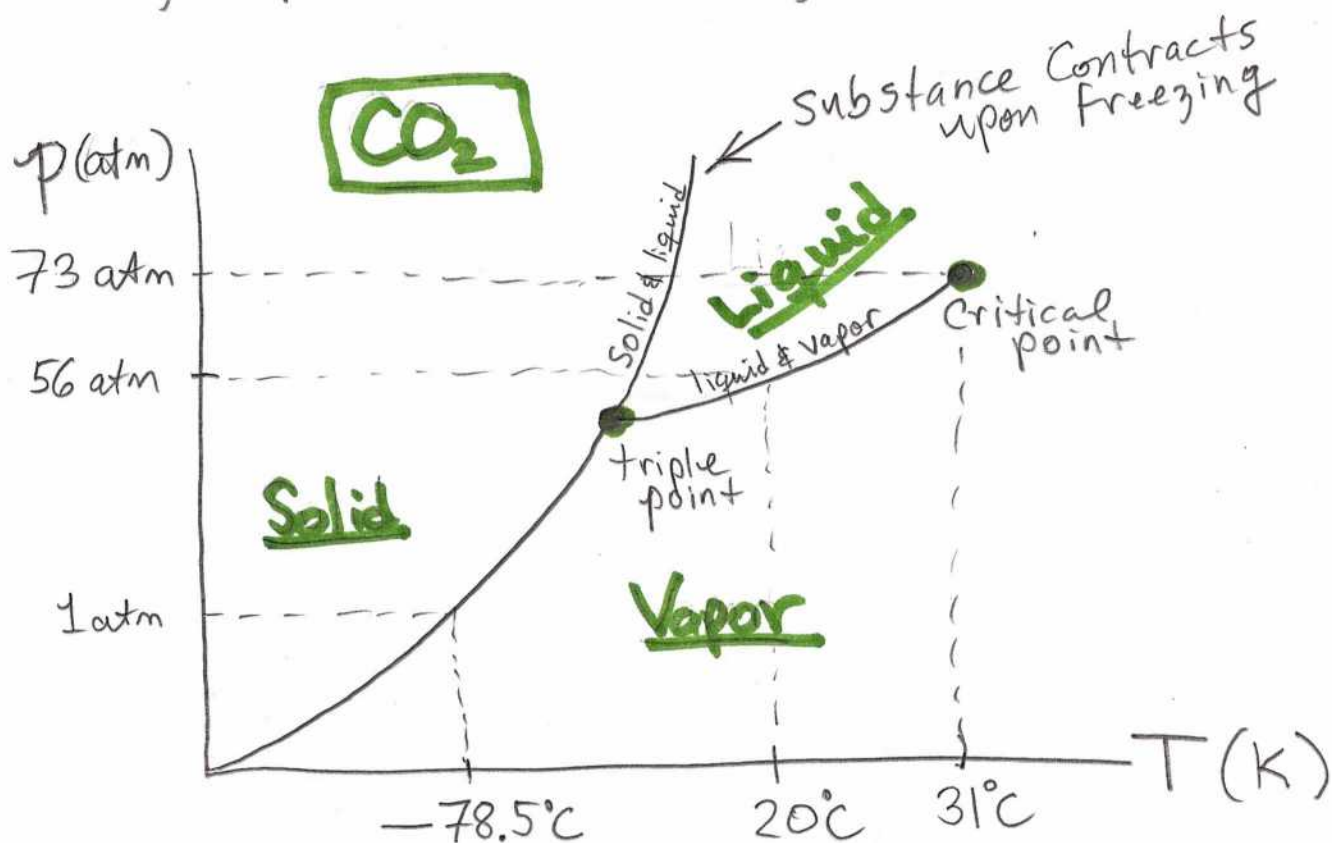
- Note:** (1) Triple Line is a "point" viewed on p-T projection \Rightarrow All phases coexist simultaneously
- (2) Critical point separates liquid-vapor region from gas, liquid, vapor regions



Simple Compressible Substance
that Expands (!) upon freezing
(H_2O , for example)

● Since this 3-D surface plot is not easy to visualize or provides for simple analysis, we always use 2-D plots.

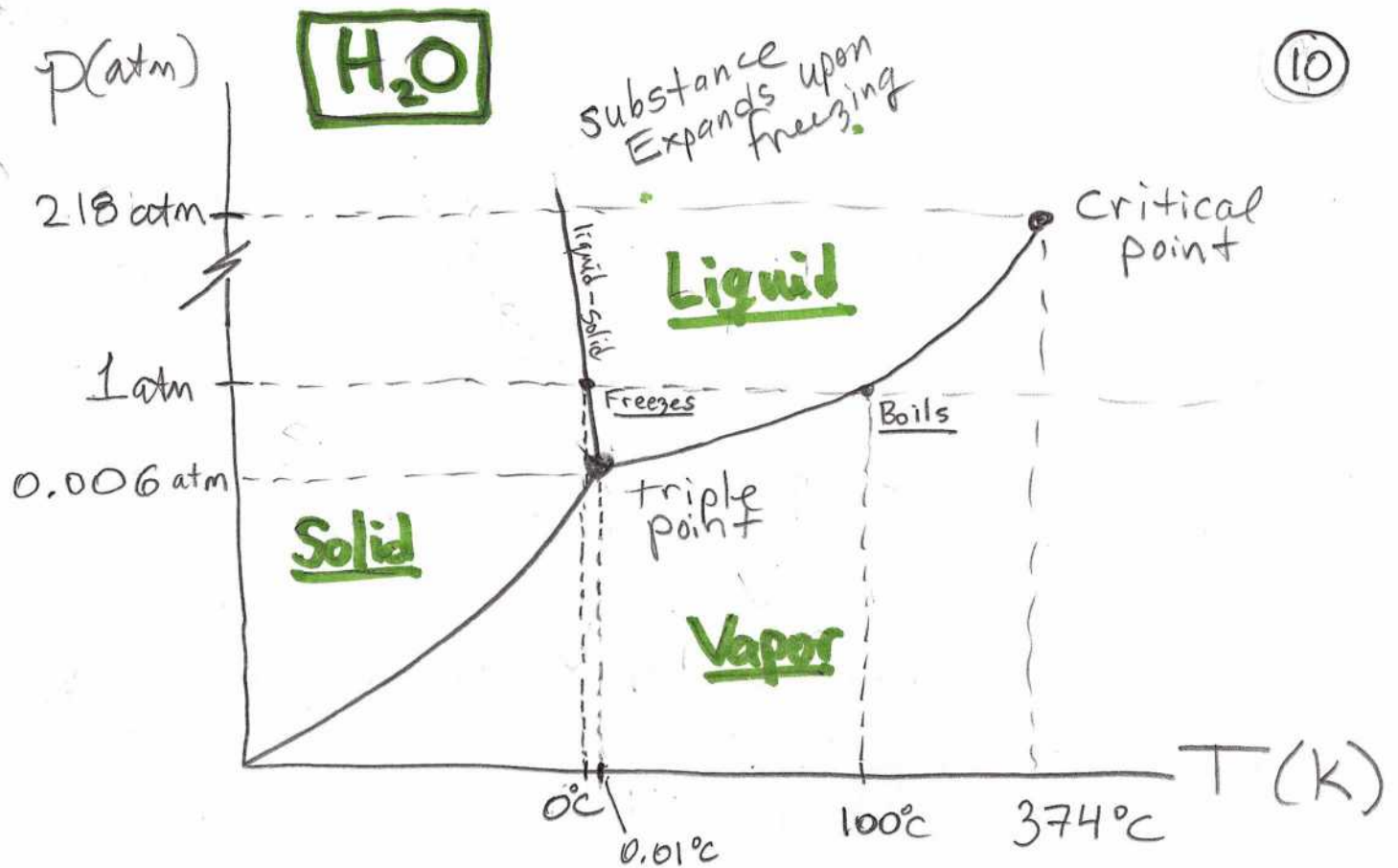
● The Phase Surface Diagrams shown here are usually represented this way:



Notice: CO_2 can only exist as a Solid at 1 atm when $T < -78.5^\circ\text{C}$

⇒ A process of Sublimation (Phase change from solid directly to Vapor) occurs at 1 atm, $T_{\text{surr}} > -78.5^\circ\text{C}$

⇒ This is why it is called "Dry Ice"

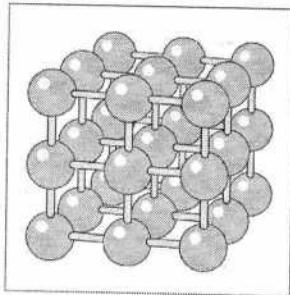


Note: We can "control" the boiling temperature of H_2O (or any simple compressible substance) simply by "controlling" the pressure! 😊

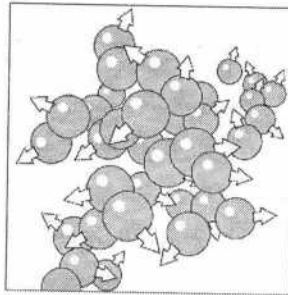
⇒ Applications for Steam Generators (Boilers)
 " " Refrigeration systems

Note: An Astronaut in space, without a space-suit, will experience problems ⇒ Body Water will immediately Boil with decompression! 😞

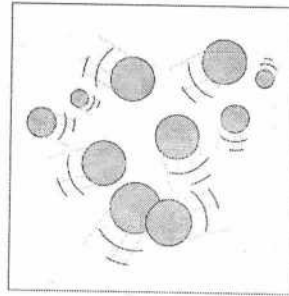
Lecture 8 | Phase Change Processes of Pure, Simple, Compressible Substances



(a)



(b)



(c)

Solid

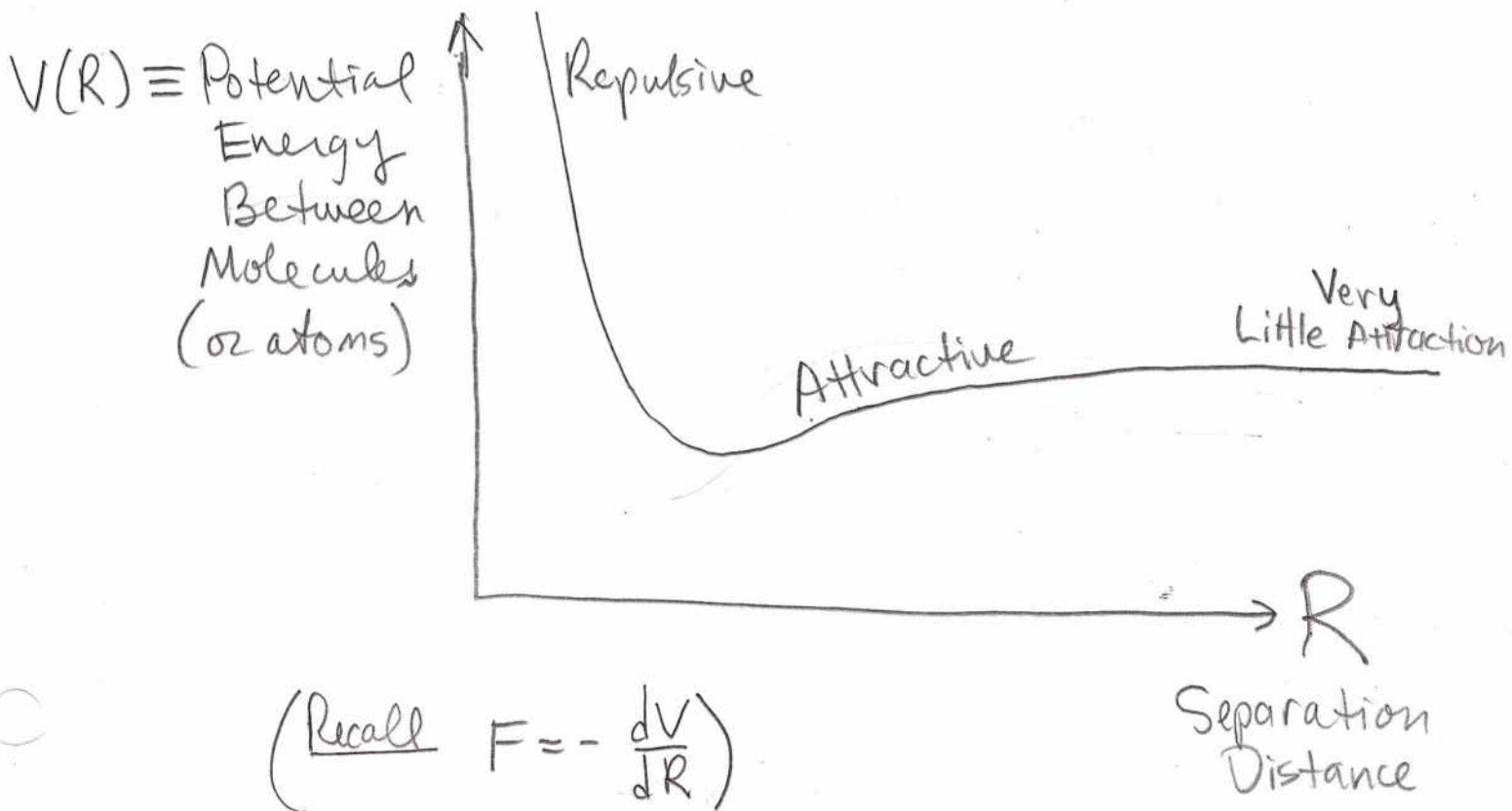
Attractive and Repulsive Forces Large

Liquid

Attractive & Repulsive forces Relatively large

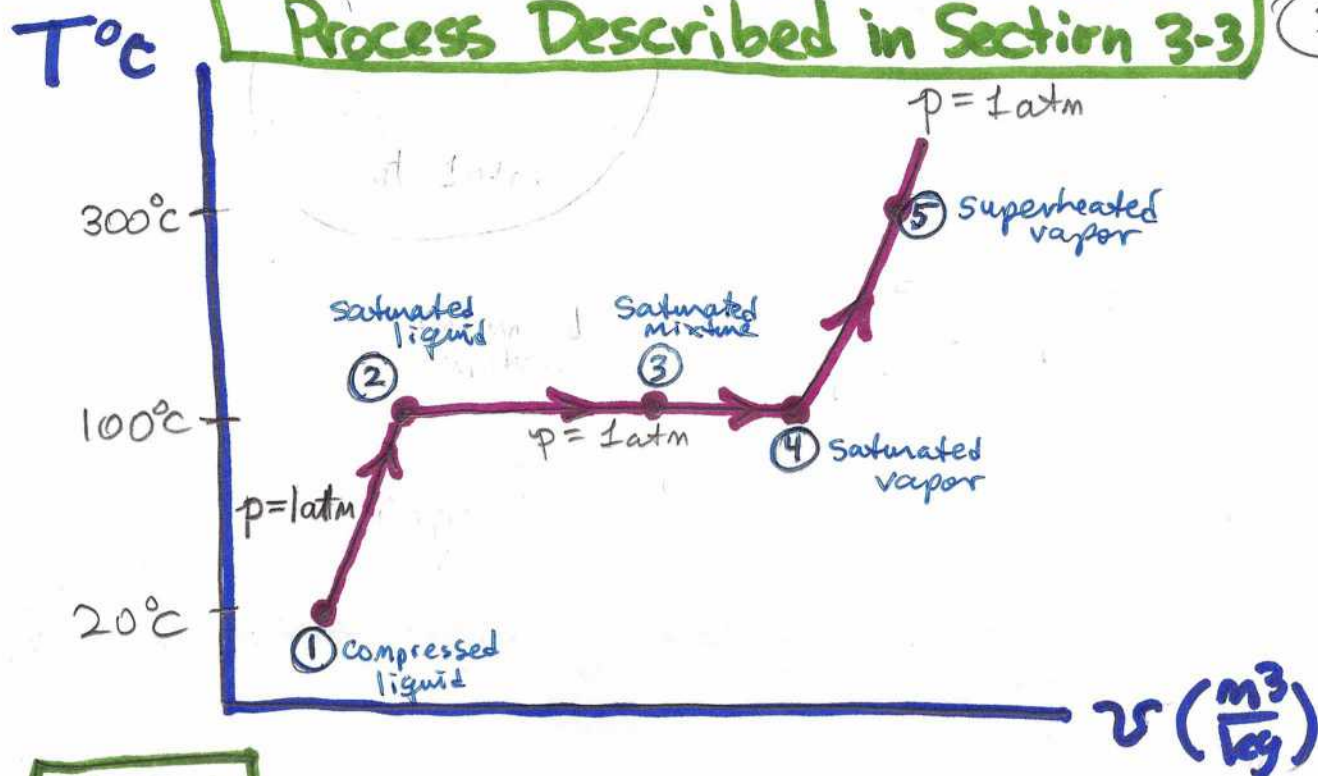
Gas

At very large separations, intermolecular forces are weak, except when they collide



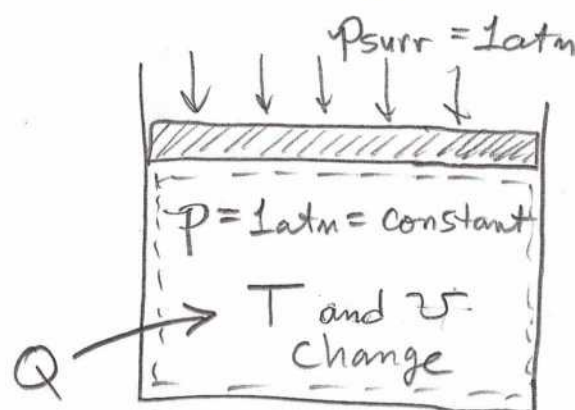
Consider the Constant-Pressure Process Described in Section 3-3

(2)



State 1

"Compressed Liquid" State
or "Subcooled Liquid"
(means that it is not about to vaporize)



State 2

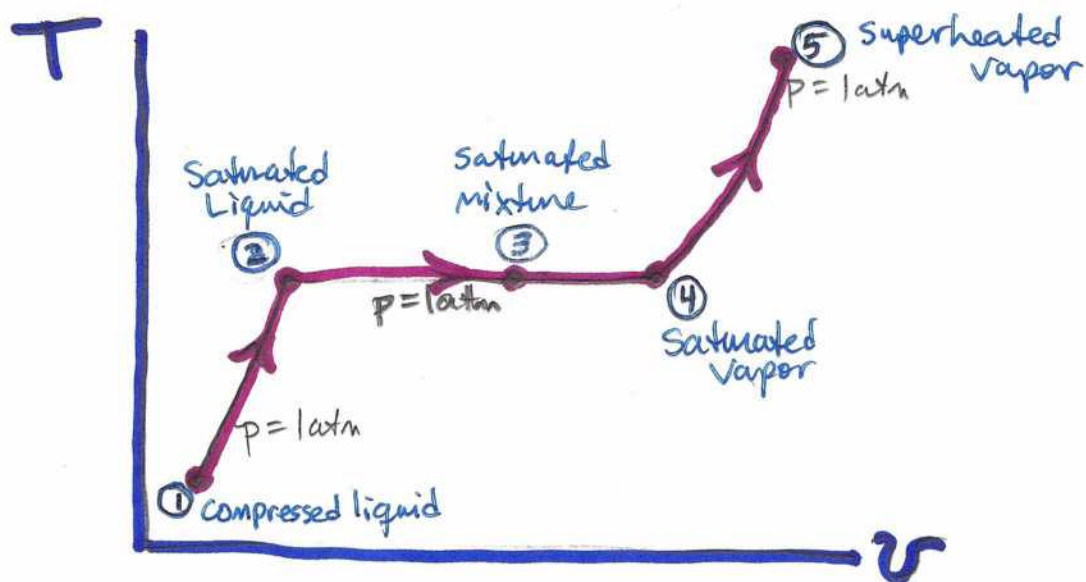
The liquid is on the verge of vaporizing

"Saturated Liquid State"

State 3

Adding more heat, Q , the liquid begins to boil, meaning there are two phases present:

(1) liquid } "Saturated Liquid-Vapor"
(2) vapor } or "Saturated Mixture"



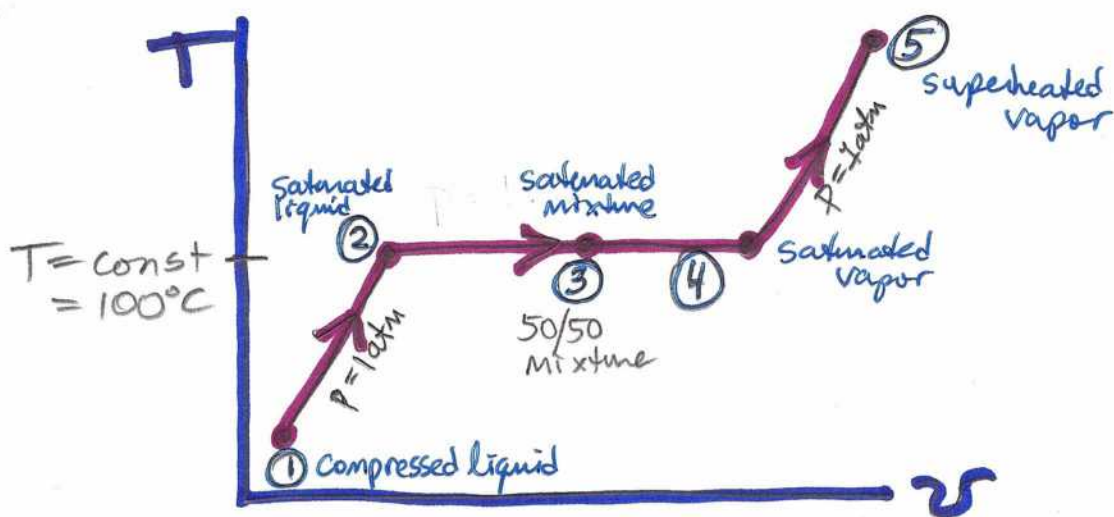
③

State 3, Continued

- Note that $T = \text{constant}$ from ② \rightarrow ④ because all the heat addition, Q , is being used to "loosen" the intermolecular forces of the liquid phase component.
- From ② \rightarrow ④, the Volume (or v) still changes as more vapor is created, while liquid disappears.
- State 3 is the "midpoint" where there is equal amounts of liquid and vapor in the cylinder.

State 4 "Saturated Vapor" State

- The Entire cylinder is filled with vapor
- Any heat transfer out will allow the vapor to condense back to liquid



State 4, continued

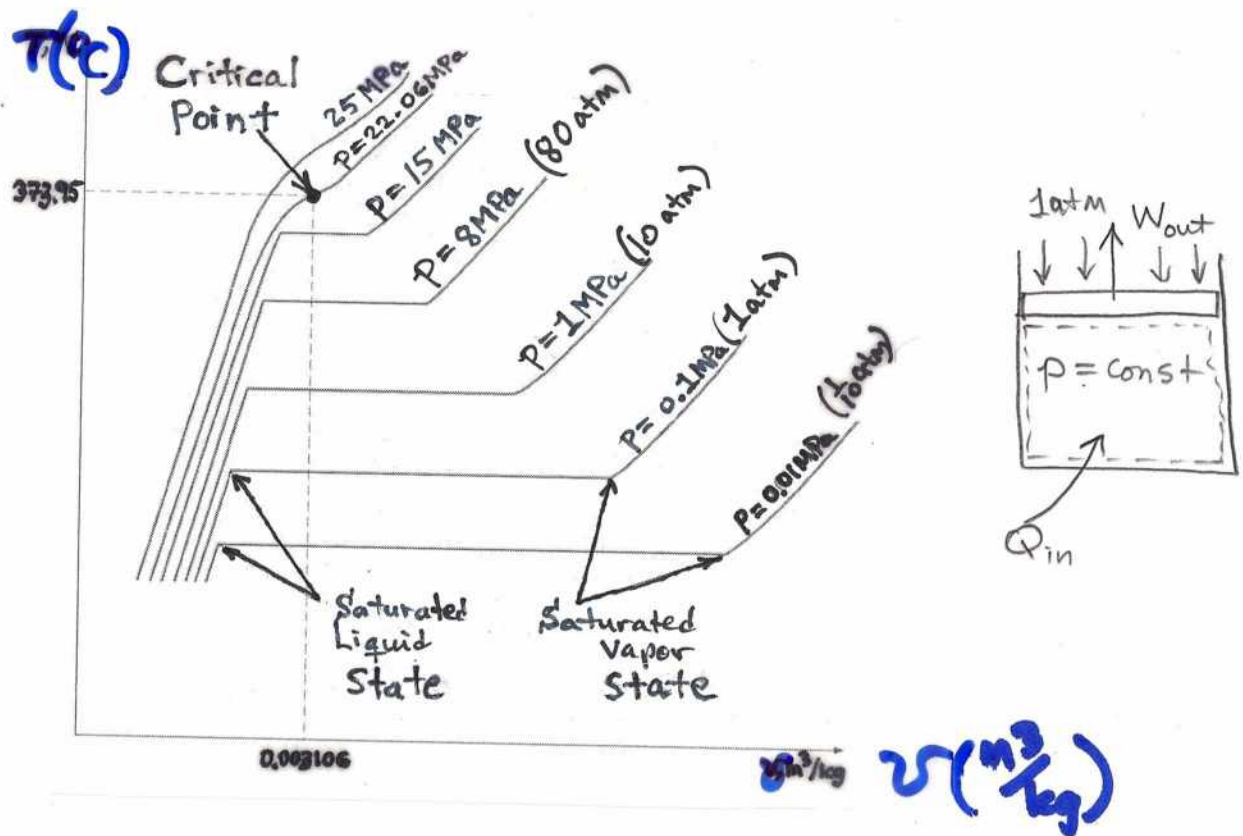
- A vapor that is just on the verge of condensing is called a "Saturated vapor"
- State 4 is a "Saturated Vapor State" since the liquid and vapor phases coexist in equilibrium here.

State 5 "Superheated vapor" state

- Once the phase change process is complete, and more heat Q is added, the constant pressure process takes us through single phase (vapor) states.
- A vapor that is not about to condense is called a "Superheated vapor"
- Water at State 5 is a "Superheated Vapor State"

(5)

If we repeat this constant pressure process "experiment" for other different pressures, say from 0.01 MPa to 25 MPa, we begin to see a pattern form on a T - v diagram:



(1) Water begins to boil (Saturated liquid State) at higher temperatures as the pressure increases.

(2) As pressure increases, the specific volume v increases at the saturated liquid state

(3) As pressure increases, the specific volume v decreases at the saturated vapor state

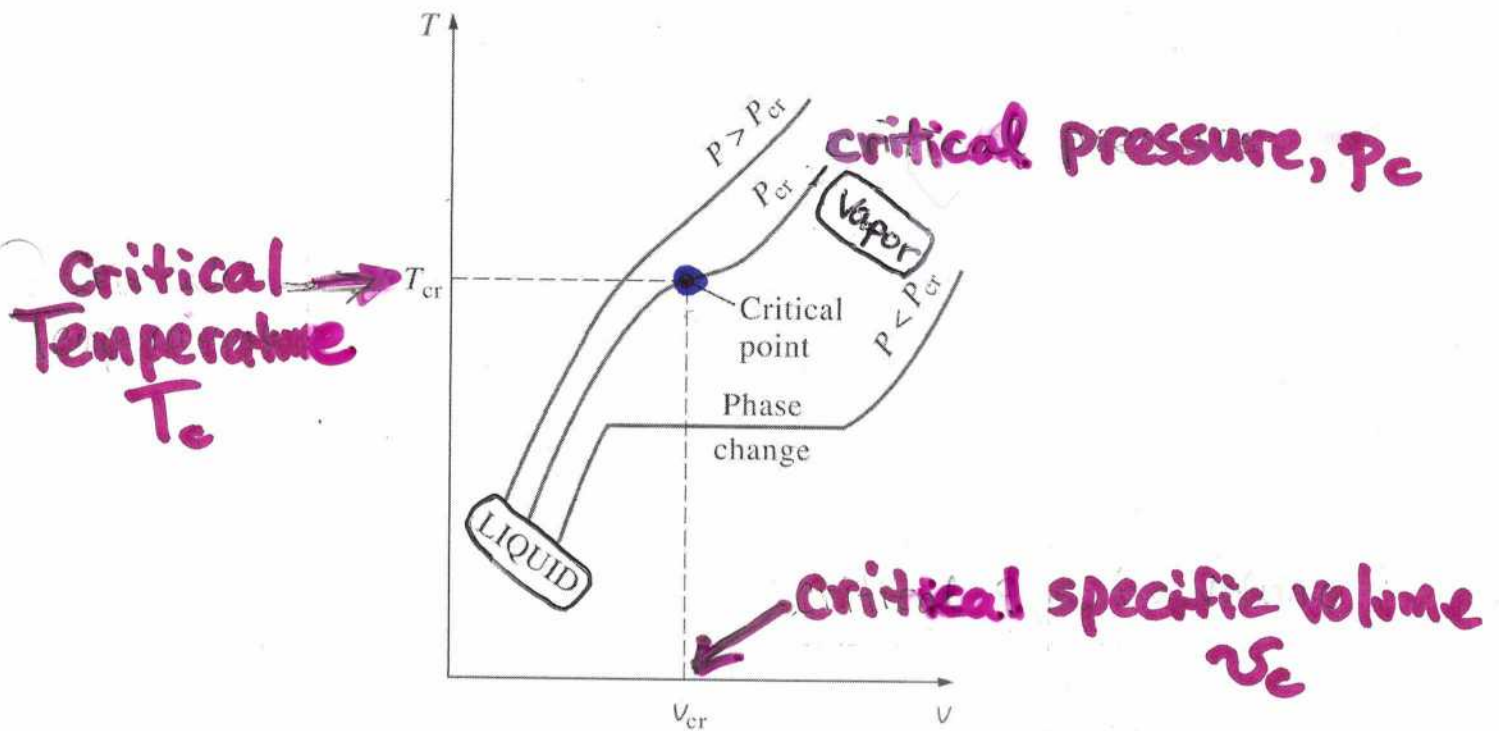
[Note: As p increases, the horizontal saturated mixture line gets shorter]

⑥

- As p increases, the length of the horizontal line decreases

$$\left(\Delta v = v_{\text{sat vapor}} - v_{\text{sat liquid}} \text{ gets smaller} \right)$$

- When the length of the horizontal line, Δv , becomes zero (a point), a special state occurs called the **"Critical Point"**



Critical Point

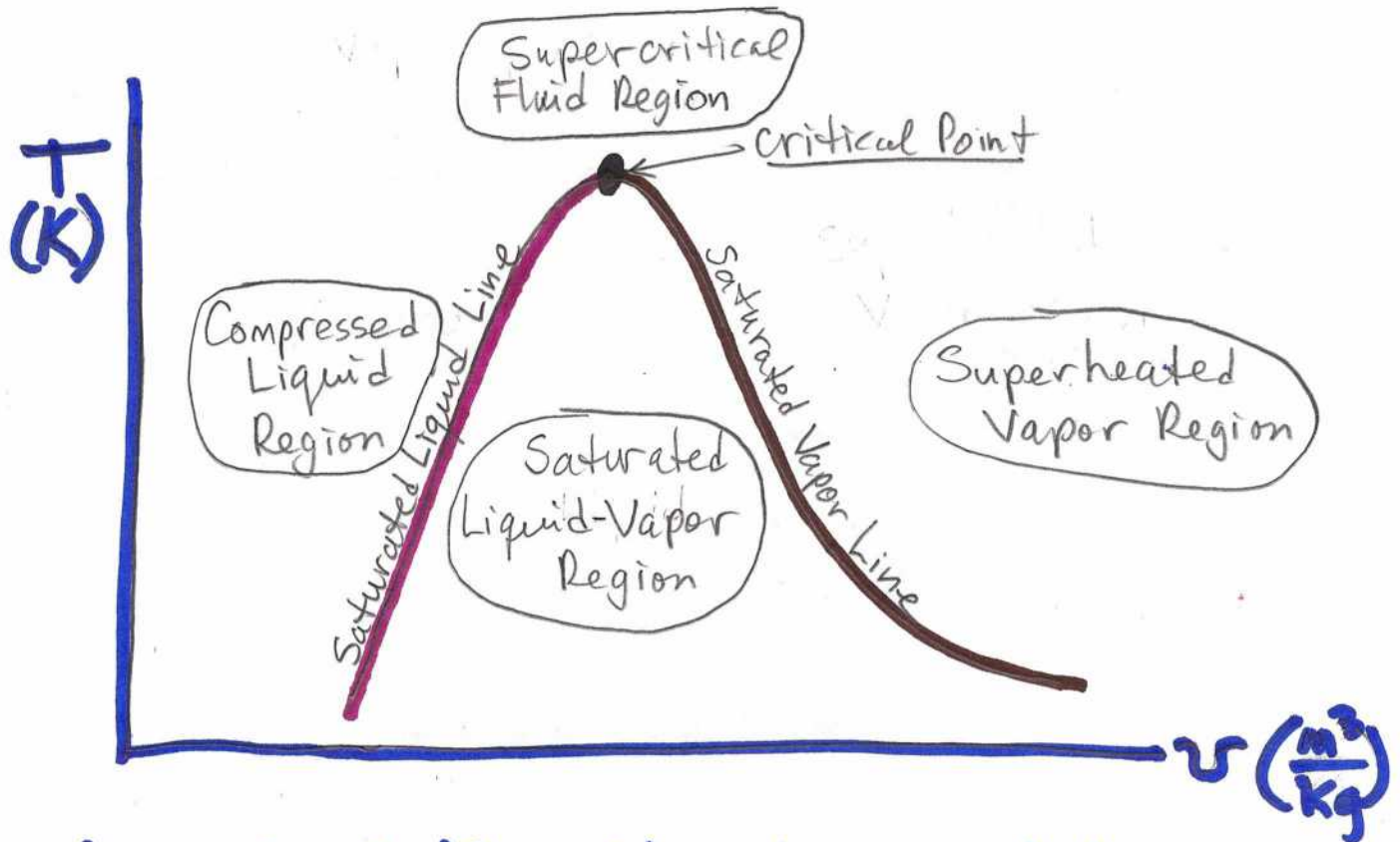
The state at which the saturated liquid and saturated vapor states are identical

\Rightarrow the phase of matter is neither truly liquid or vapor!

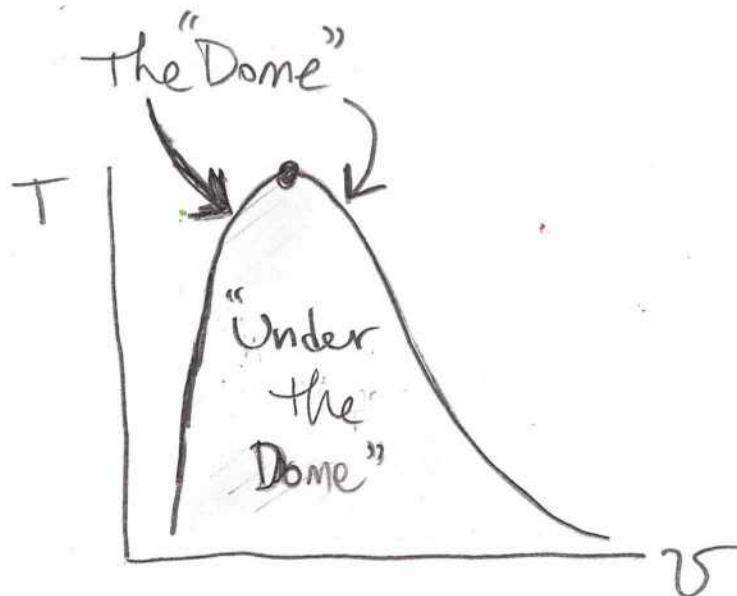
Important:

(7)

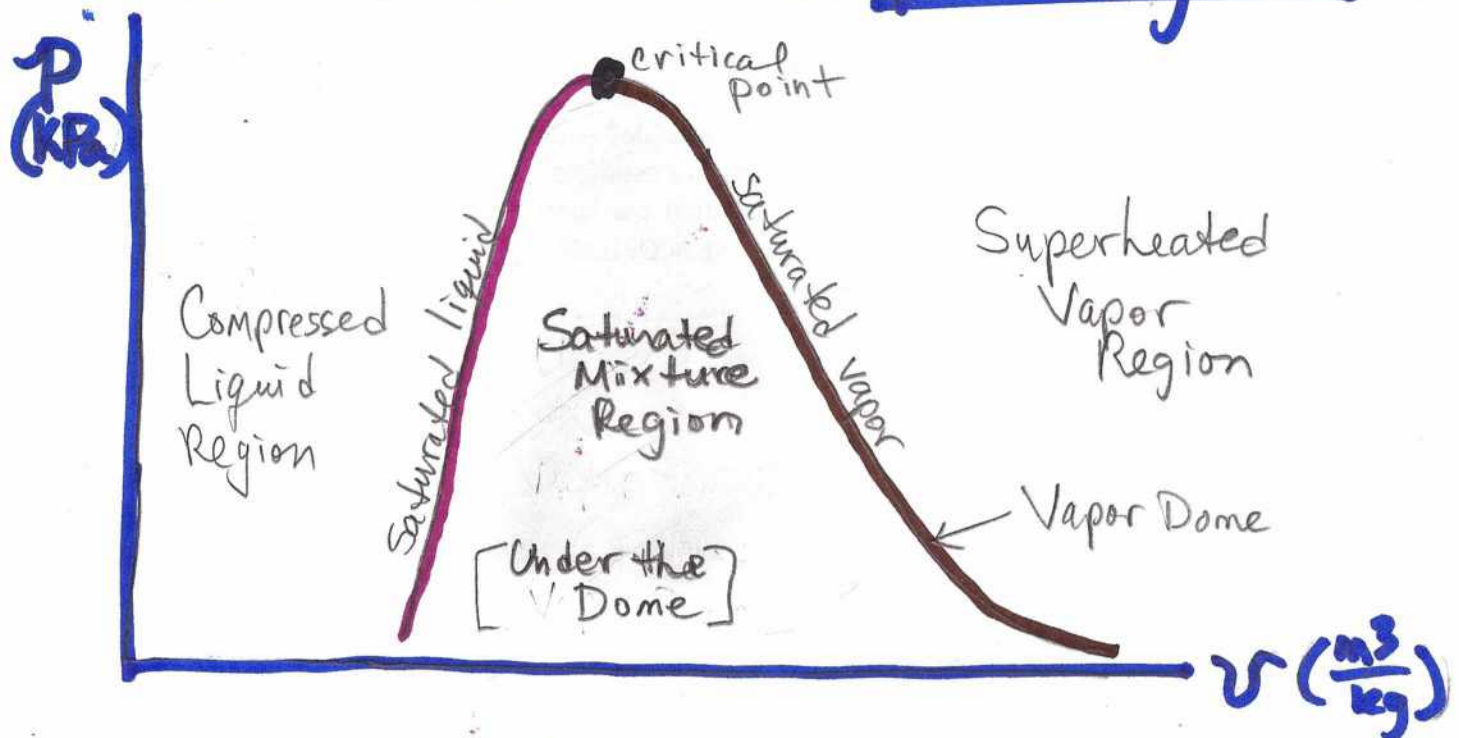
Notice that we can connect all the saturated liquid & saturated vapor states by a single line up to the critical point:



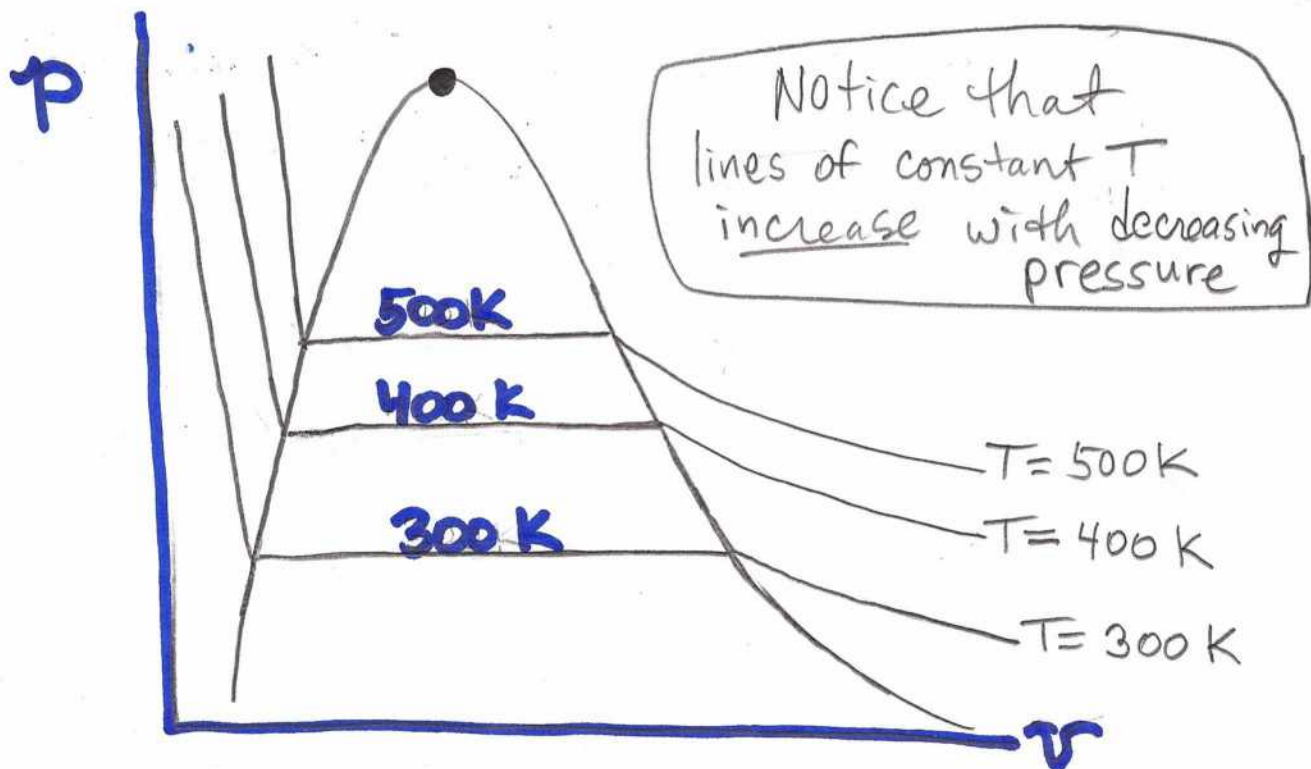
- The Saturated Liquid and saturated vapor lines form a "Dome", so it is commonly called "The Dome"



- A third way of looking at the p - v - T surface is called a p - v diagram:



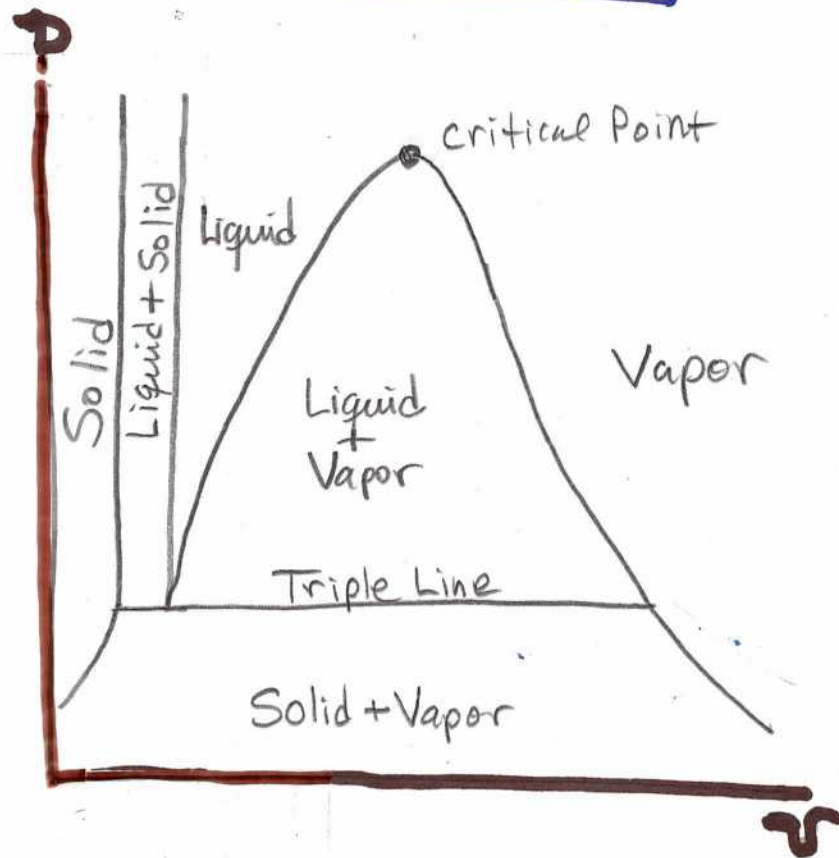
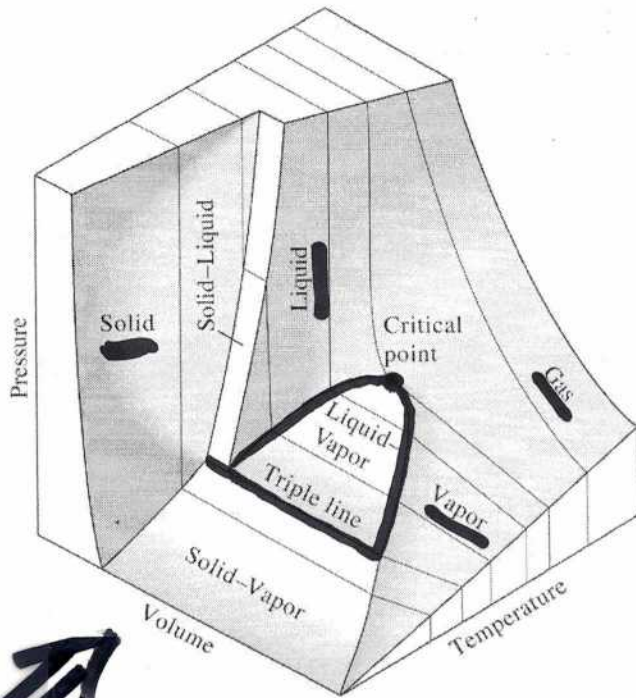
- On a p - v diagram we are interested in lines of constant temperature:



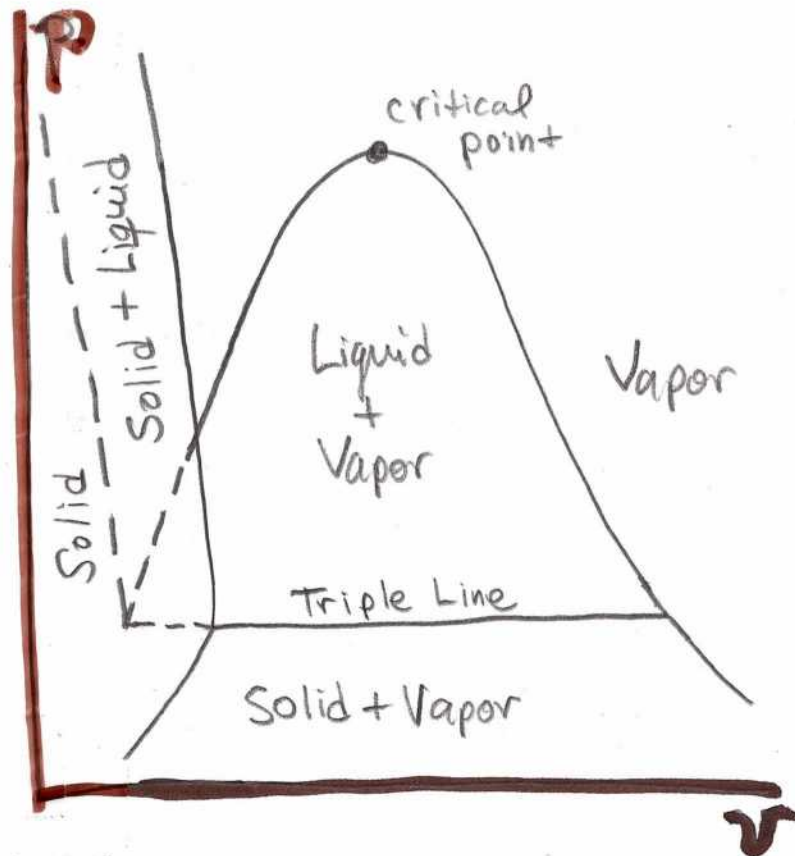
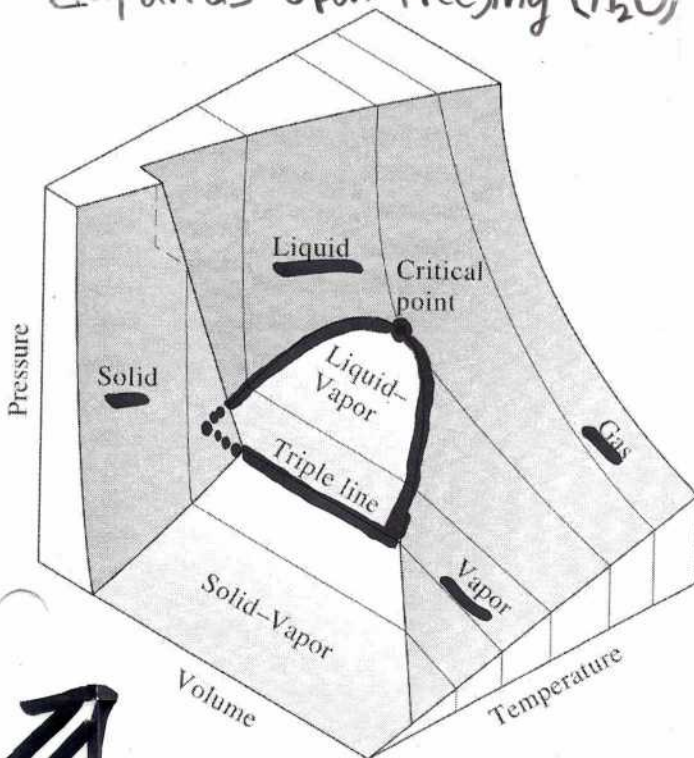
Solid, Liquid, & Vapor Regions on a P-V Diagram

9

Contracts Upon Freezing (CO_2)



Expands Upon Freezing (H_2O)



Lecture 9

Introduction to the Property Tables and Enthalpy & Entropy

- In the previous lecture we discussed the general idea of a simple compressible substance and how the properties p , v , and T are related to each other, called an "Equation of State"
- To describe the state of a system comprised of a simple, compressible substance, all you need are any two intensive properties. For example:

$$(1) p = p(T, v)$$

$$(2) T = T(p, v)$$

$$(3) v = v(p, T) \text{ etc.}$$

These are examples of an "Equation of State".

- What about Internal energy per unit mass (an intensive property)?

$$(1) u = u(p, T) \quad \left(\frac{\text{kJ}}{\text{kg}}\right) \text{ or } \left(\frac{\text{Btu}}{\text{lb}_m}\right)$$

$$(2) u = u(T, v)$$

$$(3) u = u(p, v)$$

⇒ This is also an "Equation of State" for the same simple compressible system

Enthalpy

Enthalpy is your Friend!

Enthalpy is given the symbol H (kJ) or h ($\frac{\text{kJ}}{\text{kg}}$) and is defined as

$$(1) \quad H \equiv U + pV \quad (\text{kJ}) \text{ or } (\text{Btu})$$

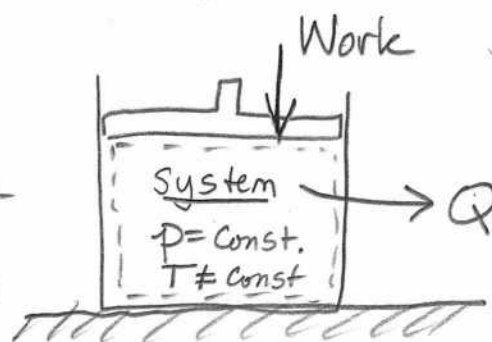
$$(2) \quad h \equiv u + pv \quad \left(\frac{\text{kJ}}{\text{kg}}\right) \text{ or } \left(\frac{\text{Btu}}{\text{lb}_m}\right)$$

- The combination of $u + pv$ actually simplifies your life when dealing with problems involving:

(1) Constant pressure processes:

Example:

Closed
piston-
cylinder
System



(2) Open Systems with mass flow in and out

$\Rightarrow u$ and pv both vary, so it is much simpler to keep track of one property instead of three!



Entropy

3

Entropy can be viewed as a measure of molecular disorder within a System and/or Surrounding environment, and is given the symbols

(1) "Big S" $\left(\frac{\text{kJ}}{\text{K}}\right)$ or $\left(\frac{\text{Btu}}{\text{R}}\right)$

(2) "little s" $\left(\frac{\text{kJ}}{\text{kg-K}}\right)$ or $\left(\frac{\text{Btu}}{\text{lb}_m\text{-R}}\right)$

- Don't worry about the physical meaning of Entropy right now \rightarrow we will deal with it later.
- Right now, just treat it as another property that is listed in the tables and is computed from the tables just like u and h.

Bottom Line: Even a Simple compressible System has an Equation of State too complicated to be described by a single, analytic equation, so we need to use property Tables listed in Appendix 1 & 2 of your text. These Tables allow you to compute u, h, s, p, T, and v given any two of these intensive properties.

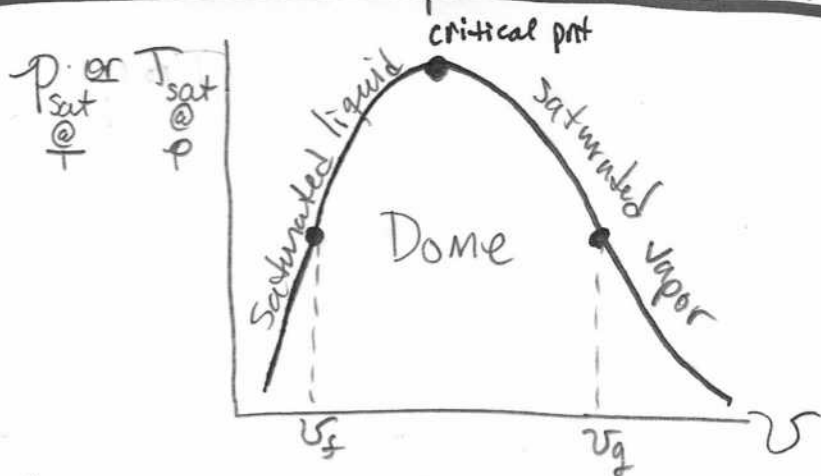
Examples: $u = u(p, T)$, $h = h(T, v)$, $s = s(T, p)$, $p = p(T, v)$

Property Tables for H_2O and R-134a

(4)

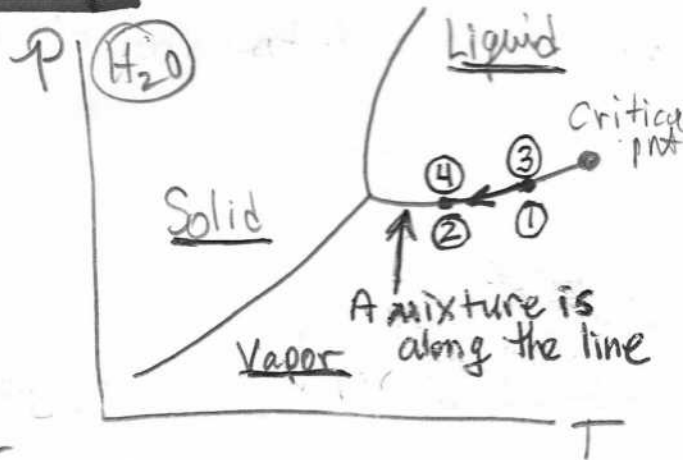
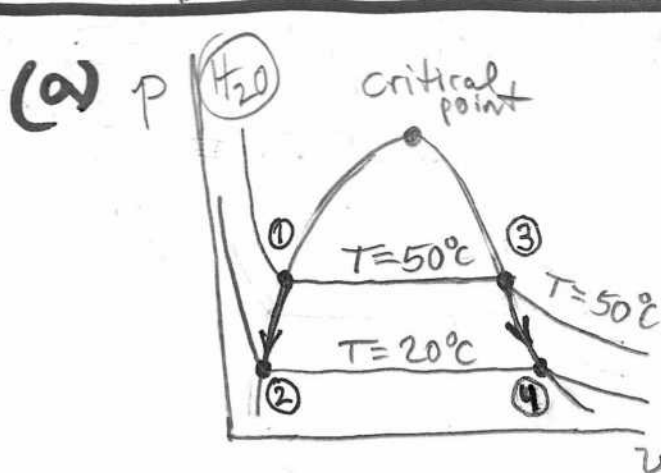
These notes focus on H_2O , but the concepts are exactly the same for R-134a.

(I) Tables A-4 (SI) and A-4E (English) list the Equation of State for saturated liquid and saturated vapor states only.



(In other words, for the line defining the Dome)

Two Important Observations:



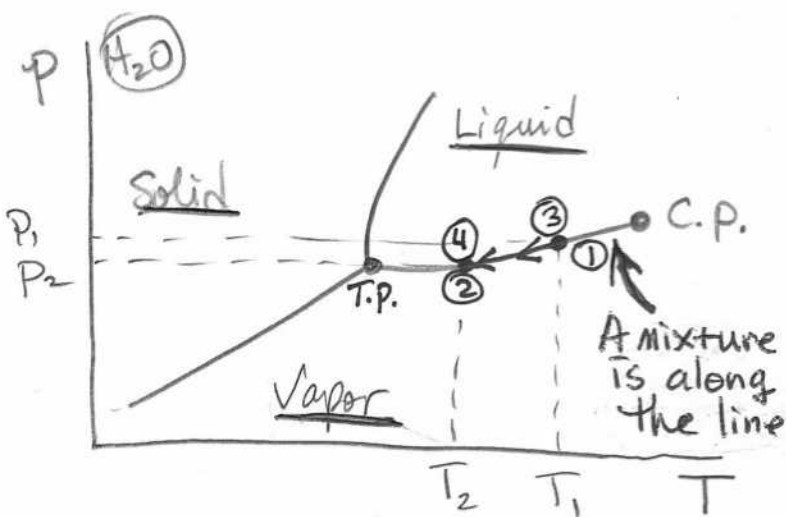
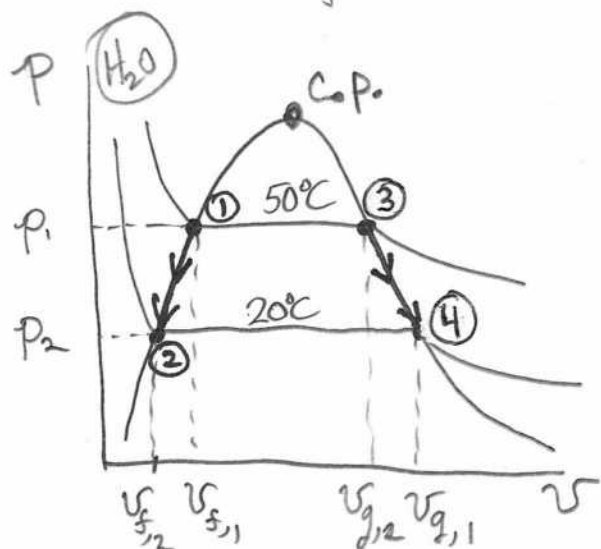
- From Table A-4, let State ① be defined by

$$T_1 = 50^\circ\text{C}, \quad p_1 = 12.352 \text{ kPa}$$

$\text{@ } 12 \text{ kPa}$ $\text{@ } 50^\circ\text{C}$

Note: state ③ has the same T_{sat} & p_{sat} but $v_1 \neq v_3$

- Using a quasi-equilibrium process, let us change the System properties so that it evolves from State ① to state ② shown in the p - v and p - T diagrams:



- On the p - T diagram, we move along the path of the vaporization line — the line separating liquid and vapor (or gas) states.
- On the p - v diagram, this is represented by both paths on either side of the Dome!
- For a simple compressible System, we know that $p = p(T, v)$
- But, moving along this special line means that these are saturation property values $p_{sat@T}$ and $T_{sat@p}$.

- Important! Notice that any horizontal line (6)
across the Dome has the same value of P_{sat} and T_{sat} , including the saturation lines!

⇒ Looking at the p-T diagram, this means that as we go from States ① → ② and ③ → ④, pressure is a function of temperature only within and on the Dome!

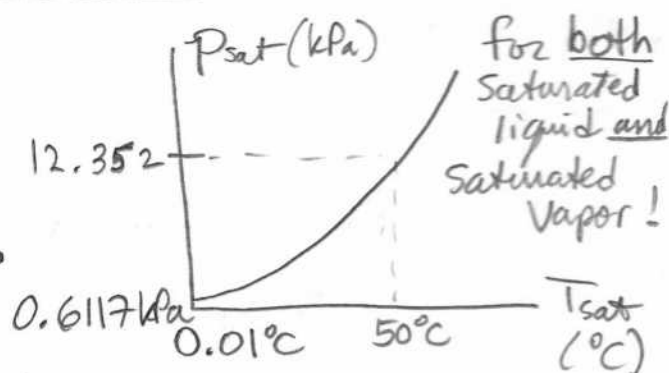
⇒ $P_{sat} = P_{sat}(T_{sat})$

⇒ Look at Table A-4 & A-5

Notice there are only two columns

for P_{sat} & T_{sat} and they have a one-to-one correspondence

⇒ for a given T_{sat} there is only ONE P_{sat}



(b) What about v , u , h , and s ? They are double-valued for a given p & T !

For a simple compressible substance,

$$v = v(p, T).$$

Along the saturated liquid line (left Dome line),

$$v_f = v_f(P_{sat}, T_{sat})$$

$$u_f = u_f(P_{sat}, T_{sat})$$

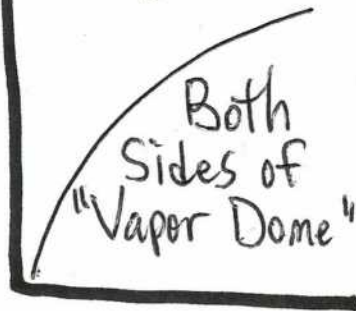
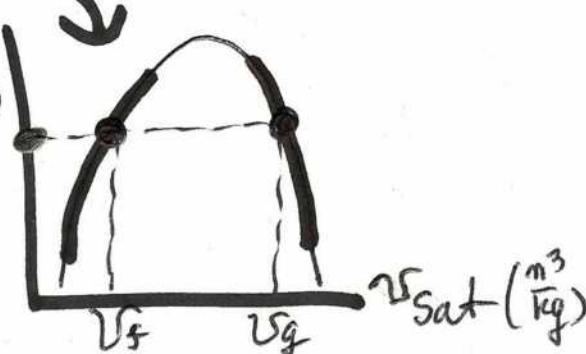
$$h_f = h_f(P_{sat}, T_{sat})$$

$$s_f = s_f(P_{sat}, T_{sat})$$

TABLE A-4

Saturated water—Temperature table

Temp., T , °C	Sat. press., P_{sat} , kPa	Specific volume, m^3/kg		Internal energy, kJ/kg			Enthalpy, kJ/kg			Entropy, kJ/kg·K		
		Sat. liquid, v_f	Sat. vapor, v_g	Sat. liquid, u_f	Evap., u_{fg}	Sat. vapor, u_g	Sat. liquid, h_f	Evap., h_{fg}	Sat. vapor, h_g	Sat. liquid, s_f	Evap., s_{fg}	Sat. vapor, s_g
0.01	0.6117	0.001000	206.00	0.000	2374.9	2374.9	0.001	2500.9	2500.9	0.0000	9.1556	9.1556
5	0.8725	0.001000	147.03	21.019	2360.8	2381.8	21.020	2489.1	2510.1	0.0763	8.9487	9.0249
10	1.2281	0.001000	106.32	42.020	2346.6	2388.7	42.022	2477.2	2519.2	0.1511	8.7488	8.8999
15	1.7057	0.001001	77.885	62.980	2332.5	2395.5	62.982	2465.4	2528.3	0.2245	8.5559	8.7803
20	2.3392	0.001002	57.762	83.913	2318.4	2402.3	83.915	2453.5	2537.4	0.2965	8.3696	8.6661
25	3.1698	0.001003	43.340	104.83	2304.3	2409.1	104.83	2441.7	2546.5	0.3672	8.1895	8.5567
30	4.2469	0.001004	32.879	125.73	2290.2	2415.9	125.74	2429.8	2555.6	0.4368	8.0152	8.4520
35	5.6291	0.001006	25.205	146.63	2276.0	2422.7	146.64	2417.9	2564.6	0.5051	7.8466	8.3517
40	7.3851	0.001008	19.515	167.53	2261.9	2429.4	167.53	2406.0	2573.5	0.5724	7.6832	8.2556
45	9.5953	0.001010	15.251	188.43	2247.7	2436.1	188.44	2394.0	2582.4	0.6386	7.5247	8.1633
50	12.352	0.001012	12.026	209.33	2233.4	2442.7	209.34	2382.0	2591.3	0.7038	7.3710	8.0748
55	15.763	0.001015	9.5639	230.24	2219.1	2449.3	230.26	2369.8	2600.1	0.7680	7.2218	7.9898
60	19.947	0.001017	7.6670	251.16	2204.7	2455.9	251.18	2357.7	2608.8	0.8313	7.0769	7.9082
65	25.043	0.001020	6.1935	272.09	2190.3	2462.4	272.12	2345.4	2617.5	0.8937	6.9360	7.8296
70	31.202	0.001023	5.0396	293.04	2175.8	2468.9	293.07	2333.0	2626.1	0.9551	6.7989	7.7540
75	38.597	0.001026	4.1291	313.99	2161.3	2475.3	313.03	2320.6	2634.6	1.0158	6.6655	7.6812
80	47.416	0.001029	3.4053	334.97	2146.6	2481.6	335.02	2308.0	2643.0	1.0756	6.5355	7.6111
85	57.868	0.001032	2.8261	355.96	2131.9	2487.8	356.02	2295.3	2651.4	1.1346	6.4089	7.5435
90	70.183	0.001036	2.3593	376.97	2117.0	2494.0	377.04	2282.5	2659.6	1.1929	6.2853	7.4782
95	84.609	0.001040	1.9808	398.00	2102.0	2500.1	398.09	2269.6	2667.6	1.2504	6.1647	7.4151
100	101.42	0.001043	1.6720	419.06	2087.0	2506.0	419.17	2256.4	2675.6	1.3072	6.0470	7.3542
105	120.90	0.001047	1.4186	440.15	2071.8	2511.9	440.28	2243.1	2683.4	1.3634	5.9319	7.2952
110	143.38	0.001052	1.2094	461.27	2056.4	2517.7	461.42	2229.7	2691.1	1.4188	5.8193	7.2382
115	169.18	0.001056	1.0360	482.42	2040.9	2523.3	482.59	2216.0	2698.6	1.4737	5.7092	7.1829
120	198.67	0.001060	0.89133	503.60	2025.3	2528.9	503.81	2202.1	2706.0	1.5279	5.6013	7.1292
125	232.23	0.001065	0.77012	524.83	2009.5	2534.3	525.07	2188.1	2713.1	1.5816	5.4956	7.0771
130	270.28	0.001070	0.66808	546.10	1993.4	2539.5	546.38	2173.7	2720.1	1.6346	5.3919	7.0265
135	313.22	0.001075	0.58179	567.41	1977.3	2544.7	567.75	2159.1	2726.9	1.6872	5.2901	6.9773
140	361.53	0.001080	0.50850	588.77	1960.9	2549.6	589.16	2144.3	2733.5	1.7392	5.1901	6.9294
145	415.68	0.001085	0.44600	610.19	1944.2	2554.4	610.64	2129.2	2739.8	1.7908	5.0919	6.8827
150	476.16	0.001091	0.39248	631.66	1927.4	2559.1	632.18	2113.8	2745.9	1.8418	4.9953	6.8371
155	543.49	0.001096	0.34648	653.19	1910.3	2563.5	653.79	2098.0	2751.8	1.8924	4.9002	6.7927
160	618.23	0.001102	0.30680	674.79	1893.0	2567.8	675.47	2082.0	2757.5	1.9426	4.8066	6.7492
165	700.93	0.001108	0.27244	696.46	1875.4	2571.9	697.24	2065.6	2762.8	1.9923	4.7143	6.7067
170	792.18	0.001114	0.24260	718.20	1857.5	2575.7	719.08	2048.8	2767.9	2.0417	4.6233	6.6650
175	892.60	0.001121	0.21659	740.02	1839.4	2579.4	741.02	2031.7	2772.7	2.0906	4.5335	6.6242
180	1002.8	0.001127	0.19384	761.92	1820.9	2582.8	763.05	2014.2	2777.2	2.1392	4.4448	6.5841
185	1123.5	0.001134	0.17390	783.91	1802.1	2586.0	785.19	1996.2	2781.4	2.1875	4.3572	6.5447
190	1255.2	0.001141	0.15636	806.00	1783.0	2589.0	807.43	1977.9	2785.3	2.2355	4.2705	6.5059
195	1398.8	0.001149	0.14089	828.18	1763.6	2591.7	829.78	1959.0	2788.8	2.2831	4.1847	6.4678
200	1554.9	0.001157	0.12721	850.46	1743.7	2594.2	852.26	1939.8	2792.0	2.3305	4.0997	6.4302

 T_{sat}
(°C) P_{sat} (kPa) T_{sat}
(°C) v_{sat} (m^3/kg)

(7)

and along the saturated vapor line (right Dome line),

$$v_g = v_g(p_{\text{sat}}, T_{\text{sat}}) \quad u_g = u_g(p_{\text{sat}}, T_{\text{sat}})$$

$$h_g = h_g(p_{\text{sat}}, T_{\text{sat}}) \quad s_g = s_g(p_{\text{sat}}, T_{\text{sat}})$$

Let's just work with v_g for now, substituting the functionality of p_{sat} & T_{sat} :

$$v_g = v_g(p_{\text{sat}}, T_{\text{sat}}), \text{ but } T_{\text{sat}} = \text{function}(p_{\text{sat}})$$

$$\Rightarrow \boxed{v_g = v_g(p_{\text{sat}} @ T_{\text{sat}})}$$

[Table A-5 for convenient
Tabular whole number values of
 p_{sat}

or by similar argument,

$$\boxed{v_g = v_g(T_{\text{sat}} @ p_{\text{sat}})}$$

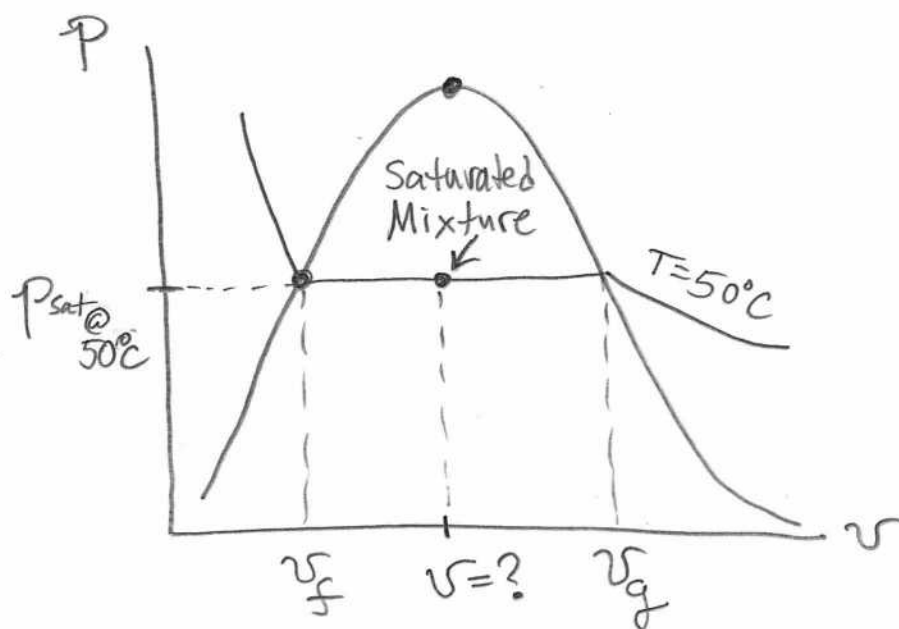
[Table A-4 for convenient
tabulation of whole number
values of T_{sat}

Also,
$$\boxed{\begin{aligned} v_f &= v_f(p_{\text{sat}} @ T_{\text{sat}}) \\ &= v_f(T_{\text{sat}} @ p_{\text{sat}}) \end{aligned}}$$

Listed in Tables A-4/A-5
and A-4E/A-5E

Q: Ok, this is great Dr. Bob, but what about computing v under the Dome for a mixture of liquid and vapor phases?!

A: This state corresponds to a two-phase saturated mixture (boiling) and we need more information,



Specifically about the proportion of the mass of vapor to that of the total mixture mass.

We define a new Property called "Quality"

Quality

The "Quality" of a saturated mixture is defined as

$$x \equiv \frac{m_{\text{vapor}}}{m_{\text{total}}}$$

and is only valid on or within the Dome so that $0 \leq x \leq 1$ (or, $0\% \leq x \leq 100\%$)

where 0% is pure saturated liquid and 100% is pure saturated vapor.

On page 127 of your textbook, you will see how to compute v for a mixture using the property x :

$$v_{avg} = v_f + x v_{fg}$$

It is also useful to compute the unknown quality, x , given T_{sat} (to determine v_f and v_{fg})

and v_{avg} :

$$x = \frac{v_{avg} - v_f}{v_{fg}}$$

Important!

The other saturated mixture properties are computed the same way:

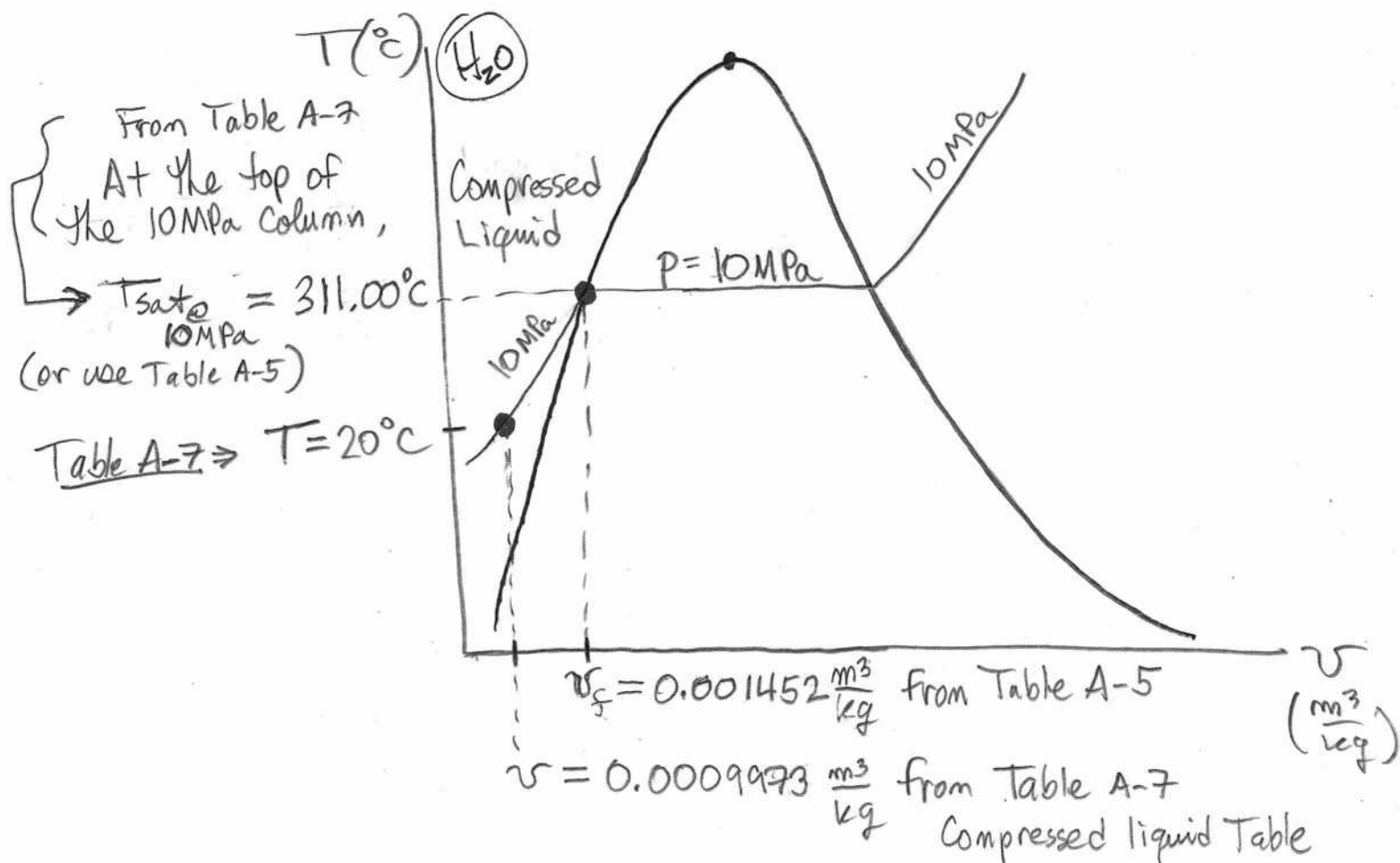
$$u_{avg} = u_f + x u_{fg}$$

$$h_{avg} = h_f + x h_{fg}$$

$$s_{avg} = s_f + x s_{fg}$$

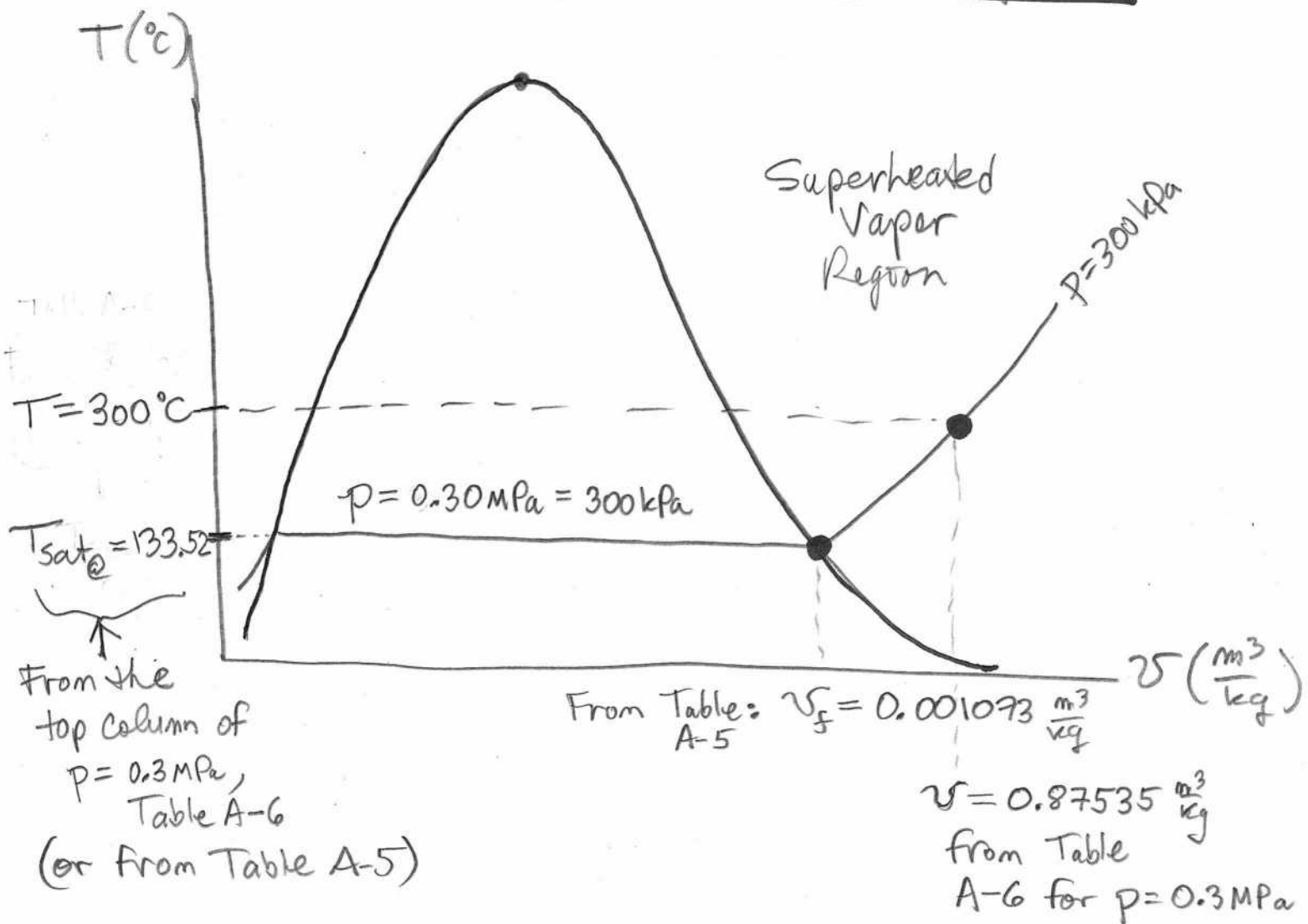
Notice that these are average property values, averaged over the entire system.

(II) Tables A-7 (SI) and A-7E (English) give you a limited range of values for the Equation of State for Compressed Liquid H_2O .



- Properties u , h , and s are also obtained from this table A-7.
- Many times the value of p lies in between the columns, as well as the temperature between the rows, so in this case you must obtain u , h , or s using Double Linear Interpolation (see supplemental handout)

(III) Tables A-6 (SI) and A-6E (English) give a very comprehensive tabular Equation of state for "superheated vapor" states of H_2O .



- Again, properties u , h , and s are obtained from Table A-7.
- Also, you may need to use Double-Linear-Interpolation for values of T and p falling in between columns and rows

"Reference" States for H_2O and R-134a

Property Tables

- Look at table A-4 (H_2O) for example, and scan across the first row.

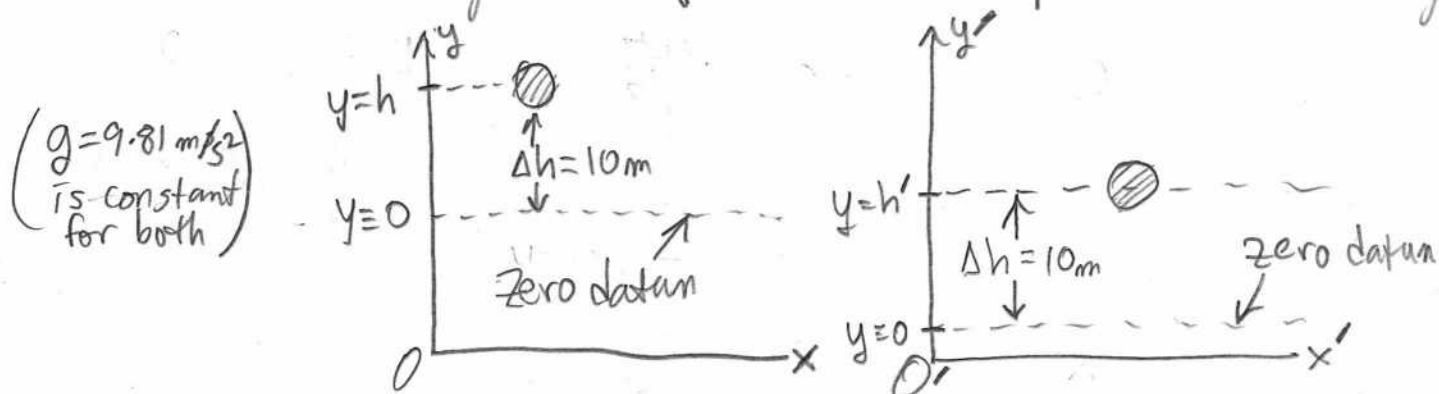
- Notice that for Reference Values

$$\left\{ \begin{array}{l} T_{sat} = 0.01^\circ C \\ P_{sat} = 0.6117 \text{ kPa} \end{array} \right\} \Rightarrow \begin{array}{l} \cdot U_f \equiv 0.000 \text{ kJ/kg} \\ \cdot S_f \equiv 0.0000 \frac{\text{kJ}}{\text{kg-K}} \end{array}$$

\Rightarrow For the saturation values $T_{sat} = 0.01^\circ C$ and $P_{sat} = 0.6117 \text{ kPa}$, the Authors of the text have arbitrarily defined $U_f \equiv 0$, $S_f \equiv 0$

- In Thermodynamics, we are only interested in changes in property values of u , h , and s , so we choose a convenient "zero datum" as a reference state for these properties.

- This is analogous to gravitational potential energy



- All H_2O tables are consistent with this datum..... CAUTION!



Table A-23 is NOT consistent with that zero datum!!

Table A-23 is constructed for Ideal gas (H_2O) Equation of State problems, and Notice that the zero reference datum is $T = 0\text{ K}$, which is $T = -273.15^\circ\text{C}$, NOT $T = 0.01^\circ\text{C}$!!

- The same arbitrary datum condition is found for R-134a (but there is no Ideal gas table for R-134a).

Bottom Line: Be careful when using tabular Equations of State because someone else's (from another textbook, Internet, etc.) may and probably is different than our textbook tables.

⇒ The discrepancy between tables can be resolved if you only use differences in property values.

Lecture 10 Using E.O.S. Tables, The Ideal Gas E.O.S., and the Gas Compressibility Factor, Z .

(I) Compressed Liquid States

(A) Tables

- Tables A-7 and A-7E provide a limited range of values for H_2O .

Example 1

Given : $\begin{cases} H_2O \\ p = 30 \text{ MPa} \\ T = 180^\circ \text{C} \end{cases}$

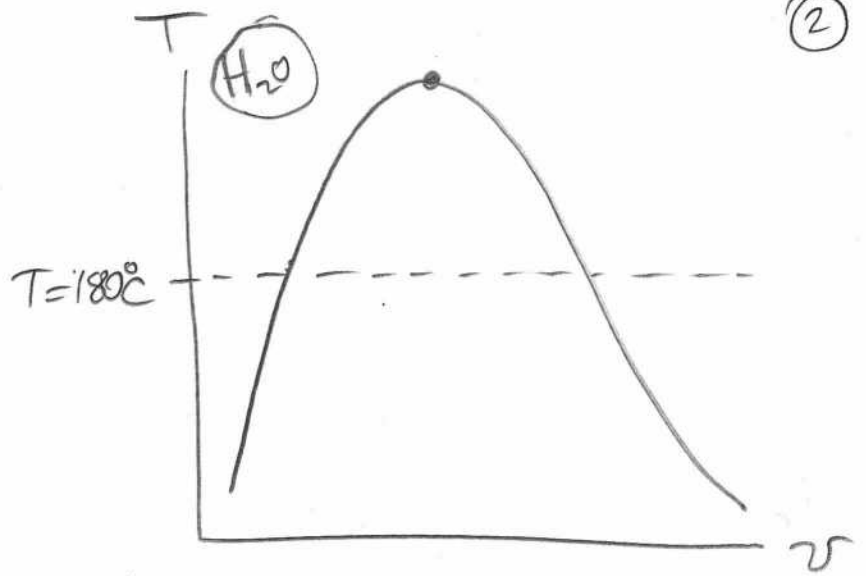
Find (a) Phase
(b) v ($\frac{m^3}{kg}$)
(c) s ($\frac{kJ}{kg \cdot K}$)

(a)

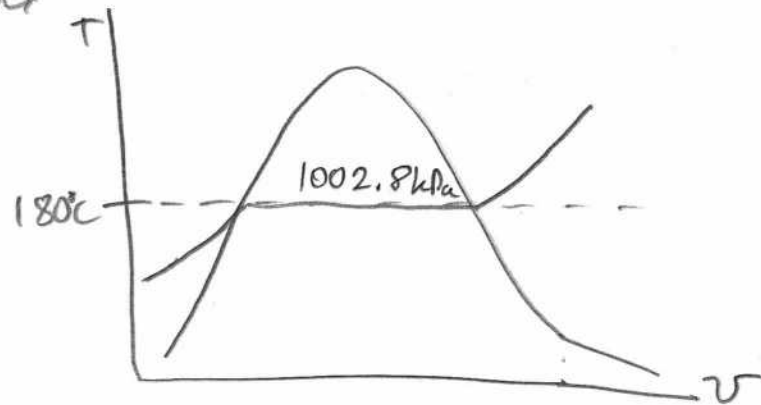
Step 1: We are given $p \neq T$, so I will choose either a $p-v$ or $T-v$ diagram to determine the phase using a graphical technique along with the tables.

I arbitrarily choose a $T-v$ diagram:

Step 2 mark $T = 180^\circ\text{C}$ on the T -axis and draw a dashed line across the diagram.

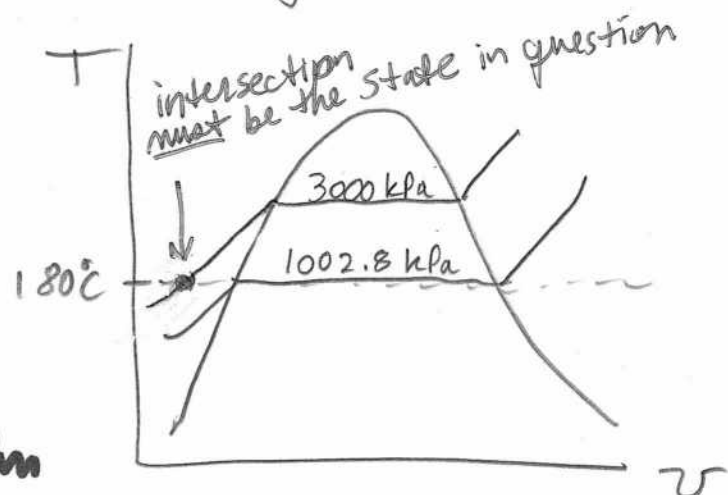


Step 3 Draw a line of constant pressure corresponding to the saturated liq./vap. temperature of $T = 180^\circ\text{C}$



Step 4 Use Table A-4 to look up $P_{\text{sat}@180^\circ\text{C}}$ we find $P_{\text{sat}@180^\circ\text{C}} = 1002.8 \text{ kPa}$, so that is the value along the constant pressure line drawn.

Step 5 We know that $p = 30 \text{ MPa} = 3000 \text{ kPa}$, so the state must lie above the line just drawn on the T - v diagram. Draw that line and ask yourself "Where does the $p = 3000 \text{ kPa}$ and $T = 180^\circ\text{C}$ line intersect?"



⇒ intersection is within the compressed liquid region of the T - v diagram!

⇒ **Phase is Compressed liquid**

(b) • Now go to Table A-7 for Compressed Liquid water and find the conditions $p = 30 \text{ MPa}$, $T = 180^\circ\text{C}$.

(3)

- Luckily, we don't have to use interpolation since the values 30 MPa , 180°C are listed.
- For these T, p we find that

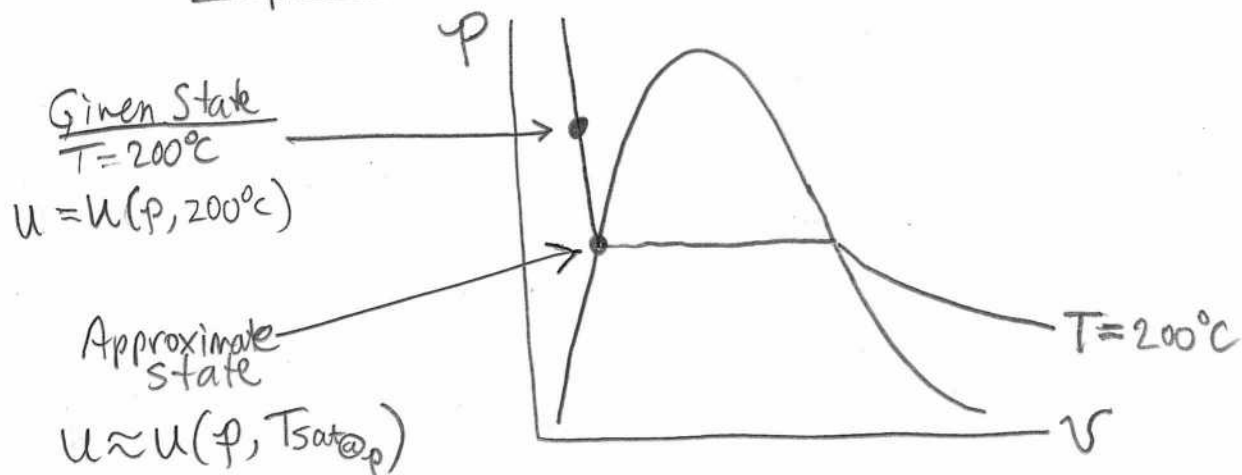
$$v = 0.0011049 \frac{\text{m}^3}{\text{kg}}$$

(c) • In the 3rd column under 30 MPa , we see that the specific entropy s is

$$s = 2.1020 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

(B) Compressed Liquid Approximation

In the absence of compressed liquid property table data (which is usually the case), a general approximation is to treat compressed liquids as a saturated liquid at the given temperature:



(4)

Q: How can we justify this approximation, physically?

A: Because Compressed liquid properties depend on T much more strongly than they do on p

⇒ Of the four intensive properties h, u, s , and v , the Enthalpy h is most sensitive to variations in pressure.

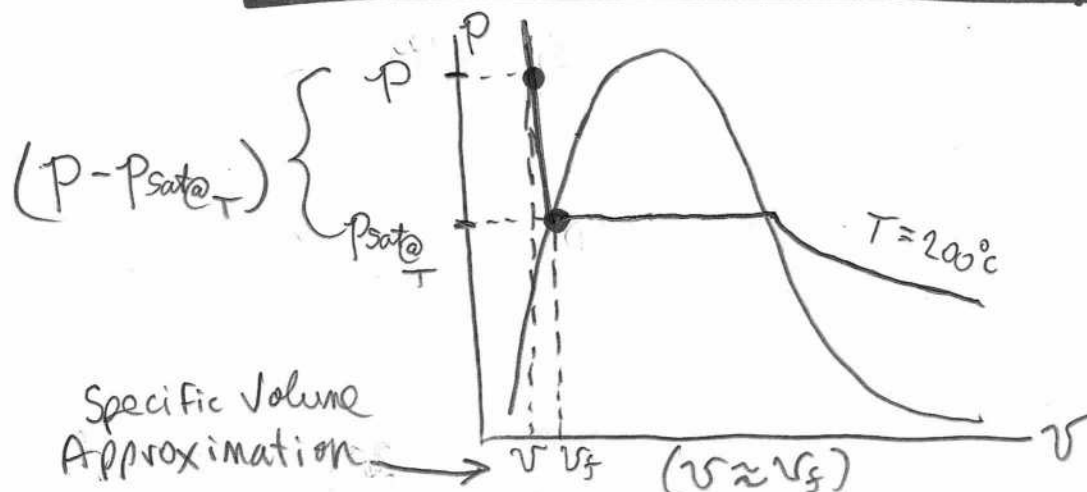
⇒ So, for a given temperature, T :

$$\begin{aligned} v &\approx v_{f@T} \\ u &\approx u_{f@T} \\ s &\approx s_{f@T} \end{aligned}$$

$$h \approx h_{f@T} + \text{pressure correction factor}$$

⇒ In a following lecture, we will find out that the correction factor is $v_{f@T}(p - p_{sat@T})$

⇒
$$h \approx h_{f@T} + v_{f@T}(p - p_{sat@T})$$



Example 2 Compressed Liquid Approximation

Given: Compressed liquid water @ 100°C , 15 MPa

Find: v , u , h using (a) saturated liq. Approx.
(b) "Exact" values in Table A-7

Assumptions

(1) $v \approx v_{f@100^\circ\text{C}}$

(2) $u \approx u_{f@100^\circ\text{C}}$

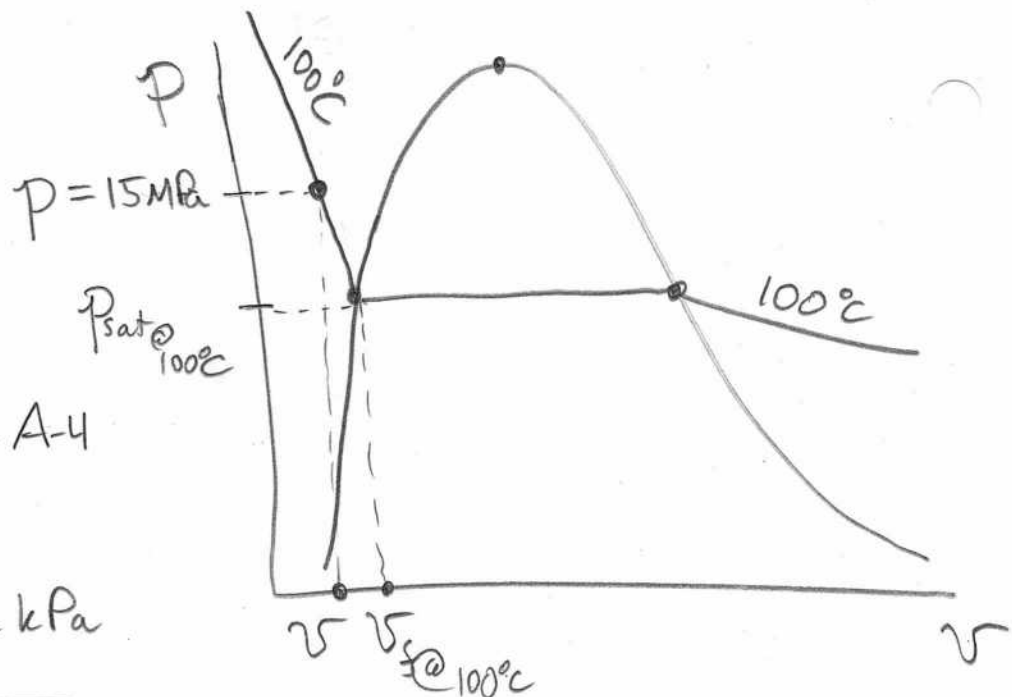
(3) $h \approx h_{f@100^\circ\text{C}} + v_{f@100^\circ\text{C}} \times (P - P_{\text{sat}@100^\circ\text{C}})$

Analysis

(a) From the Saturated water (temperature) Table A-4

@ $T = 100^\circ\text{C}$

$P_{\text{sat}@100^\circ\text{C}} = 101.42\text{ kPa}$



$v \approx v_{f@100^\circ\text{C}} = 0.001043\text{ m}^3/\text{kg}$

$u \approx u_{f@100^\circ\text{C}} = 419.06\text{ kJ/kg}$

$h \approx 419.17\text{ kJ/kg} + (0.001043\text{ m}^3/\text{kg})(15,000\text{ kPa} - 101.42\text{ kPa})$

$h \approx 434.71\text{ kJ/kg}$

15.54 kJ/kg

(6)

(b) Using the "exact" values in the Compressed liquid tables A-7,

$$\left. \begin{array}{l} p = 15 \text{ MPa} \\ T = 100^\circ\text{C} \end{array} \right\}$$

$$\left. \begin{array}{l} v = 0.001036 \text{ m}^3/\text{kg} \\ u = 414.85 \text{ kJ/kg} \\ h = 430.39 \frac{\text{kJ}}{\text{kg}} \end{array} \right\}$$

Percent errors in the approximate Analysis:

$$\epsilon_v = \left| \frac{0.001036 \frac{\text{m}^3}{\text{kg}} - 0.001043 \frac{\text{m}^3}{\text{kg}}}{0.001036 \frac{\text{m}^3}{\text{kg}}} \right| \times 100 = \underline{0.67\%}$$

$$\epsilon_u = \left| \frac{419.06 \frac{\text{kJ}}{\text{kg}} - 414.85 \frac{\text{kJ}}{\text{kg}}}{414.85 \frac{\text{kJ}}{\text{kg}}} \right| \times 100 = \underline{1.01\%}$$

$$\epsilon_h = \left| \frac{430.39 \frac{\text{kJ}}{\text{kg}} - 434.71 \frac{\text{kJ}}{\text{kg}}}{430.39 \frac{\text{kJ}}{\text{kg}}} \right| \times 100 = \underline{1.00\%}$$

If you only used $h \approx h_{s@100^\circ\text{C}}$, the error incurred is still only

$$\epsilon_h = \left| \frac{430.39 \frac{\text{kJ}}{\text{kg}} - 419.17 \frac{\text{kJ}}{\text{kg}}}{430.39 \frac{\text{kJ}}{\text{kg}}} \right| \times 100 = \underline{2.61\%}$$

I want you to always use the approx. for h shown here, not because of accuracy, but because you will need this to compute liquid pump power/work input requirements.

(II) Saturated Liquids/Mixtures/Vapors

- For these phase conditions, the Equation of State is too complicated to be expressed as an analytic formula, so for

(i) $H_2O \Rightarrow$ Tables A-4 through A-5 (SI)
A-4E - A-5E (Eng.)

(ii) R-134a \Rightarrow Tables A-11 through A-12 (SI)
A-11E - A-12E (Eng.)

have been tabulated.

(A) H_2O

Example 3 Given: $\begin{cases} T = 223^\circ C \\ v = 0.02 \frac{m^3}{kg} \end{cases}$

Find: (a) Phase

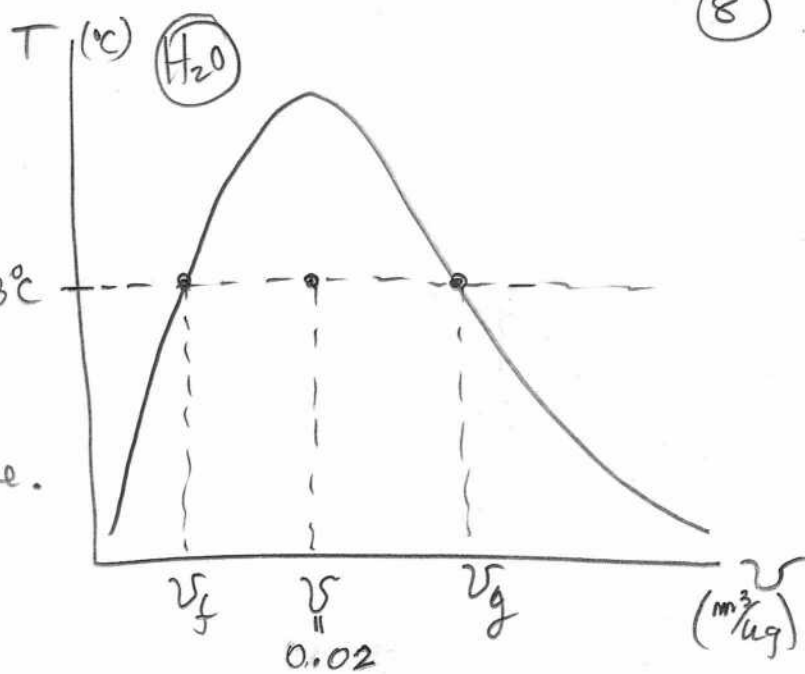
(b) p (kPa)

(c) h (kJ/kg)

Solution

- (a) Step 1: We are given T and v , so let's plot a T - v diagram (next page). On that diagram, mark $T = 223^\circ C$ on the vertical axis. (Obviously, $T = 223^\circ C < T_{crit} = 374^\circ C$ for H_2O — see Table A-1 to confirm this)

Step 2: Now look at Table A-4 $\Rightarrow 223^\circ\text{C}$ lies in between 220°C and 225°C , so we need to perform linear interpolation for all of our answers here.



Let's start by comparing the given $v = 0.02 \text{ m}^3/\text{kg}$

with the saturated liquid/vapor values in Table A-4 at $T \approx 223^\circ\text{C}$.

\Rightarrow we see that $v_f \leq v \leq v_g$

\Rightarrow **Phase = Saturated Mixture**

(b) Step 3: Let's find the pressure of the saturated mixture state using linear interpolation:

$$(i) \frac{T_{\text{given}} - 220^\circ\text{C}}{225^\circ\text{C} - 220^\circ\text{C}} = \frac{223 - 220}{225 - 220} = \underline{\underline{0.6}}$$

Difference
in tabular values

$\Rightarrow 223^\circ\text{C}$ is 60% above 220°C and 40% below 225°C

(ii) Use this fraction to compute P :

$$P = \underset{(\text{at } 220^\circ\text{C})}{2319.6 \text{ kPa}} + (0.6) \underset{(\text{at } 225^\circ\text{C})}{(2549.7 - 2319.6 \text{ kPa})}$$

$$\boxed{P = 2457.7 \text{ kPa}} \quad (@223^\circ\text{C})$$

(c)
Step 4

9
We need to find h , which we can compute using $h = h_f + x h_{fg}$, but we don't know x . However, we have enough info (because we were given two Intensive properties of a pure, simple compressible substance) to find x . Problem is that we need to interpolate to find both v_f and v_g at $223^\circ\text{C} \Rightarrow$ Use the fraction 0.6 between the rows to save time!

$$v_f = v_{f@220^\circ\text{C}} + (0.6)(v_{f@225^\circ\text{C}} - v_{f@220^\circ\text{C}})$$

$$v_f = 0.001190 + (0.6)(0.001199 - 0.001190)$$

$$\boxed{v_f = 0.0011954 \text{ m}^3/\text{kg}}$$

$$v_g = v_{g@220^\circ\text{C}} + (0.6)(v_{g@225^\circ\text{C}} - v_{g@220^\circ\text{C}})$$

$$v_g = 0.086094 + (0.6)(0.078405 - 0.086094 \frac{\text{m}^3}{\text{kg}})$$

$$\boxed{v_g = 0.0814806 \frac{\text{m}^3}{\text{kg}}}$$

thus,

$$x = \frac{v - v_f}{v_g - v_f} = \frac{0.02 \frac{\text{m}^3}{\text{kg}} - 0.0011954}{0.0814806 \frac{\text{m}^3}{\text{kg}} - 0.0011954}$$

$$\boxed{x = 0.2364 (23.64\%)}$$

(@223°C)

(10)

Step 5: Now use x @ 223°C to compute h_f & h_{fg} at 223°C . But we again need to interpolate!
Again, use the 0.6 fraction to same time:

$$\frac{h_{f@223^\circ\text{C}} - h_{f@220^\circ\text{C}}}{h_{f@225^\circ\text{C}} - h_{f@220^\circ\text{C}}} = 0.6 \quad \left[\begin{array}{l} \text{finding } h_f @ 223^\circ\text{C} \\ \text{by interpolation} \end{array} \right]$$

$$\Rightarrow h_{f@223^\circ\text{C}} = h_{f@220^\circ\text{C}} + (0.6)(h_{f@225^\circ\text{C}} - h_{f@220^\circ\text{C}})$$

$$h_f = 943.55 \frac{\text{kJ}}{\text{kg}} + (0.6)(966.76 \frac{\text{kJ}}{\text{kg}} - 943.55 \frac{\text{kJ}}{\text{kg}})$$

$$\boxed{h_f = 957.48 \frac{\text{kJ}}{\text{kg}} @ 223^\circ\text{C}}$$

$$h_{fg} = 1857.4 \frac{\text{kJ}}{\text{kg}} + (0.6)(1835.4 - 1857.4 \frac{\text{kJ}}{\text{kg}})$$

[Finding h_{fg} by interpolat.]

$$\boxed{h_{fg} = 1844.2 \frac{\text{kJ}}{\text{kg}} @ 223^\circ\text{C}}$$

Thus $h_{e@223^\circ\text{C}} = h_{f@223^\circ\text{C}} + x_{e@223^\circ\text{C}} h_{fg@223^\circ\text{C}}$

$$h_{e@223^\circ\text{C}} = 957.48 \frac{\text{kJ}}{\text{kg}} + (0.2364)(1844.2 \frac{\text{kJ}}{\text{kg}})$$

$$\boxed{h = 1393.4 \frac{\text{kJ}}{\text{kg}}} \quad \left(@ 223^\circ\text{C}, P = 2457.7 \text{ kPa} \right. \\ \left. 23.64\% \text{ quality, } x \right)$$

(B) R-134a

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Example 4 Given: $\begin{cases} T = -10^\circ\text{F} \\ S = 0.15 \frac{\text{Btu}}{\text{lbm} \cdot \text{R}} \end{cases}$

Find: (a) Phase

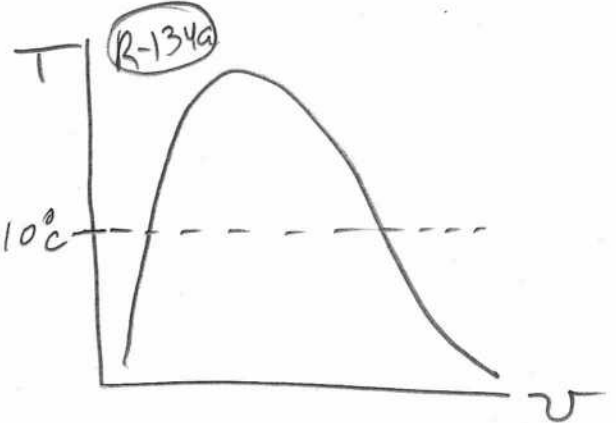
(b) p (psia)

(c) v (ft^3/lbm)

(d) u ($\frac{\text{Btu}}{\text{lbm}}$)

Solution

(a) Step 1: We are given T , so draw a T - v diagram with a dashed line across @ -10°C



Step 2: We are given -10°C
 $S = 0.15 \frac{\text{Btu}}{\text{lbm} \cdot \text{R}}$, so without any other information, look at the saturated liq./vap table A-11E and compare the given value to that of S_f and S_g near $T = -10^\circ\text{C}$.

$\Rightarrow -10^\circ\text{C}$ lies exactly on the row and so we find that

$$S_f = 0.02092 \frac{\text{Btu}}{\text{lbm} \cdot \text{R}} < S = 0.15 \frac{\text{Btu}}{\text{lbm} \cdot \text{R}} < S_g = 0.22660 \frac{\text{Btu}}{\text{lbm} \cdot \text{R}}$$

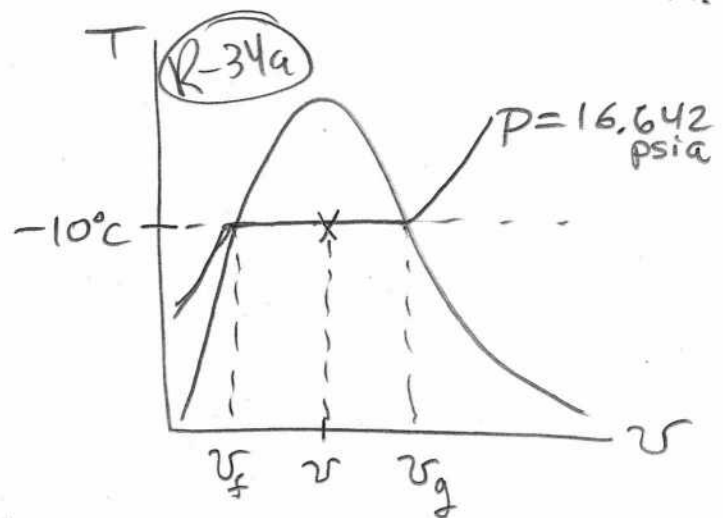
\Rightarrow **Phase = saturated mixture**

(b) Step 3: We can now immediately see that $P = 16.642 \text{ psia}$ because it is a sat. mix. (12)

(c) Step 4: to find v ,
we use

$$v = v_f + x(v_g - v_f)$$

but we don't know the quality, x , of the mixture.



We get the quality from given information:

$$x = \frac{s - s_f}{s_{fg}} = \frac{0.15 - 0.02092 \frac{\text{Btu}}{\text{lb}_m \cdot \text{R}}}{0.20569 \frac{\text{Btu}}{\text{lb}_m \cdot \text{R}}}$$

$$\boxed{x = 0.6275} \quad (62.75\% \text{ vapor / total mass})$$

Thus,
$$v = v_{f@-10^\circ\text{C}} + x(v_{g@-10^\circ\text{C}} - v_{f@-10^\circ\text{C}})$$

$$v = 0.01171 \frac{\text{ft}^3}{\text{lb}_m} + (0.6275)(2.7091 - 0.01171 \frac{\text{ft}^3}{\text{lb}_m})$$

$$\boxed{v = 1.70445 \frac{\text{ft}^3}{\text{lb}_m}}$$

(d) Step 5: to find the specific internal energy, u ,
we use

$$u = u_{f@-10^\circ\text{C}} + (0.6275)(u_{fg@-10^\circ\text{C}})$$

$$u = 9.073 \frac{\text{Btu}}{\text{lb}_m} + (0.6275)(84.191 \frac{\text{Btu}}{\text{lb}_m})$$

$$\boxed{u = 61.903 \frac{\text{Btu}}{\text{lb}_m}}$$

II Superheated Vapors

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- Superheated vapor states may or may not obey the Ideal Gas E.O.S. ($pv = RT$) and therefore there is no simple analytic formulas for u , h , and s .
- A superheated vapor that has an EOS that does not obey the Ideal Gas EOS. is called a "Real" gas. We will talk more about this in (IV)
- For H_2O and R-134a (refrigerant), Superheated Vapor states are tabulated in Tables A-6 (H_2O , SI), A-6E (H_2O English), A-11 (R-134a, SI), and A-11E (R-134a, English).

Example 5

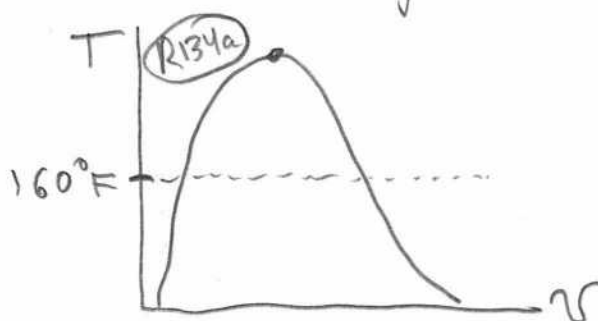
Given: $\begin{cases} \text{R-134a} \\ p = 160 \text{ psi} \\ T = 160^\circ\text{F} \end{cases}$

Find: (a) Phase
(b) v (m^3/kg)
(c) u (kJ/kg)

Analysis

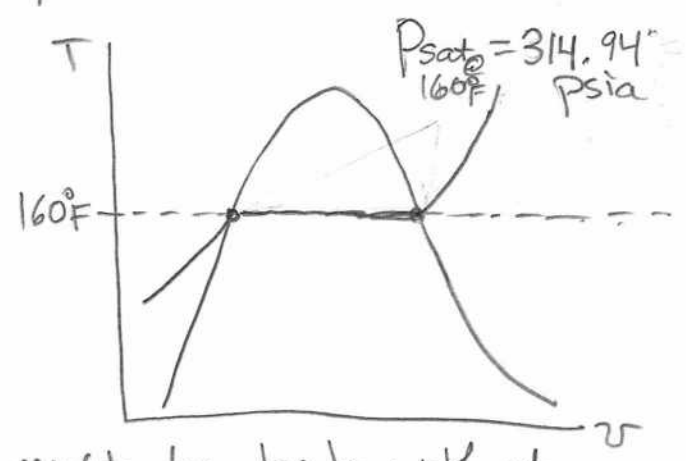
(a) **Step 1** We are given R-134a @ 160 psi, 160°F . It doesn't matter if you draw a $p-v$ or $T-v$ diagram. I arbitrarily choose a $T-v$ diagram.

- Draw a dashed line across the $T-v$ diagram for the given temperature of the state in question.



Step 2 Now look at the Saturated Refrigerant A-11E to find the Saturation pressure $P_{sat@160^\circ F}$ at the given $T=160^\circ C$ and draw a line of constant pressure on the $T-v$ diagram:

$(P_{sat@160^\circ F} = 314.94 \text{ psia})$

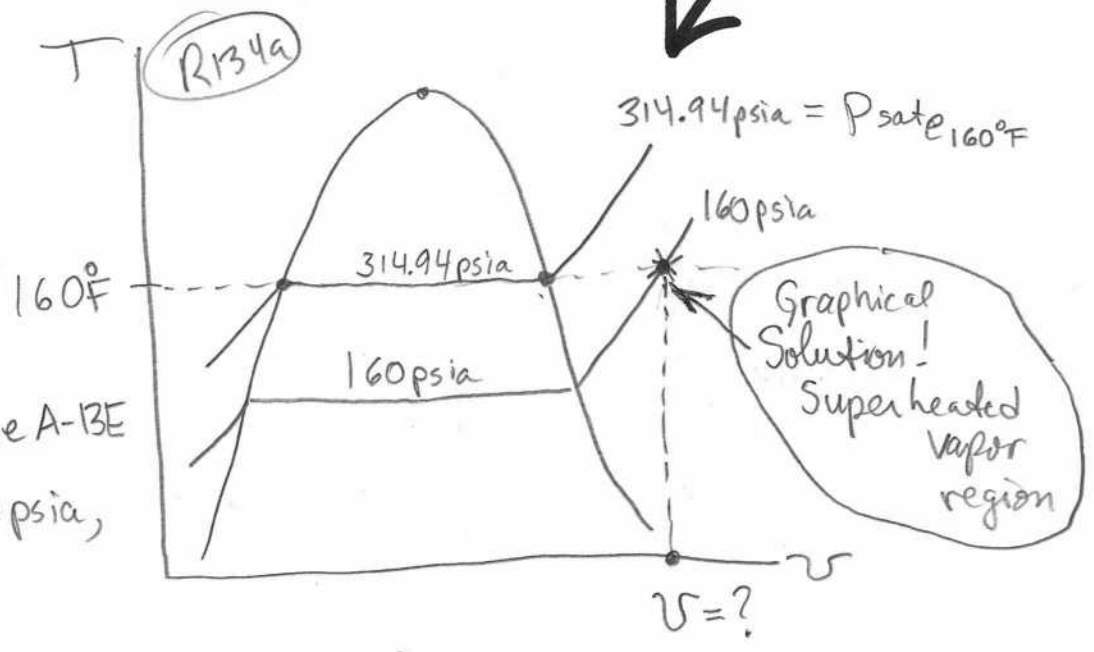


Step 3 The pressure at the given state is 160 psia.

This means that a line of Constant pressure at 160 psia must be below that of 314.94 psia. so, now draw that line of constant pressure and note where $T=160^\circ F$ dashed line intersects the $p=160 \text{ psia}$ line \Rightarrow That is the graphical solution of the phase region.

Step 4

Now that we know the phase is a superheated vapor, go to Table A-13E and find $p=160 \text{ psia}$, $T=160^\circ F$.



Step 5 From Table A-13E, we find that

- (b) $v = 0.34790 \frac{\text{ft}^3}{\text{lbm}}$
- (c) $u = 120.13 \frac{\text{Btu}}{\text{lbm}}$

Example 6

Given

$$\begin{cases} \text{H}_2\text{O} \\ p = 2000 \text{ kPa} \\ h = 3072 \text{ kJ/kg} \end{cases}$$

15

Find: (a) Phase

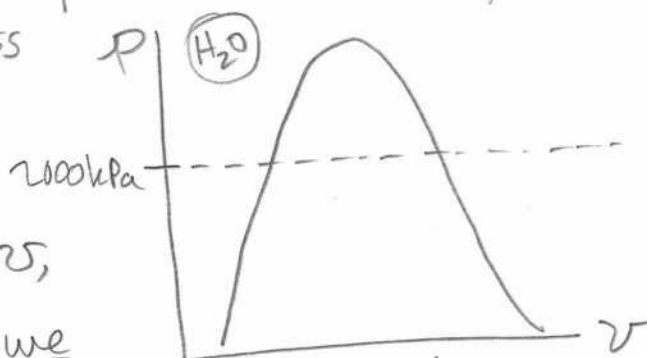
(b) s ($\frac{\text{kJ}}{\text{kg} \cdot \text{K}}$)

(d) T ($^{\circ}\text{C}$)

(c) v (m^3/kg)

Analysis:

(a) **Step 1** We are given p , so draw a p - v diagram with a dashed line across at 2000 kPa.



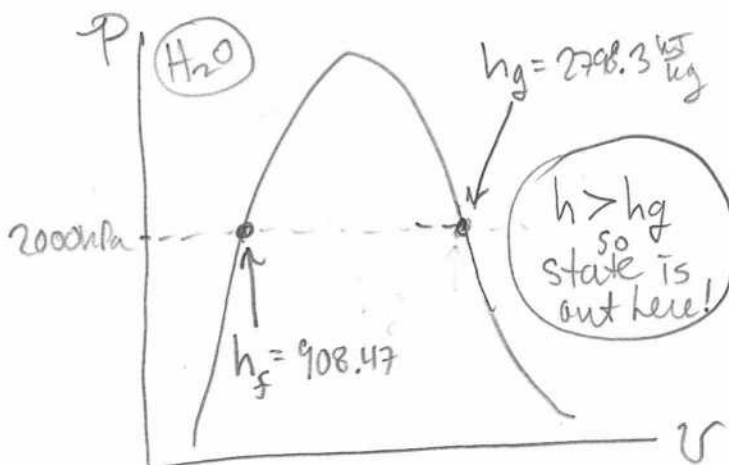
Step 2 We don't know v , and we don't have T . So we must eliminate the possibility of the phase being a saturated mixture or compressed liquid by using the given h along with Table A-5:

Step 3 Looking at Table A-5, for $p = 2000 \text{ kPa}$, we see that $h_f = 908.47 \text{ kJ/kg}$ and $h_g = 2798.3 \text{ kJ/kg}$,

and the given

$$h > h_g$$

⇒ **must be a superheated vapor!**



Step 4 Go to the Superheated vapor (H_2O) Table A-6 and find $p = 2000 \text{ kPa} = 0.2 \text{ MPa}$ look down the h -column and find where $h = 3072 \frac{\text{kJ}}{\text{kg}}$ The closest we find is $h = 3072.1 \frac{\text{kJ}}{\text{kg}} \approx 3072.0 \frac{\text{kJ}}{\text{kg}}$ (ignore the $0.1 \frac{\text{kJ}}{\text{kg}}$ for this example) we now have:

(b) $s = 7.8941 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$

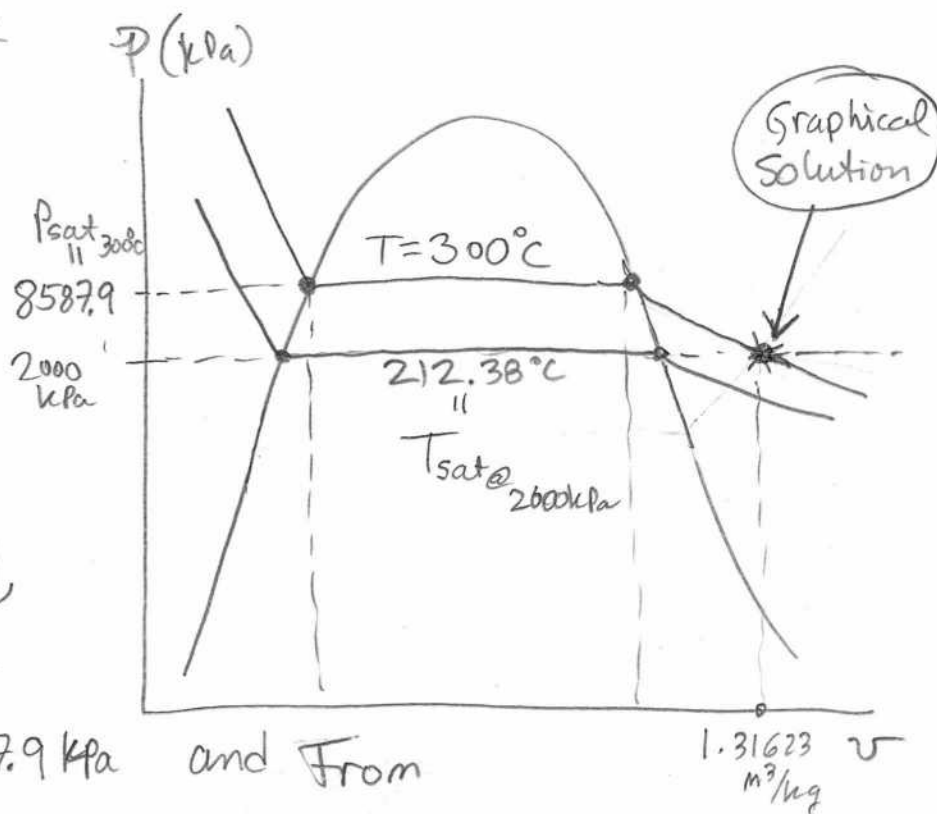
(c) $v = 1.31623 \frac{\text{m}^3}{\text{kg}}$

(d) $T = 300^\circ\text{C}$

- If you now go back to Table A-4, for $T = 300^\circ\text{C}$, we

find $P_{\text{sat}@ } 300^\circ\text{C}} = 8587.9 \text{ kPa}$ and From

Table A-5, $T_{\text{sat}@ } 2000 \text{ kPa}} = 212.38^\circ\text{C}$



- These results are drawn on the p - v diagram, and, show that lines of constant temperature on a p - v diagram increase with increasing pressure.

(IV) Gases and Vapors

(A) "Ideal" Gases

- Any gas that obeys the Ideal Gas Equation of State is said to be "an Ideal gas" and exhibits "Ideal gas behavior".

Ideal Gas E.O.S.

An "Ideal Gas" is a fictitious substance that obeys a relationship between pressure-Temperature-Volume in any one of the following forms:

$$(1) \quad pV = RT$$

$$(2) \quad pV = \frac{R}{m} T$$

$$(3) \quad p \frac{V}{m} = RT$$

$$(4) \quad pV = mRT$$

$$(5) \quad pV = NRT$$

⋮

etc.

Vapor

A "Vapor" implies that a gas is not far from a State of Condensation
 \Rightarrow A vapor is not an Ideal Gas!

In the Ideal Gas E.O.S.,

(18)

$$\begin{aligned} R &\equiv \text{Universal Gas Constant} \\ &= 8.31447 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}} \\ &= 8314.47 \frac{\text{J}}{\text{kmol} \cdot \text{K}} \end{aligned}$$

$$M \equiv \text{Molecular Mass of the gas substance in } \frac{\text{kg}}{\text{kmol}}$$

(see Table A-1)

It is convenient to define

$$R \equiv \frac{R}{M} = \text{"Specific" Gas Constant valid for a particular substance}$$

$$\text{units} = \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

(continued)

Moles

1 "Mole" is equal to 6.022×10^{23} particles which is called "Avogadro's number":

$$N_A = 6.022 \times 10^{23} \frac{\text{particles}}{\text{mole}}$$

Note: • A "mole" is like a "dozen" eggs

- 1 mole of eggs = 6.022×10^{23} eggs

Mole Number N

Mole Number is simply the number of moles of a substance.

- $N = 3 \text{ moles of argon atoms}$

$$N = 3 \text{ mol} \times N_A$$

$$N = 3 \text{ mol} \times 6.022 \times 10^{23} \frac{\text{atoms}}{\text{mol}}$$

$$N = 1.8066 \times 10^{24} \text{ argon atoms}$$

(Continued....)

Example 7 From Table A-1

with $R = 8.31447 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}$

(1) "Air"

$$R_{\text{air}} = \frac{R}{M_{\text{air}}} = \frac{8.31447 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}}{28.97 \frac{\text{kg}}{\text{kmol}}}$$

$$R_{\text{air}} = 0.2870 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

(2) Helium

$$R_{\text{He}} = \frac{R}{M_{\text{He}}} = \frac{8.31447 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}}{4.003 \frac{\text{kg}}{\text{kmol}}}$$

$$R_{\text{He}} = 2.0769 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

(3) Krypton

$$R_{\text{Kr}} = \frac{R}{M_{\text{Kr}}} = \frac{8.31447 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}}{83.80 \frac{\text{kg}}{\text{kmol}}}$$

$$R_{\text{Kr}} = 0.09921 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

(continued....)

Mass ↔ Mole Conversion

$$m = MN$$

\uparrow mass (kg) \uparrow molecular mass $\left(\frac{\text{kg}}{\text{kmol}}\right)$ \nwarrow Mole Number (mol)

Example 6

(1) Mass of argon in System: $m = 5 \text{ kg}$

(2) Table A-1 : $M_{\text{Ar}} = 39.948 \frac{\text{kg}}{\text{kmol}}$

(3) Number of moles of argon in System

$$N = \frac{m}{M} = \frac{5 \text{ kg}}{39.948 \text{ kg/kmol}}$$

$$N = 0.12516 \text{ kmol argon}$$

Note: This conversion was used to obtain $pV = NRT$ from $pV = mRT$:

$$pV = (MN)RT$$

$$pV = M N \frac{R}{M} T$$

$$pV = NRT$$

Example 8 Ideal Gas EOS.

- 50 kg of Methane gas is contained inside a rigid tank and is maintained at 200°C . The Tank has a volume of 15 m^3 .
- What is the pressure of the Methane inside the tank?
- Assume: the Methane behaves as an Ideal gas.

Solution

The Ideal gas EOS is

$$pv = RT$$

where $v = \frac{m}{V}$

So

$$p \frac{V}{m} = RT$$

The pressure is

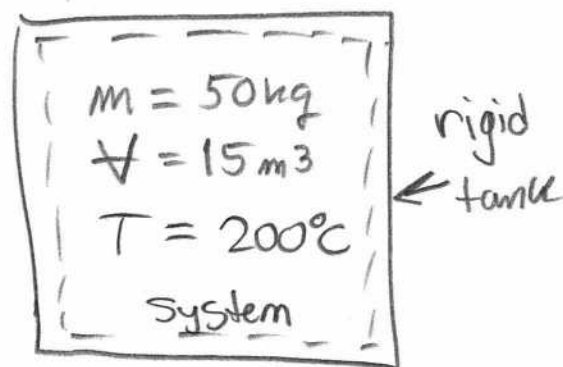
$$p = \frac{mRT}{V}$$

From Table A-1, $M_{\text{CH}_4} = 16.043 \frac{\text{kg}}{\text{kmol}}$ and $R_{\text{CH}_4} = 0.5182 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

$$p = \frac{(50 \text{ kg}) \left(0.5182 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \right) (200^{\circ}\text{C} + 273)}{15 \text{ m}^3} = 817.03 \frac{\text{kJ}}{\text{m}^3}$$

$$1 \frac{\text{kJ}}{\text{m}^3} = 1 \frac{(\text{kN})\text{m}}{\text{m}^3} = 1 \frac{\text{kN}}{\text{m}^2} = 1 \text{ kPa} \quad \text{so} \Rightarrow$$

$$p = 817.03 \text{ kPa}$$



Example 9

Constant pressure process of an Ideal gas

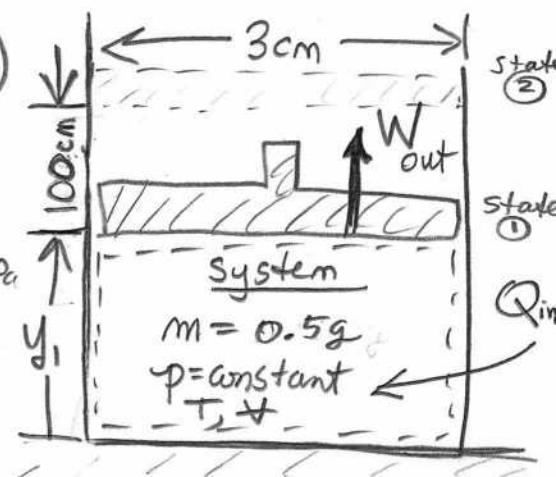
23

A vertically-oriented piston-cylinder assembly contains 5 grams of Xenon at 250°C , 1atm . The diameter of the piston-cylinder assembly is 3cm . Heat is conducted through the cylinder wall, into the gas, and the piston rises such that the process is quasiequilibrium and at constant pressure. The piston face rises 100cm from its original position. Assuming Xenon behaves as an Ideal gas, what is the final temperature of the Xenon gas in $^\circ\text{C}$?

Solution (Draw a Picture!!)

Given: State ① $\begin{cases} m_1 = 0.005\text{kg} \\ T_1 = 250^\circ\text{C} = 523\text{K} \\ P_1 = 1\text{atm} = 101.325\text{kPa} \end{cases}$

State ② $\begin{cases} m_2 = m_1 = 0.005 \\ P_2 = P_1 = 1\text{atm} \end{cases}$

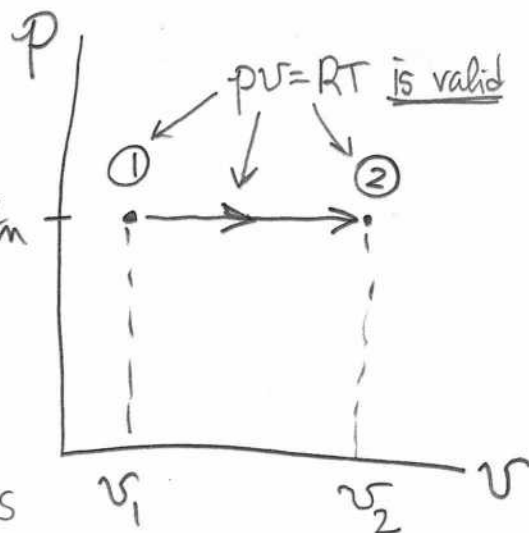


Assume: Ideal Gas Behavior

Analysis

Since the gas is assumed to be Ideal, and quasiequilibrium,

the $\boxed{pv = RT}$ EOS is valid at States ① & ② and all along the process path.



at State ①, $P_1 V_1 = RT_1$

and at State ②, $P_2 V_2 = RT_2$

The process from ① → ② is constant pressure, so

$$P_1 \equiv P_2$$

Key Concept to Understand

Using the Ideal Gas EOS., substitution gives

$$\frac{RT_1}{V_1} = \frac{RT_2}{V_2}$$

or

$$T_2 = \left(\frac{V_2}{V_1} \right) T_1$$

By definition, $V_1 = \frac{V_1}{m}$ and $V_2 = \frac{V_2}{m}$ so

$$T_2 = \left(\frac{V_2}{m} \times \frac{m}{V_1} \right) T_1$$

$$T_2 = \left(\frac{V_2}{V_1} \right) T_1$$

We know that the volume is related to piston surface area and height, by:

$$V_1 = \pi r^2 y_1 = \frac{\pi D^2}{4} y_1$$

$$V_2 = \frac{\pi D^2}{4} y_2$$

So

$$T_2 = \left(\frac{\frac{\pi D^2}{4} y_2}{\frac{\pi D^2}{4} y_1} \right) T_1$$

so

$$T_2 = \left(\frac{y_2}{y_1} \right) T_1$$

Q: What is y_1 ? We know $y_2 = 100 \text{ cm} + y_1$

At state ①, $p_1 v_1 = RT_1$ so that

$$v_1 = \frac{RT_1}{p_1} = \frac{(0.06332 \frac{\text{kJ}}{\text{kg} \cdot \text{K}})(523 \text{ K})}{101.325 \text{ kPa}}$$

$$v_1 = 0.326833 \frac{\text{m}^3}{\text{kg}} \equiv \frac{V_1}{m}$$

$$\Rightarrow V_1 = \left(0.326833 \frac{\text{m}^3}{\text{kg}} \right) (0.005 \text{ kg})$$

$$V_1 = 0.00163416 \text{ m}^3 \times \frac{(100 \text{ cm})^3}{1 \text{ m}}$$

$$V_1 = 1634.16 \text{ cm}^3$$

But, $V_1 = \frac{\pi D_1^2}{4} y_1$

so $y_1 = \frac{4 V_1}{\pi D_1^2} = \frac{4 \times 1634.16 \text{ cm}^3}{\pi \times (3 \text{ cm})^2}$

$$y_1 = 231 \text{ cm}$$

$$\Rightarrow y_2 = 100 \text{ cm} + 231 \text{ cm}$$

$$y_2 = 331 \text{ cm}$$

Thus,

$$T_2 = (523 \text{ K}) \left(\frac{331 \text{ cm}}{231 \text{ cm}} \right) = 749.4 \text{ K} \Rightarrow T_2 = 476^\circ \text{C}$$

(B) "Non-Ideal" or "Real" Gases

Non-Ideal (or "Real") Gas

A Real Gas obeys the Equation of State

$$pV = ZRT$$

where

$Z \equiv$ the Compressibility Factor and is found from the Figure A-15

\Rightarrow For a given temperature, as the gas density increases, the intermolecular force (potential energy) interactions can no longer be ignored and must be accounted for by "correcting" the Ideal gas E.O.S. with " Z ":

$$v_{\text{real}} = Z \left(\frac{RT}{p} \right)$$

\Rightarrow

$$v_{\text{real}} = Z v_{\text{Ideal}}$$

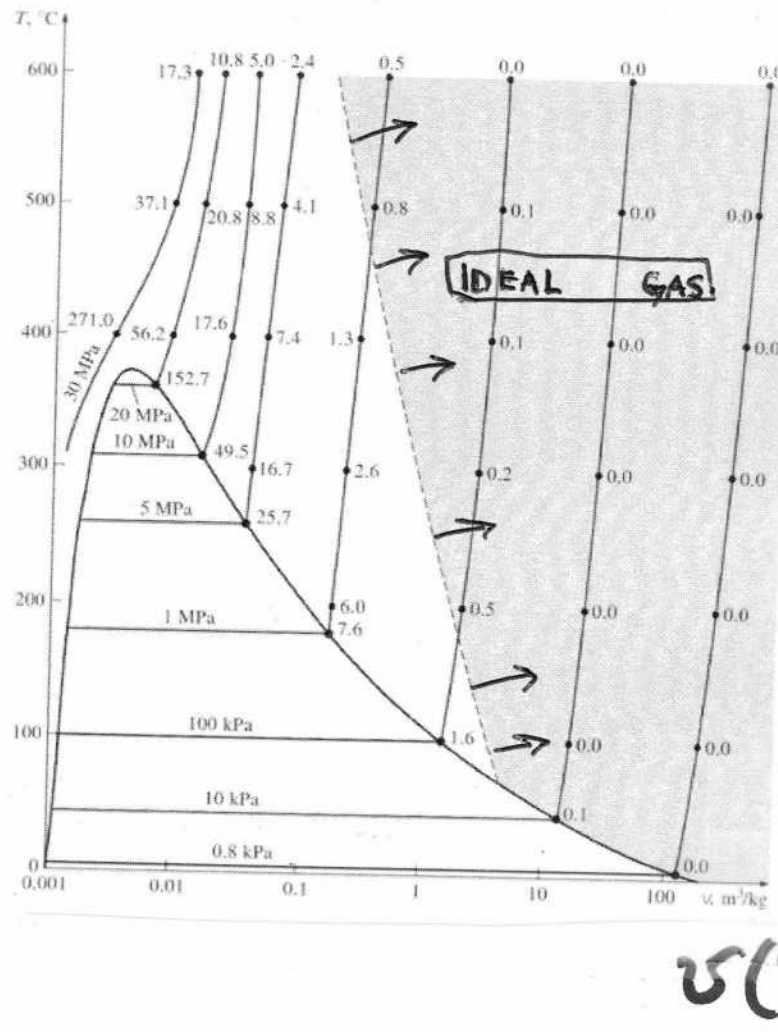
• Note

(1) $Z = 1 \Rightarrow$ Ideal Gas

(2) $Z < 1$
 or
 $Z > 1$ } Non-Ideal, or
 "Real" Gas

When do we use the Ideal Gas EOS?

T
(°C)



(page 137 of text)
This plot
Shows the
% Error
in
 H_2O
Specific
Volume
using
Ideal Gas
EOS

v (m³/kg)

The Ideal Gas EOS is only for Conditions of:

- High T
- Low p
- Low Density $\rho \Rightarrow$ High v

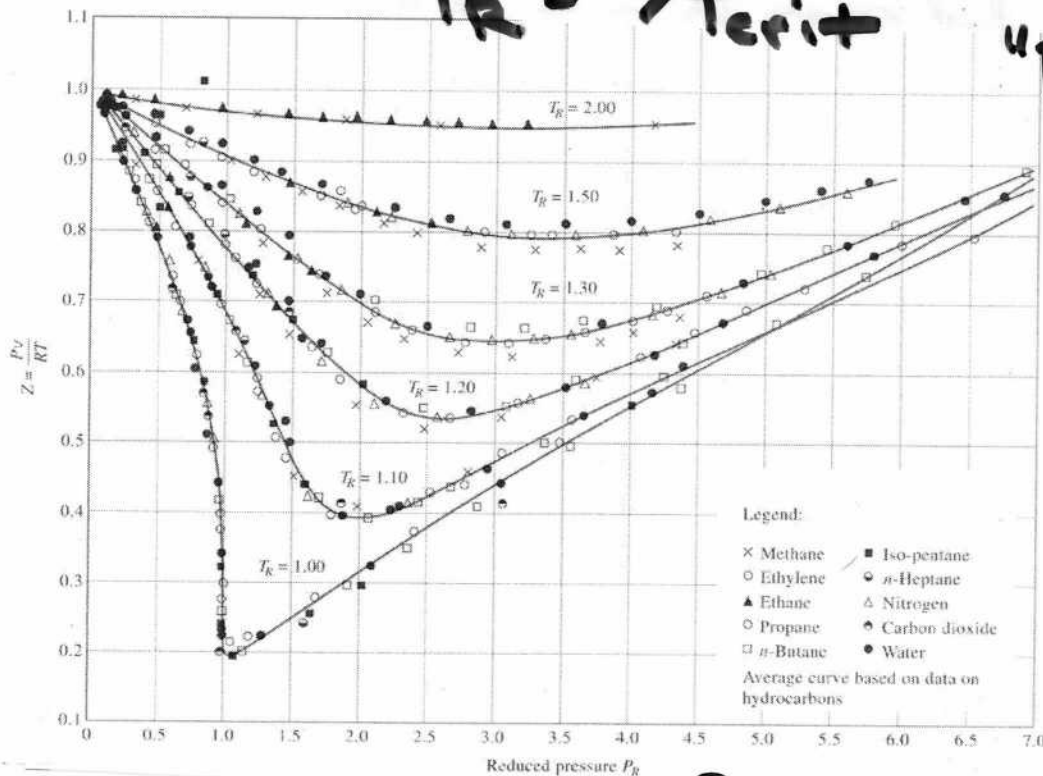
When these conditions are ~~not~~ true,
then we must use "Compressibility Chart"
Figure A-15

$$T_R = T/T_{crit}$$

Shows the
"Principle of
Corresponding
States"

Figure
3-49
in text

Z



$$P/P_{crit}$$

"Reduced" T_R , P_R , and v_R :

(1) $T_R \equiv T/T_{crit}$

(2) $P_R \equiv P/P_{crit}$

(3) $v_R \equiv \frac{v_{actual}}{\frac{RT_{crit}}{P_{crit}}} = \frac{v_{actual}}{v_{crit}}$

Many gases show the same behavior
when normalized by their T_{crit} & P_{crit} :

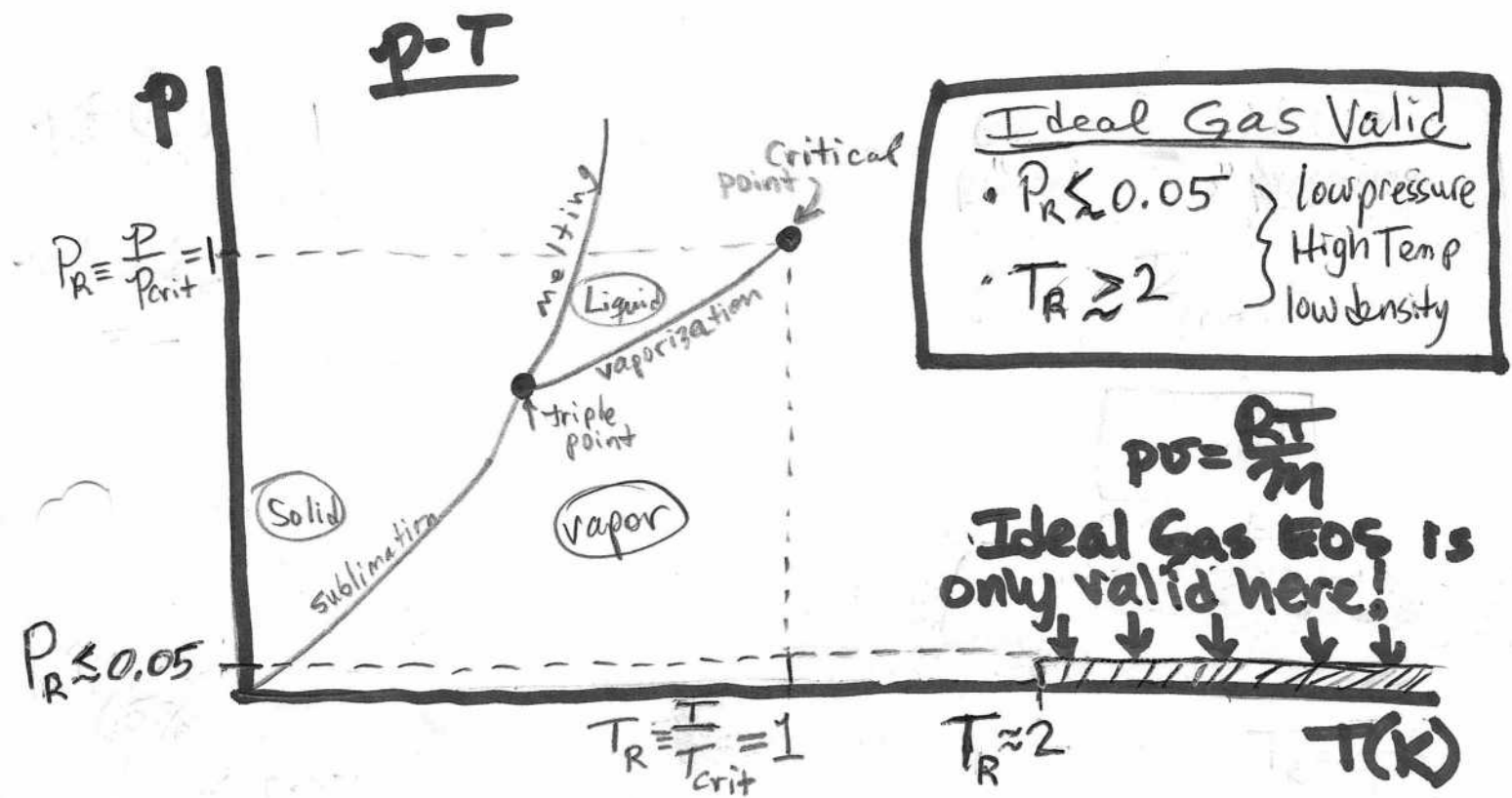
⇒ use

$$Z = \frac{Pv}{RT}$$

Chart A-15

- In order to more easily visualize where the Ideal Gas EOS is valid, let us look at p - T , T - v , and p - v diagrams and compare

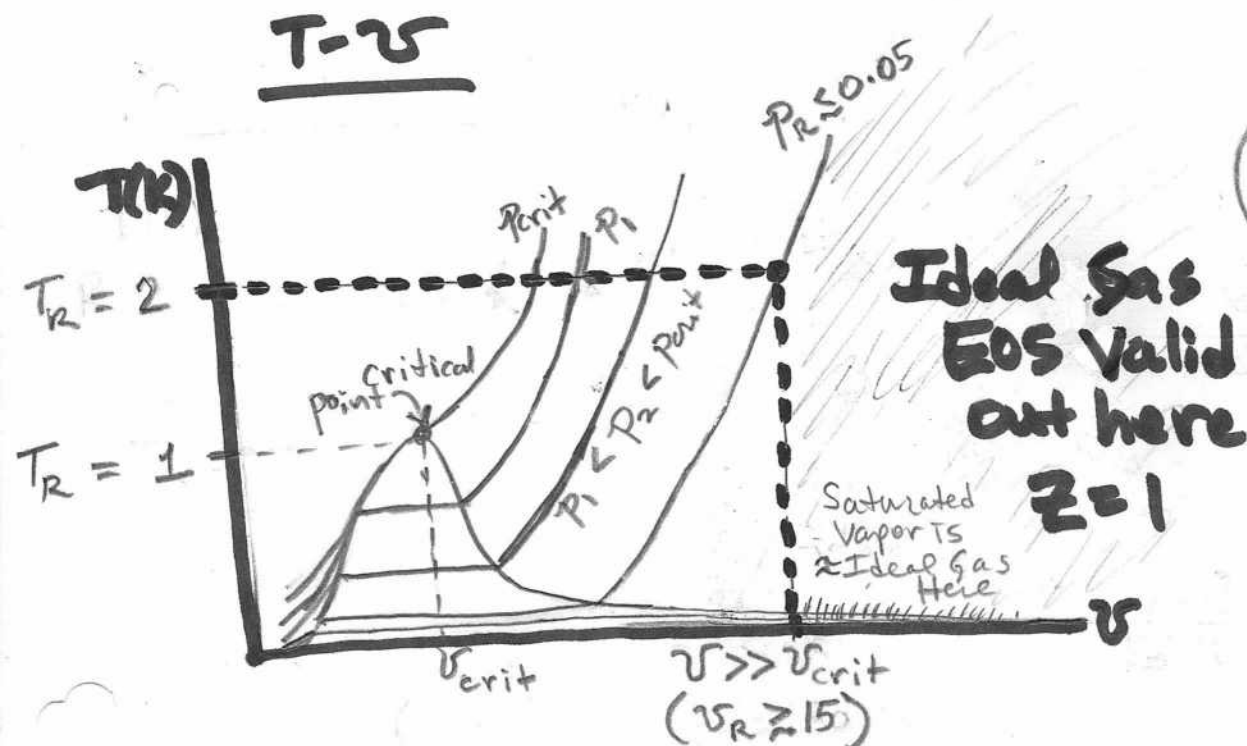
- Also read Section 3-7 in your textbook.



- As you look at this p - T diagram, also look at Figure 3-49 that shows z - p_R - T_R relationship. (page 138)

- It is also instructive to look at the $T-v$ and $p-v$ diagrams to visualize where the Ideal gas EOS is valid:

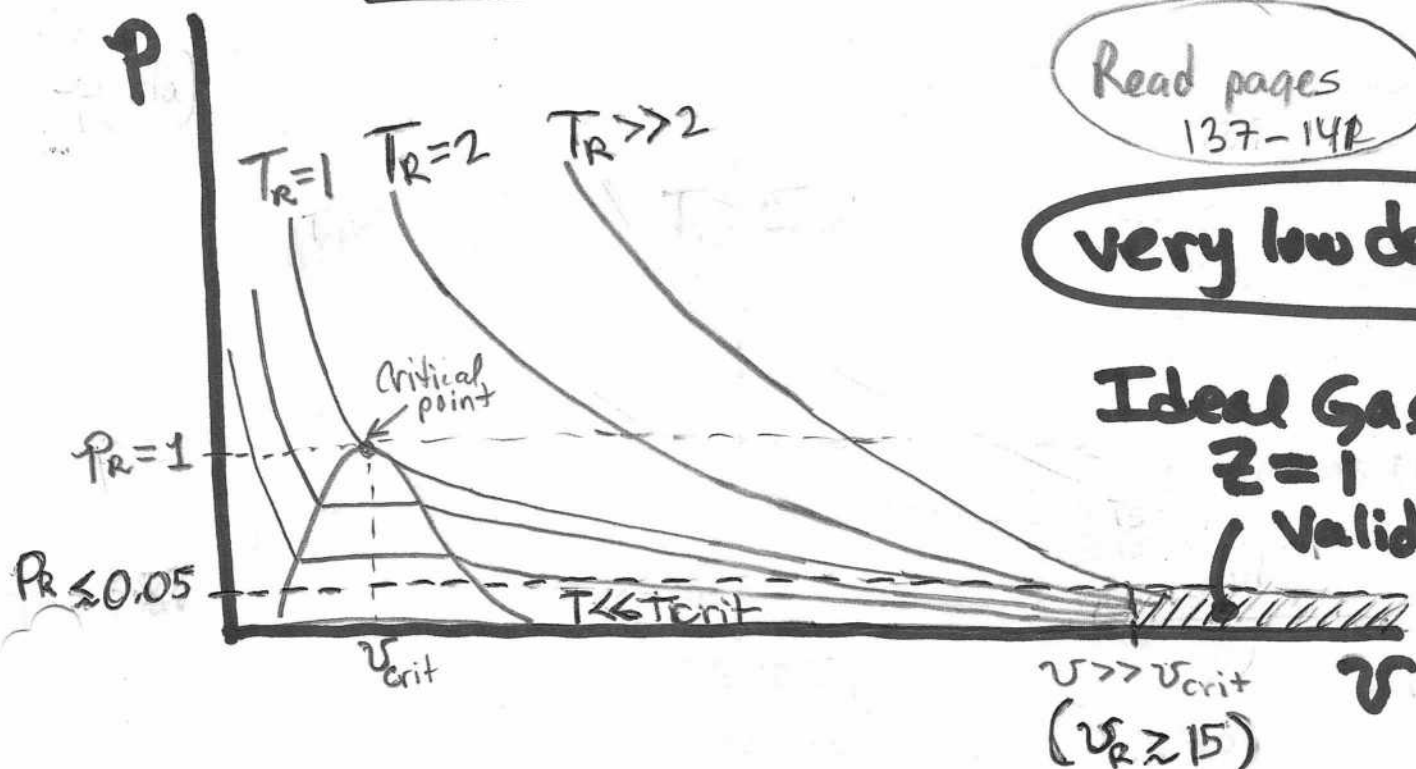
$T-v$



Read pages
137-141

very low density

$p-v$



Read pages
137-141

very low density

Ideal Gas
 $z=1$
Valid

The Ideal Gas E.O.S. ($Z=1$) is valid only for Low Density gases

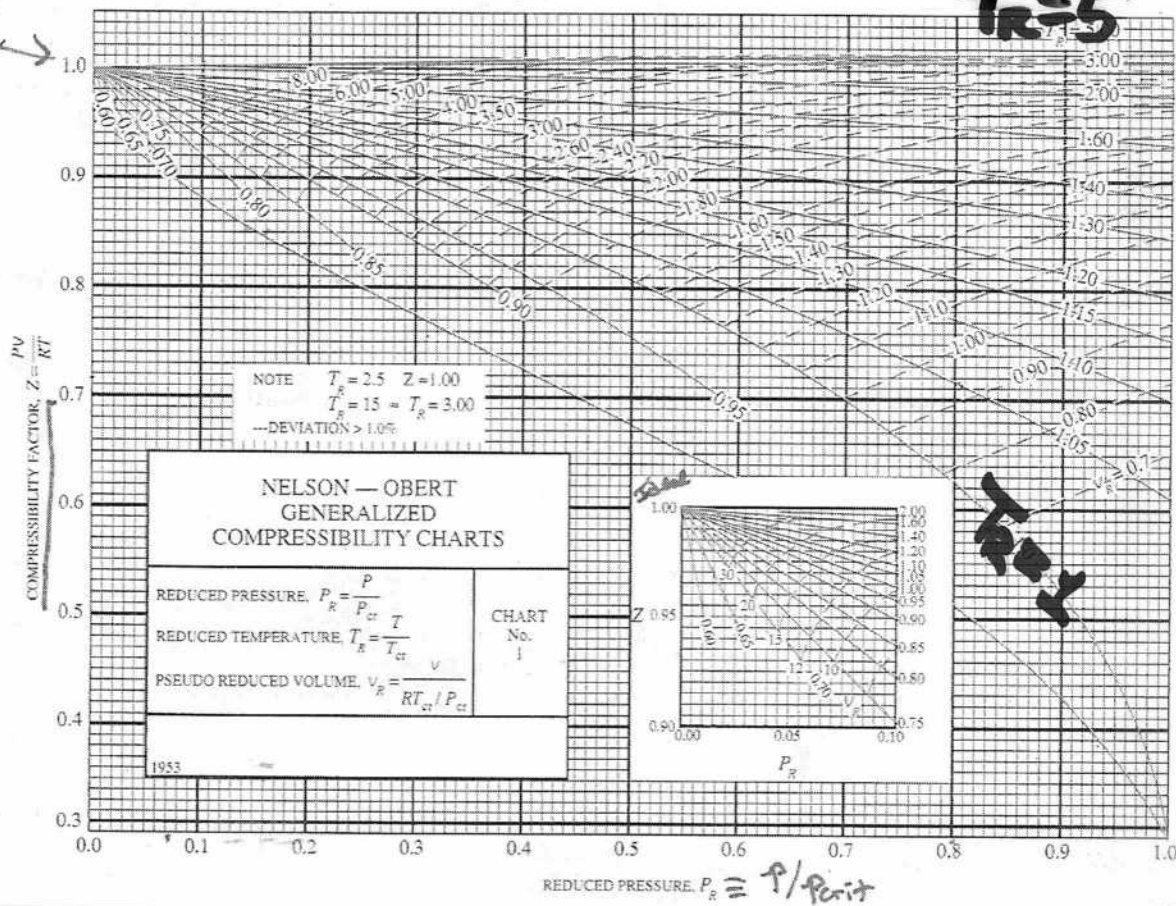
IDEAL GAS \Rightarrow

$$\left. \begin{array}{l} P \ll P_{crit} \quad (P_R \approx 0.05) \\ T \gg T_{crit} \quad (T_R \geq 2) \\ v \gg v_{crit} \quad (v_R \geq 15) \end{array} \right\} \Rightarrow Z \approx 0.98$$

Within $\sim 2\%$ error

Why?

- The physical reason for a gas (or gas mixture) behaving such that the Ideal-Gas Law is valid to within 1-2% relative error in specific volume is that in this situation, intermolecular attractive forces are negligible (during collisions, they act as if they were hard "billiard balls" (perfectly repulsive force on contact) and the collision is perfectly elastic.
- While this is beyond the scope of the class, the principal reason for the difficulty in not being able to formulate a "universal" equation of state is that the influence of the intermolecular forces on the momentum transfer of the molecules (or atoms) cannot be calculated without precise knowledge of the magnitude of these forces. Therefore, the E.O.S. of a "Real Gas" must be established empirically. See Section 3-8, pg. 144-147, especially Van der Waal's EOS which is only qualitatively correct

(a) Low pressures, $0 < P_R < 1.0$ $T_R = 5$ Ideal Gas
↓ Z  P_{crit}

- Generalized Compressibility Chart for Non-Ideal Gas Behavior ($Z \neq 1$)

You can use the Ideal Gas E.O.S. only when:

- $T_R \geq 2$
- $V_R \geq 15$
- $P_R \leq 0.05$

Such that

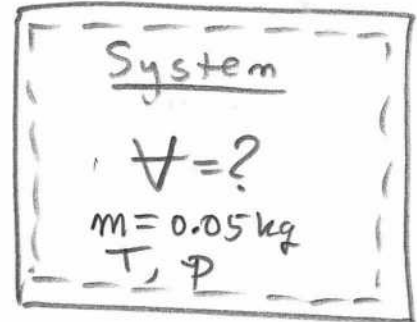
$$0.98 \leq Z \leq 1.02 \quad (\text{correspond to error} \leq 2\%)$$

Example 10 H_2O Real Gas Behavior

- H_2O is enclosed within a rigid tank at $200^\circ C$ and 1000 kPa . The mass of the H_2O is 0.05 kg . What is the Volume of the Tank?

Solution

Given: $\begin{cases} T = 200^\circ C = 473 \text{ K} \\ p = 1000 \text{ kPa} = 1 \text{ MPa} \\ m = 0.05 \text{ kg} \end{cases}$



Find: V

Analysis

- First, we need to determine the Phase of the System. While we don't know V , we can draw a T - v diagram using

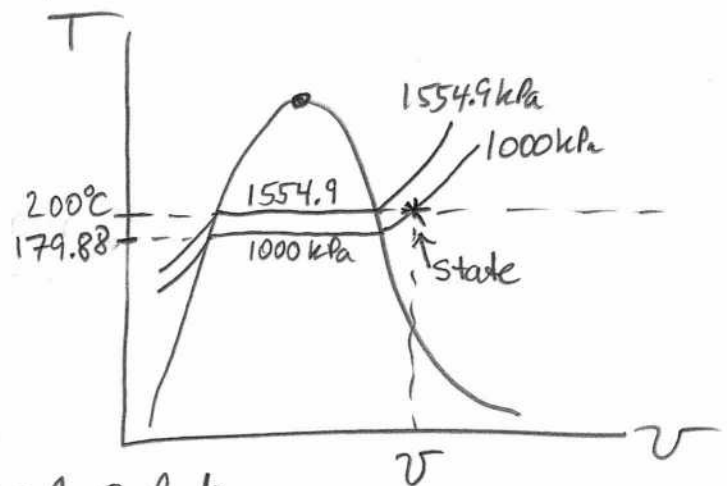
Table A-4 @ $200^\circ C$,

$P_{\text{sat}} @ 200^\circ C = 1554.9 \text{ kPa}$ and

from Table A-5, for 1000 kPa ,

$T_{\text{sat}} @ 1000 \text{ kPa} = 179.88^\circ C$, so we

so we see from the graphical solution that the Phase is a superheated vapor.



- Next, we need to know if we can use the Ideal gas EOS in order to find V .

From Table A-1 for water, the critical point properties are: (34)

$$\begin{cases} T_{\text{crit}} = 647.1 \text{ K} \\ p_{\text{crit}} = 22.06 \text{ MPa} \end{cases}$$

The reduced temperature and pressure for this given state are

$$T_R \equiv \frac{T}{T_{\text{crit}}} = \frac{473 \text{ K}}{647.1 \text{ K}} \Rightarrow$$

$$T_R \approx 0.73$$

$$P_R \equiv \frac{P}{P_{\text{crit}}} = \frac{1 \text{ MPa}}{22.06 \text{ MPa}} \Rightarrow$$

$$P_R \approx 0.045$$

Recall from the notes that in order to use the Ideal Gas EOS, we must have $T_R \geq 2$ and $P_R \geq 0.05$, such that $0.98 \leq Z \leq 1.02$.

\Rightarrow It looks like the H_2O exhibits real-gas behavior.

\Rightarrow Using Figure A-15 [see inset of Figure A-15(a)]

$$Z \approx 0.95$$

which falls just outside the range of Ideal-gas validity [this range is an engineer's discretion, of course]

\Rightarrow using

$$pV = ZRT$$

with

$$R = 0.4615 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \text{ (Table A-1)}$$

$$p \frac{V}{m} = ZRT$$

(35)

$$\boxed{V_{\text{real}} = \frac{ZmRT}{p}}$$

plugging in values,

$$V_{\text{real}} \approx \frac{(0.95)(0.05 \text{ kg})(0.4615 \frac{\text{kJ}}{\text{kg}\cdot\text{K}})(473\text{K})}{1000 \text{ kPa}}$$

$$\boxed{V_{\text{real}} \approx 0.01037 \text{ m}^3}$$

This corresponds to a Sphere Diameter of $V = \frac{4}{3}\pi(\frac{D}{2})^3$

$$\Rightarrow V_{\text{real}} = \frac{\pi D^3}{6}$$

$$\Rightarrow D_{\text{real}} = \left[\frac{6V}{\pi} \right]^{1/3} = \left[\frac{6 \times 0.01037 \text{ m}^3}{\pi} \right]^{1/3}$$

$$\boxed{D_{\text{real}} \approx 0.270 \text{ m}} \quad (27 \text{ cm diameter})$$

If we were to use the Ideal gas EOS., we would have obtained

$$\boxed{V_{\text{Ideal}} \approx 0.01091 \text{ m}^3} \quad (\text{Error} \approx 5\%)$$

The Diameter corresponds to

$$\boxed{D_{\text{Ideal}} \approx 0.275 \text{ m}} \quad (27.5 \text{ cm diameter})$$

In this case, the size of the tank is not very sensitive to the real gas effects.

Lecture 11 Specific Heats C_p and C_v

①

- We have just learned how to compute the properties of a system at a fixed Thermodynamic State;

$$\{p, T, v, u, h, s, x\}$$

- We also learned that we only need two intensive Properties in order to determine any other intensive Property at the Same Thermodynamic State, as long as the System is comprised of a pure, simple compressible Substance:

$$z = f(x, y)$$

- We learned about p-v-T diagrams for a substance; this is an "Equation of State" relating pressure, Temperature, and specific volume for any phase of matter.
- We learned that the property tables in the Book's Appendix are a more encompassing Equation of State than just a p-v-T relationship because they relate p-v-T-u-h-s for solids, liquids, saturated liquids & vapors, and superheated vapors.

- Question : How did the Authors relate $p-v-T$ to $u-h-s$ in order to create the full Equation of State tables?

- A More important Question : In thermodynamics we are only interested in computing changes in System Properties (recall the arbitrary, convenient "zero datum" reference state?)

⇒ How do changes in $p-T-v$ relate to changes in $u-h-s$?

i.e., $\boxed{\{\Delta p, \Delta T, \Delta v\} \overset{??}{\rightleftharpoons} \{\Delta u, \Delta h, \Delta s\}} \quad ??$

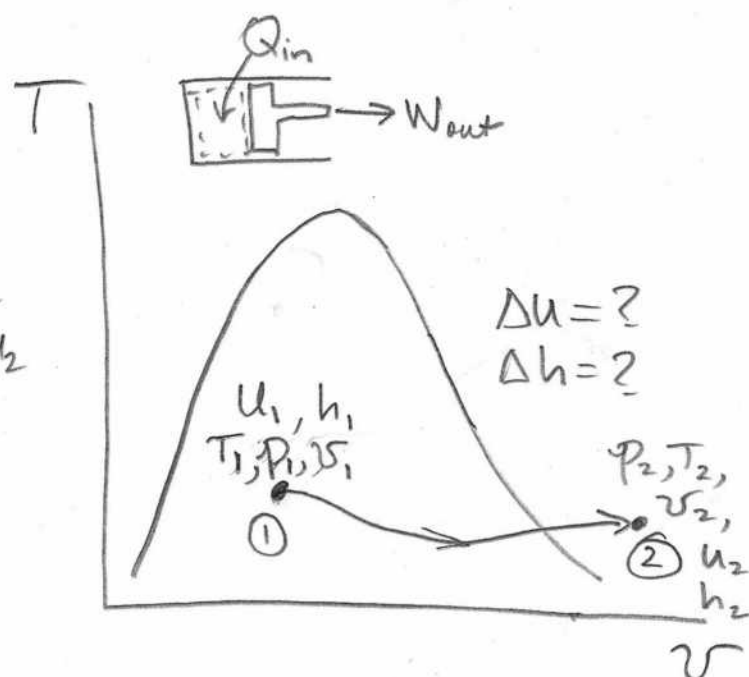
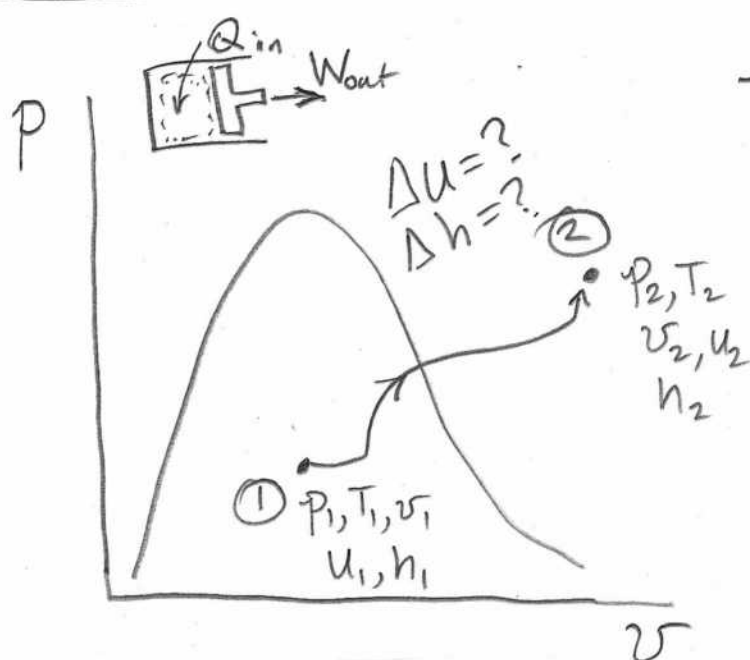
- For now forget about Entropy, S , as we will get to that later.
- Right now we want to know how to relate u & h of a system at two different States, to T , p , and v at those States no matter if the process was quasi-equilibrium or non-equilibrium.

Bottom Line: we want to know

the functions that describe:

$$(1) \Delta U \equiv U_2(T_2, P_2) - U_1(T_1, P_1)$$

$$(2) \Delta h \equiv h_2(P_2, v_2) - h_2(P_1, v_1)$$



Q: Where are we going with this?

A: Once we understand how to relate changes in Properties, we will then discuss (separately) Interactions with the System in the form of Work & Heat.

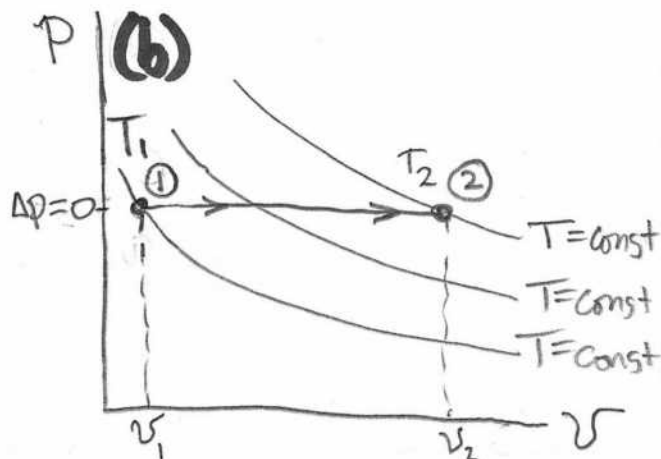
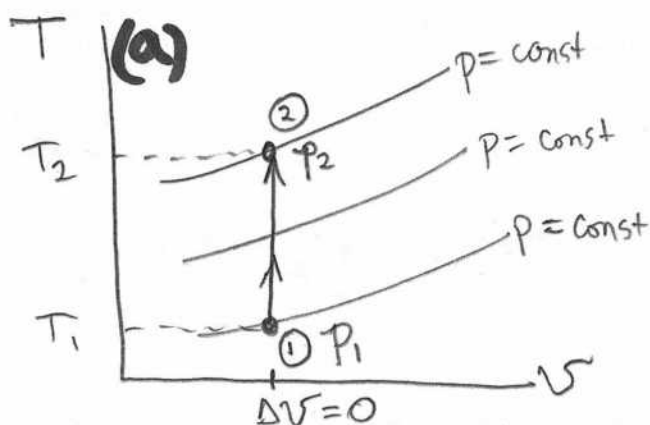
⇒ Work & Thermal Energy Transfer DRIVE Changes in System Properties and Conservation Equations Connect them All!

Specific "Heat"

Specific Heat is the amount of Energy Transfer required to raise the temperature of a unit mass of a substance by 1° (either Celcius or Fahrenheit)

- Notice that Specific "Heat" is a very bad nomenclature because the Net Energy transfer could be accomplished by Q , W , or a combination of both!
- It really should be called "Specific Energy Transfer" but we are stuck with terminology left over from the 1800's when Thermodynamics was still being researched.
- We are interested in two types of specific heats constrained (temporarily) by:

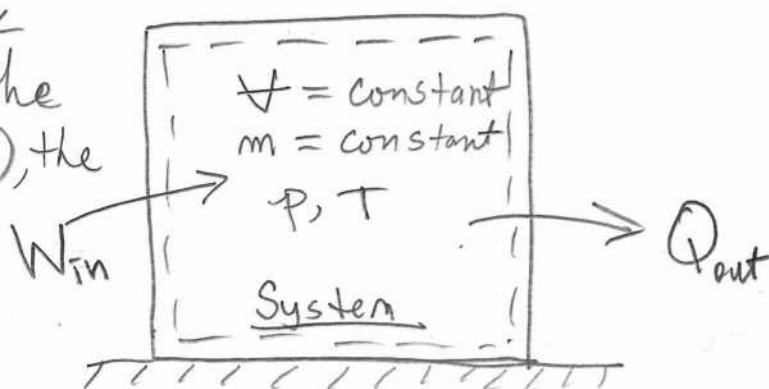
- or
- (a) Constant Volume Process ($\Delta v \equiv 0$)
 - (b) Constant pressure Process ($\Delta p \equiv 0$)



A) Specific Heat at Constant Volume, c_v

- Consider a rigid box in which the System material inside is a pure, simple, compressible substance. For now, Consider a quasiequilibrium process.

- As heat and/or work are performed on the System (Quasiequilibrium), the temperature and pressure will change.



- We require two independent, intensive Properties to describe the State of a pure, Simple compressible substance. We can write this using the Chain rule of Calculus:

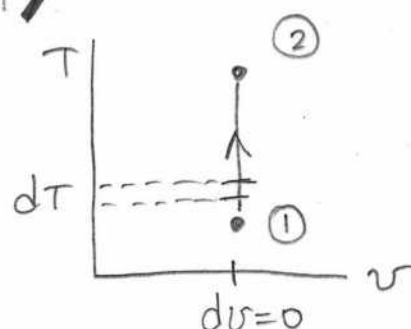
$$du = \left(\frac{\partial u}{\partial T} \right)_v dT + \left(\frac{\partial u}{\partial v} \right)_T dv \quad [\text{an "exact" differential}]$$

- For this process, the System Volume doesn't change (Constant Volume process) so $dv \equiv 0$ and

$$du = \left(\frac{\partial u}{\partial T} \right)_v dT + \left(\frac{\partial u}{\partial v} \right)_T dv$$

$$du = \left(\frac{\partial u}{\partial T} \right)_v dT$$

(infinitesimal step \Rightarrow quasiequilibrium process)



(6)

Constant Volume Specific Heat, c_v

$$C_v \equiv \left(\frac{\partial u}{\partial T} \right)_v$$

$$\left(\frac{\text{kJ}}{\text{kg} \cdot ^\circ\text{C}} \right) \text{ or } \left(\frac{\text{Btu}}{\text{lb}_m \cdot ^\circ\text{F}} \right)$$

If $u = u(T)$ only, Then

$$du = C_v(T) dT$$

$$\Rightarrow C_v = C_v(T)$$

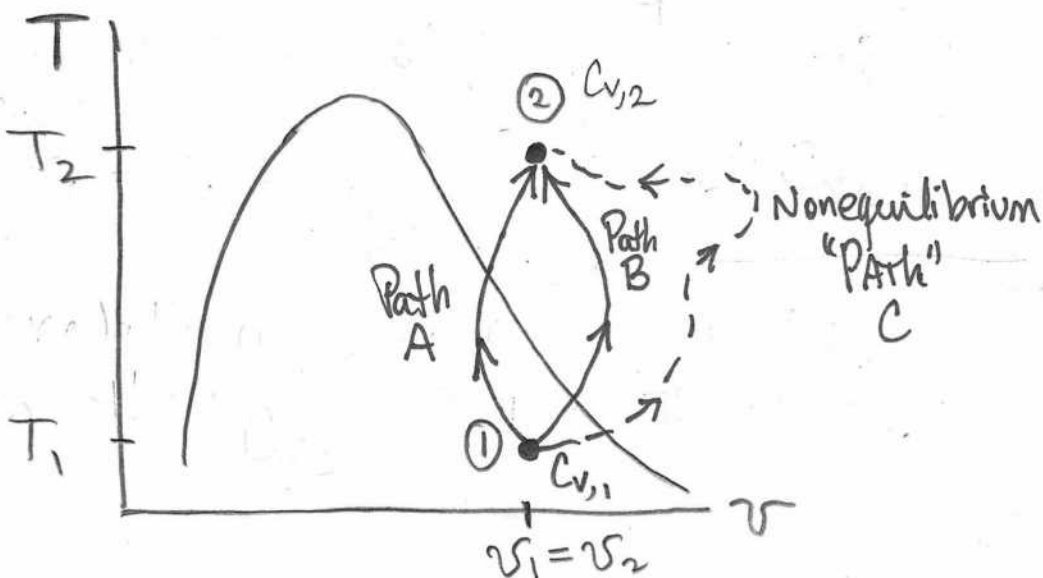
- In General, it's a function of two intensive prop.:

$$C_v = C_v(p, T) = C_v(p, v) = C_v(T, v), \text{ etc.}$$

- Although we defined C_v for a constant specific volume, quasiequilibrium process, C_v is itself an intensive property of the System and therefore C_v can be defined at any two States ① & ②, independent of the process path:

Note:

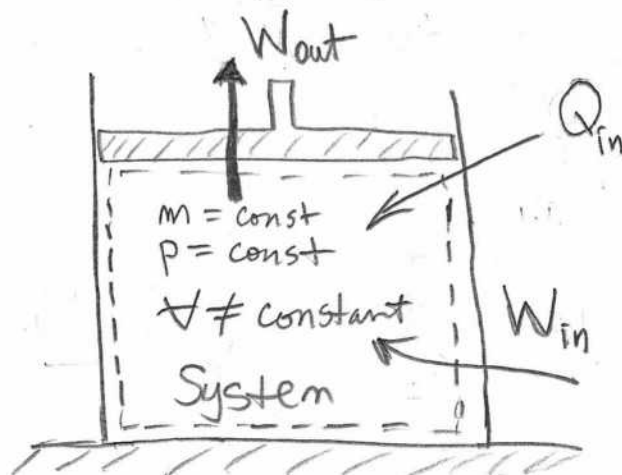
We use C_v for Constant Volume processes
 \Rightarrow This is meant to illustrate that C_v IS a Property of a State



(B) Specific Heat at Constant Pressure, C_p

- Consider a Piston-cylinder device in which the material (gas, liquid, or solid) is a pure, simple Compressible substance.

- As Heat and/or Work are performed on the System, the Volume is allowed to change so that the System pressure remains Constant



- The System's Enthalpy per unit mass is

$$h = u + p\upsilon \quad \left(\frac{\text{kJ}}{\text{kg}}\right) \text{ or } \left(\frac{\text{Btu}}{\text{lb}_m}\right)$$

- The advantage of this property is that you don't need to keep track of both u and " $p\upsilon$ " work when either the boundary moves (Closed Systems) or when mass is flowing (Open Systems)
- Specific Enthalpy, h , is an intrinsic property

$$h = h(p, \upsilon) = h(T, \upsilon) = h(p, T) \text{ etc.}$$

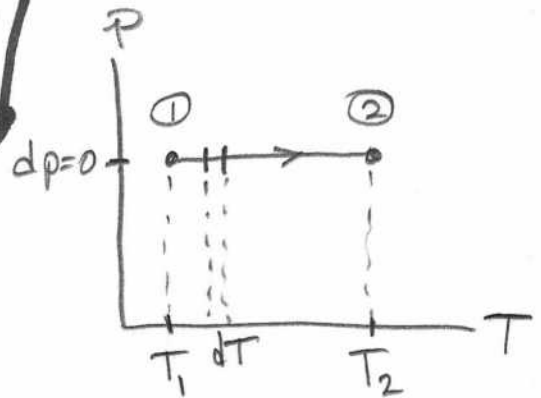
We can again use the chain rule of Calculus:

[An "exact" differential]

$$dh = \left(\frac{\partial h}{\partial T} \right)_p dT + \left(\frac{\partial h}{\partial p} \right)_T dp$$

so that

$$dh = \left(\frac{\partial h}{\partial T} \right)_p dT$$



Constant Pressure Specific Heat

$$C_p \equiv \left(\frac{\partial h}{\partial T} \right)_p$$

$$\left(\frac{\text{kJ}}{\text{kg} \cdot ^\circ\text{C}} \right) \text{ or } \left(\frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{F}} \right)$$

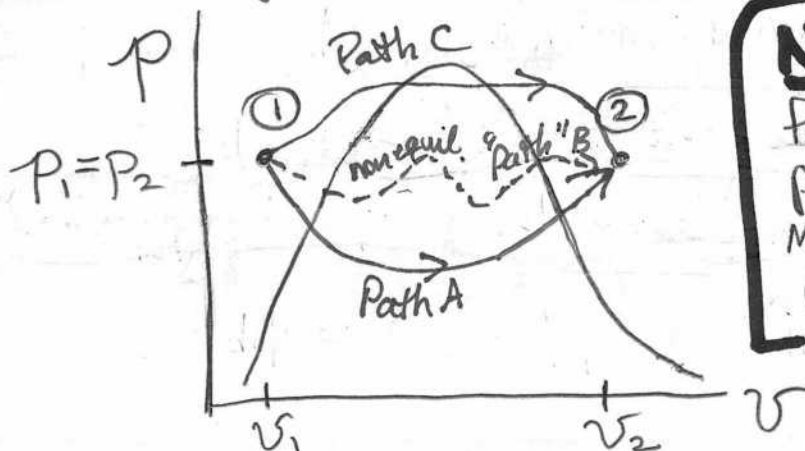
If $h = h(T)$ only, then $dh = C_p(T) dT$

$$\rightarrow C_p = C_p(T)$$

- In General, it's a function of two independent intensive properties:

$$C_p = C_p(p, T) = C_p(p, v) = C_p(T, v)$$

- Again, C_p can be defined at any two States ① & ②, independent of the Process path:



Note: We use C_p for constant pressure processes \rightarrow this is meant to illustrate that C_p IS a Property of a State

Specific Heat Ratio

The ratio of C_p to C_v is defined as

$$K \equiv \frac{C_p}{C_v} > 1 \text{ always}$$

and is another Property of a System

Q: Why is K always greater than 1?

A: Consider two closed Systems. One has rigid sides but the other has a piston on top.

The mass of substance inside is the same.

$C_v = 5 \frac{\text{kJ}}{\text{kg} \cdot ^\circ\text{C}}$

$W_{in} + Q_{in} = 5 \text{ kJ}$

$V = \text{Constant}$
 $m = 1 \text{ kg}$
 $\Delta T = 1^\circ\text{C}$

Constant Volume Process

$C_p = 7 \frac{\text{kJ}}{\text{kg} \cdot ^\circ\text{C}}$

$P = \text{Constant}$
 $m = 1 \text{ kg}$
 $\Delta T = 1^\circ\text{C}$

Constant Pressure Process

$Q_{in} + W_{in} = 7 \text{ kJ}$

- For both processes, we added just enough Net Energy ($Q_{in} + W_{in}$) to raise the System temperature by $\Delta T = 1^\circ\text{C}$
- However, it takes more Energy to achieve $\Delta T = 1^\circ\text{C}$ for the constant pressure process because Extra Energy must be supplied to compensate for the work required to raise the piston.

Lecture 12 Using C_v & C_p to Compute ΔU and Δh for Solids, Liquids, Gases

- In this lecture we will be computing ΔU and Δh of a system for various phases of a pure, simple compressible substance using C_p & C_v .

Incompressible Substance

A Fictitious substance for which

$$\text{Density } \rho = \frac{1}{v} \equiv \text{Constant}$$

Note:

(1) For many practical applications ^{ME 315 # 316}, Solids and Liquids can be approximated as Incompressible Substances.

(2) The "Incompressible Approximation" implies that:

- The Volume of a fixed mass doesn't change much during a process.
- The Energy required to change the mass' Volume is much larger than the Energies to change other Properties (Temperature, Shape, Velocity, etc.)

Incompressible Approx. for ① Solid Phase Δu & Δh Calculations

②

- In ME 315 and 316, we will consider matter in the Solid Phase to be approximately Incompressible.
- Experimentally and Mathematically, it can be shown that the Specific Heats at Constant Pressure and Constant Volume for an Incompressible Substance are
 - (a) Identical to each other
 - (b) A Function of Temperature Only

$$\Rightarrow \boxed{C_p(T) \approx C_v(T) \equiv C(T)}$$

- For solids then,

$$dh = d(u + pv)$$

$$dh = du + p dv + v dp$$

≈ 0 for a solid
(incompressible Approx)

≈ 0 for a solid
(by observation of experiments)

so

$$dh = du = C(T) dT$$

$$\boxed{\Delta h = \Delta u = \int_{T_1}^{T_2} C(T) dT}$$

- If $C(T)$ doesn't change much during the process, then

$$C(T) \approx C_{avg} = \text{constant}$$

So

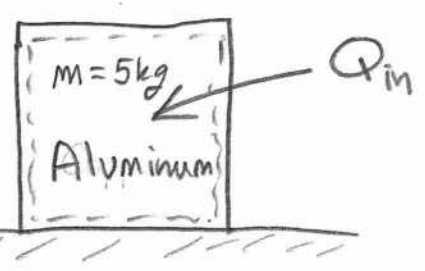
$$U_2 - U_1 = C_{avg} \int_{T_1}^{T_2} dT$$

$$U_2 - U_1 = C_{avg} (T_2 - T_1)$$

$$\Delta U = C_{avg} (T_2 - T_1)$$

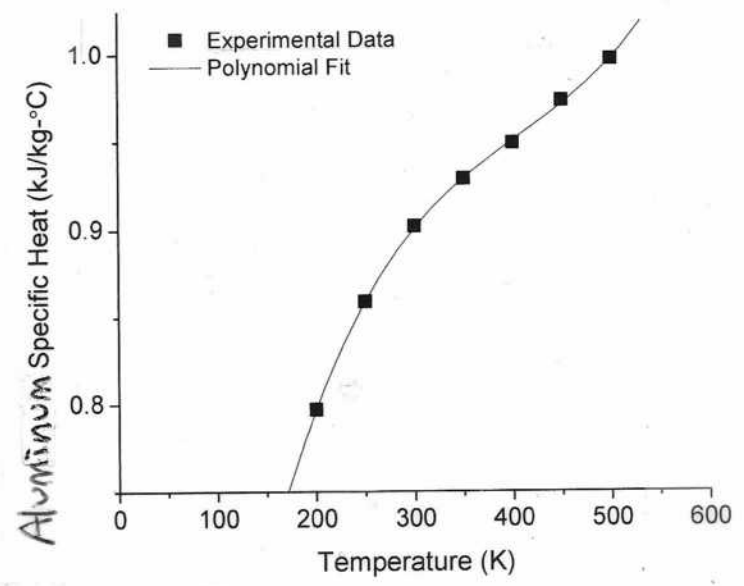
Example 1 Heating a block of Aluminum

A 5kg solid block of aluminum is heated from 27°C to 177°C . What is the change in the aluminum block's Internal energy per unit mass, Δu ?



Perform the calculations in two ways:

- (a) Using $C(T)$ obtained from Appendix A-3b in textbook
- (b) Approximate C using an average value



Solution

- Assumptions**
- (1) Incompressible solid $\left\{ \begin{array}{l} (a) C = C(T) \\ (b) C \approx C_{avg} = \text{const} \end{array} \right.$
 - (2) Closed System
 - (3) $\Delta KE = 0$, $\Delta PE = 0$
sys sys

Analysis

(a) The data listed in Table A-3b for aluminum has been plotted in the graph above, and a 3rd order polynomial curve fit gives:

$$C(T) = a_0 + a_1 T + a_2 T^2 + a_3 T^3$$

where

$$a_0 = 0.21771 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$a_1 = 0.00466 \frac{\text{kJ}}{\text{kg} \cdot \text{K}^2}$$

$$a_2 = -1.05238 \times 10^{-5} \frac{\text{kJ}}{\text{kg} \cdot \text{K}^3}$$

$$a_3 = 8.66667 \times 10^{-9} \frac{\text{kJ}}{\text{kg} \cdot \text{K}^4}$$

use Kelvin Here Due to Powers!

Why use Kelvin Here?

$$(60^\circ\text{C} - 20^\circ\text{C})^2 \neq (60^\circ\text{C})^2 - (20^\circ\text{C})^2$$

The change in internal energy, using the specific heat for an incompressible material is

$$\Delta U = \int_{T_1}^{T_2} C(T) dT$$

substituting in the polynomial,

$$\Delta U_{\text{Fit}} = \int_{T_1}^{T_2} (a_0 + a_1 T + a_2 T^2 + a_3 T^3) dT$$

$$\Delta U_{\text{Fit}} = a_0 \int_1^2 dT + a_1 \int_1^2 T dT + a_2 \int_1^2 T^2 dT + a_3 \int_1^2 T^3 dT$$

(5)

$$\Delta U = a_0(T_2 - T_1) + a_1 \cdot \frac{1}{2}(T_2^2 - T_1^2) + a_2 \cdot \frac{1}{3}(T_2^3 - T_1^3) + a_3 \cdot \frac{1}{4}(T_2^4 - T_1^4)$$

Each term is

$$(i) a_0(T_2 - T_1) = 0.21771 \frac{\text{kJ}}{\text{kg} \cdot ^\circ\text{C}} (177^\circ\text{C} - 27^\circ\text{C})$$

$$= 32.6565 \frac{\text{kJ}}{\text{kg}}$$

$$(ii) \frac{a_1}{2}(T_2^2 - T_1^2) = \frac{0.00466 \frac{\text{kJ}}{\text{kg} \cdot ^\circ\text{C}^2}}{2} [(177+273)^2 - (27+273)^2]$$

Use Kelvin!!

$$= 0.00233 \frac{\text{kJ}}{\text{kg} \cdot \text{K}^2} [(450\text{K})^2 - (300\text{K})^2]$$

$$= 262.125 \frac{\text{kJ}}{\text{kg}}$$

$$(iii) \frac{a_2}{3}(T_2^3 - T_1^3) = \frac{-1.05238 \times 10^{-5} \frac{\text{kJ}}{\text{kg} \cdot \text{K}^3}}{3} [(450\text{K})^3 - (300\text{K})^3]$$

$$= -224.946 \frac{\text{kJ}}{\text{kg}}$$

$$(iv) \frac{a_4}{4}(T_2^4 - T_1^4) = \frac{8.66667 \times 10^{-9} \frac{\text{kJ}}{\text{kg} \cdot \text{K}^4}}{4} [(450\text{K})^4 - (300\text{K})^4]$$

$$= 106.397 \frac{\text{kJ}}{\text{kg}}$$

So,

$$\Delta U_{\text{fit}} = 32.6565 \frac{\text{kJ}}{\text{kg}} + 262.125 \frac{\text{kJ}}{\text{kg}} - 224.946 \frac{\text{kJ}}{\text{kg}} + 106.397 \frac{\text{kJ}}{\text{kg}}$$

$$\Delta U_{\text{fit}} = 176.2 \frac{\text{kJ}}{\text{kg}}$$

(b) Approximate the Specific heat using an average value ^⑥

From the Table A-3b, between 300 K and 450 K,

$$c \approx c_{avg} = \frac{0.902 + 0.929 + 0.949 + 0.973}{4} \frac{\text{kJ}}{\text{kg} \cdot ^\circ\text{C}}$$

$$c_{avg} = 0.938 \frac{\text{kJ}}{\text{kg} \cdot ^\circ\text{C}}$$

then, $\Delta U_{avg} = c_{avg} (T_2 - T_1)$

$$\Delta U_{avg} = \left(0.938 \frac{\text{kJ}}{\text{kg} \cdot ^\circ\text{C}} \right) (177^\circ\text{C} - 27^\circ\text{C})$$

$$\Delta U_{avg} = 140.7 \frac{\text{kJ}}{\text{kg}}$$

The Error incurred using this averaging of c is

$$\text{Error} = \frac{176.2 \frac{\text{kJ}}{\text{kg}} - 140.7 \frac{\text{kJ}}{\text{kg}}}{176.2 \frac{\text{kJ}}{\text{kg}}}$$

$$\text{Error} \approx 20\%$$

Note: Using a Wiser choice of c_{avg} will reduce the error.

Incompressible Approx. for (II) Liquid Phase ΔU & Δh Calculations

(A) ΔU

- Assuming that the liquid substance is incompressible, then the internal specific energy change for a constant volume process is

$$du \equiv c_v(T) dT$$

where again it can be shown that c_v is a function of temperature only for an incompressible substance.

$$\text{Thus, } \Delta U = U_2 - U_1 = \int_{(1)}^{(2)} c(T) dT$$

Using an average value for the specific heat,

$$\int_{(1)}^{(2)} c_{avg} dT = c_{avg} \int_{(1)}^{(2)} dT = c_{avg} (T_2 - T_1) = c_{avg} \Delta T$$

so that

$$\Delta U \approx c_{avg} \Delta T$$

(Assuming the liquid is incompressible, $v = \text{constant}$)

(B) Δh

- On page 2 of this Lecture it was shown that

$$dh = du + p dv + v dp$$

Special Case 1 Incompressible substance Approx. for Flowing liquids (Heating/Cooling)

In this case, the flowing liquid is cooled/heated at constant pressure so

$$dp \approx 0$$

$$\Rightarrow dh \approx du + p dv$$

The compressibility of flowing liquid (water) in these types of systems is very small,
so again the flow may be approximated to be incompressible:

$$dv \approx 0$$

$$\Rightarrow dh \approx du \approx c(T) dT$$

Using an average value for specific heat,

$$\Delta h \approx \Delta u \approx C_{avg} \Delta T$$

Special Case 2

Incompressible substance Approx. for pumping liquids to a compressed state

In this case, liquid is pumped from one place (state) to another (state) such that **the temperature is \approx constant (isothermal process)**

$$\Rightarrow dh \approx du + p d\cancel{v} + v dp$$

(incompressible approx)

$$dh \approx c_v d\cancel{T} + v dp$$

(isothermal process)

so

$$\boxed{dh \approx v dp}$$

with $v \equiv \text{constant}$, integrating between States ① and ② of the liquid gives

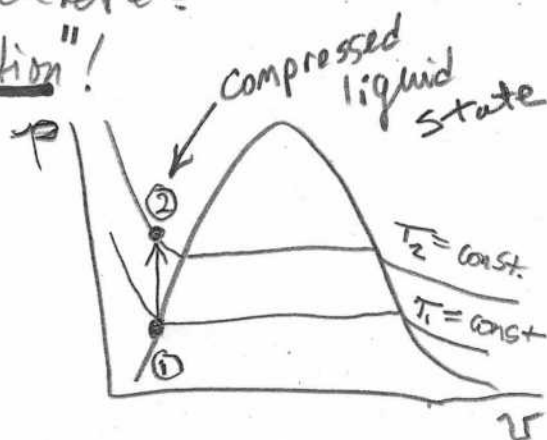
$$\boxed{\Delta h = v \Delta p}$$

Q: Where have we seen this before?

A: The "Compressed Liquid Approximation"!

State ① = Saturated Liquid,


$$\boxed{h(p_2, T_2) \approx h_{f@T_{sat①}} + v_{f@T_{sat①}} (p_2 - p_{sat①})}$$

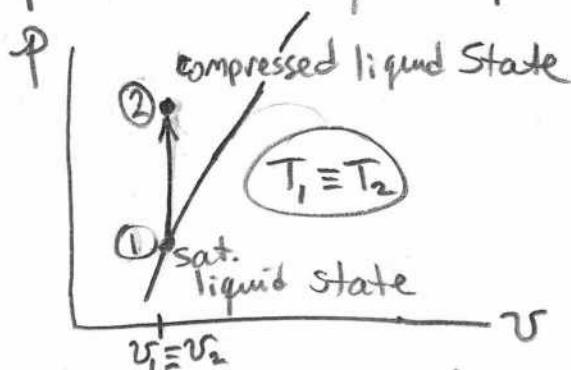


Stop!! Think!

(10)

Q: What is contradictory about "Case 2"?

A: We are assuming that the liquid being pumped is incompressible and Isothermal while pumping within the compressible liquid phase region !! 



⇒ The assumption of the process being both incompressible and isothermal violates the 2nd Law of thermodynamics (which we will learn later) ----- but only "slightly" because unlike superheated vapor, compressed liquid properties are not much different from corresponding saturated liquid values.

⇒ In actuality, the liquid's temperature would rise as the specific volume decreased (density ↑)

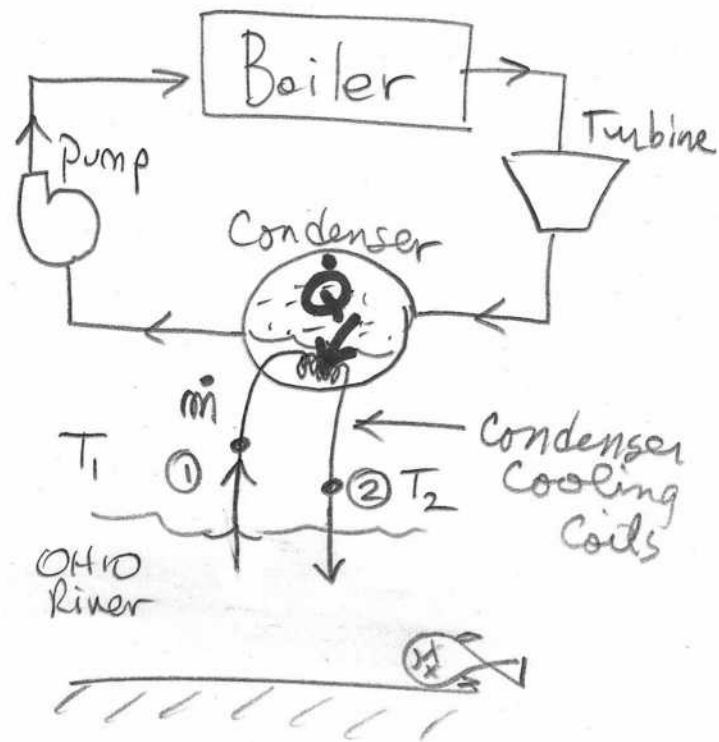
See page 557, 2nd Paragraph. The answer is "no!" ---- If water were truly incompressible then its temperature would not increase and the 2nd Law of thermo would not be violated but incompressible substances are purely fictitious (I don't know anything about neutron stars)

Example 2 Condenser Cooling Coil

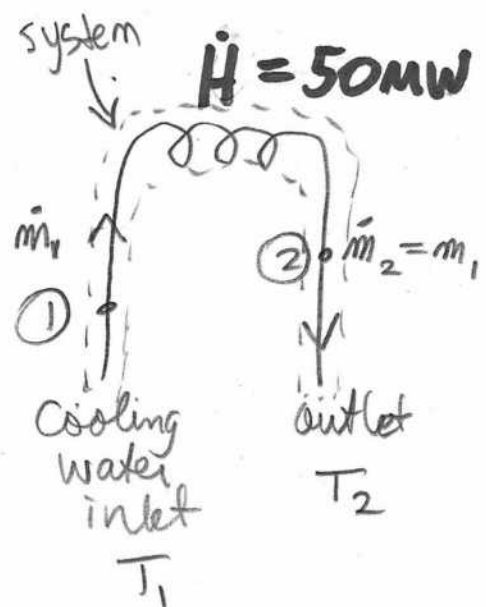
(11)

- An electricity generating power plant operates on a thermodynamic cycle with H_2O as the working fluid. Heat must be rejected to the Ohio river for the plant to operate.

The condenser rejects this heat through cooling coils as shown \rightarrow



- It is known that the rate of Thermal energy transfer into the cooling water results in an Enthalpy rate of 50MW. The E.P.A. Mandates that ΔT must be no greater than $50^\circ C$.



What is the minimum cooling water flow rate?

Solution

(12)

Given: $\dot{H} = 50 \text{ MW}$
 $\Delta T_{\min} = 50^\circ \text{C}$

$$\left(\dot{H} = \frac{dH}{dt} \equiv \text{constant} \right)$$

Find: \dot{m}_{water}

- Assume:
- (1) incompressible working fluid (the cooling water)
 - (2) Average specific heat of water is $C = 4.18 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$ (Table A-3(a))
 - (3) Negligible pressure change in steady flow coolant

Analysis For Flowing liquids where the pressure change along the pipe length is negligible (cooling & heating applications)

$$\Delta h \approx C_{p,\text{avg}} \Delta T \quad \left(\frac{\text{kJ}}{\text{kg}} \right)$$

On an Energy basis,

$$\Delta H_{\text{avg}} = \dot{m} C_{p,\text{avg}} \Delta T \quad (\text{kJ})$$

We will learn in Chapter 5 for Steady Flows

$$\dot{H} \approx \dot{m} C_{p,\text{avg}} \Delta T \quad (\text{kW or MW, etc})$$

$$\Rightarrow \dot{m} = \frac{\dot{H}}{C_{p,\text{avg}} \Delta T} = \frac{50 \times 10^6 \text{ J/s}}{\left(4.18 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \times 1000 \frac{\text{J}}{\text{kJ}} \right) (50^\circ \text{C})}$$

$$\dot{m} \approx 240 \text{ kg/s}$$

III) Ideal Gas $\Delta u, \Delta h$

(12)

In 1843, Joule experimentally demonstrated that for an Ideal Gas, the Internal Energy is a function of Temperature only

$$u = u(T) \quad \underline{\text{Ideal Gas}} \quad \left(\frac{\text{kJ}}{\text{kg}}\right)$$

For an Ideal gas $pv = RT$

plug in

$$h \equiv u(T) + pv$$

$$h = u(T) + RT$$

$h(T) = \text{function of temperature only}$

$$h = h(T) \quad \underline{\text{Ideal Gas}} \quad \left(\frac{\text{kJ}}{\text{kg}}\right)$$

The specific heats of an Ideal gas are

$$C_p \equiv \left(\frac{\partial h}{\partial T}\right)_p = \left(\frac{dh}{dT}\right)_p = \text{function of } T \text{ only}$$

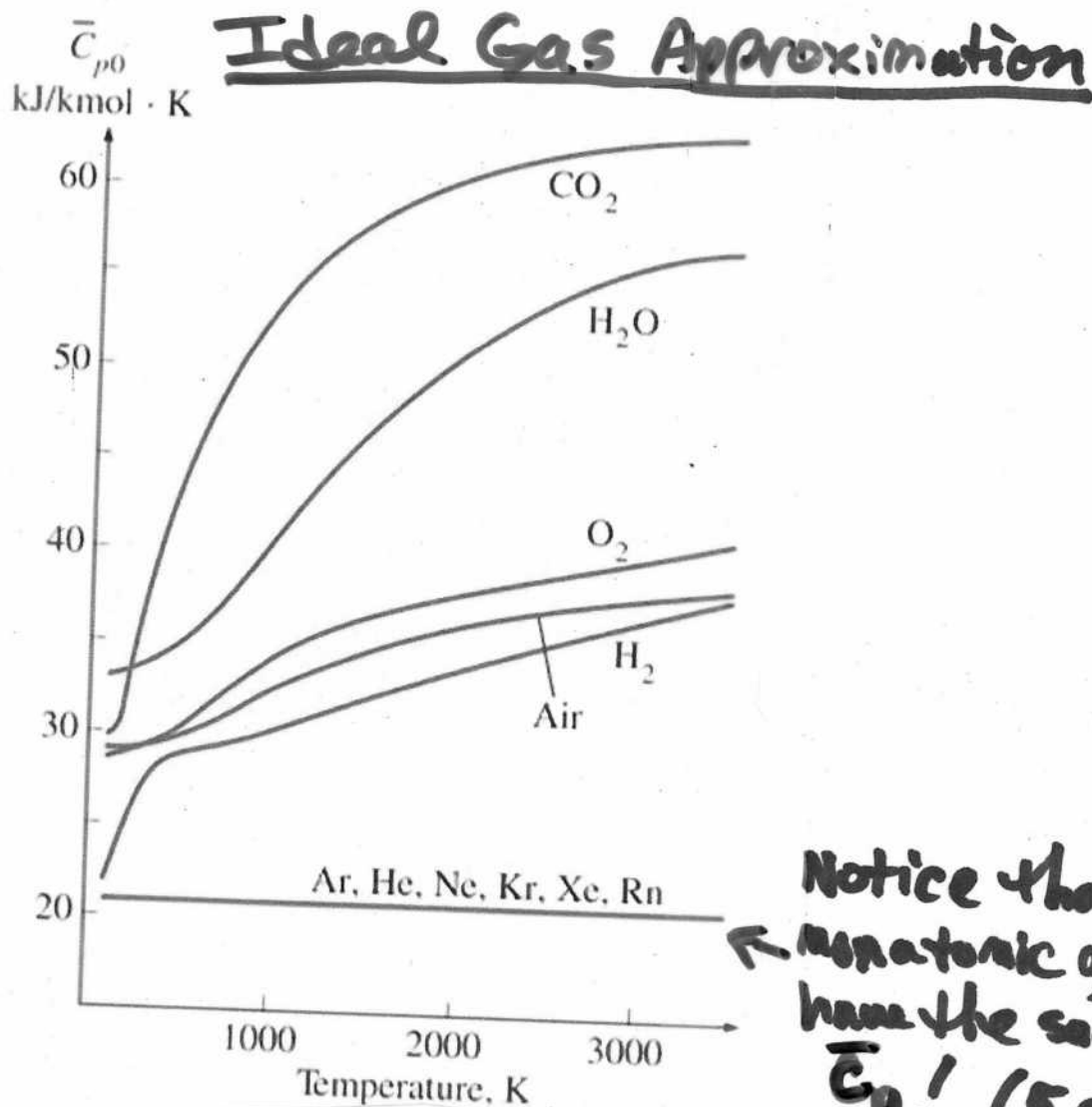
$$\Rightarrow \quad \underline{C_p = C_p(T) \quad \text{Ideal Gas}} \quad \left(\frac{\text{kJ}}{\text{kg-K}}\right)$$

$$C_v \equiv \left(\frac{\partial u}{\partial T} \right)_v = \left(\frac{du}{dT} \right)_v = \text{function of } T \text{ only}$$

(13)

$$\Rightarrow \boxed{C_v = C_v(T) \text{ Ideal Gas}} \left(\frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right)$$

On a Molar basis, we put a "bar" over the top to indicate that the property has units of per-unit mol instead of per-unit-mass:



Notice that
 ← monatomic gases
 have the same
 $\bar{C}_p!$ (EQ)
 on a molar basis

Note : Recall from Lecture 11 That

- $du = C_v dT + \left(\frac{\partial u}{\partial v}\right)_T dv$
- $dh = C_p dT + \left(\frac{\partial h}{\partial p}\right)_T dp$

In Sections

I Solid phase : $du = dh \approx c(T) dT$ because

- Incompressible Approx ($\Delta v \approx 0$) is a good assumption
- Small change in pressure ($\Delta p \approx 0$) is a good assumption

II Liquid Phase :

(i) $du \approx c(T) dT$ because

- Incompressible Approx ($\Delta v \approx 0$) is a good assumption

(ii) Since $h \equiv u + pv$, then $dh = du + p dv + v dp$

- Two special cases were discussed

(1) Incompressible flowing liquid for heating & cooling

- The pressure gradient in the fluid flowing along the pipe is "small" $\Rightarrow \Delta p \approx 0$

- Incompressible Flow $\Rightarrow \Delta v \approx 0$

$$\Rightarrow dh = du = c(T) dT$$

(2) Incompressible substance approx. for pumping liquids

- Incompressible ($\Delta v \approx 0$)
- Isothermal process ($du = c_v dT$)

$$\Rightarrow dh \approx v dp \Rightarrow h(p, T) \approx h_{sat} + v_{sat} (p - p_{sat})$$

III Ideal Gas ($pv = RT, z = 1$)

- An Ideal Gas is often compressible !!
- $u = u(T), h = h(T)$ because intermolecular forces are weak (low density)

(I, II) have physically different reasons than (III) for $u = u(T)$ and $h = h(T)$

For an Ideal Gas,

(A) Δu

$$du = C_v(T) dT$$

$$\Rightarrow \boxed{u_2 - u_1 = \int_{T_1}^{T_2} C_v(T) dT}$$

See Tables
A-17 thru
A-25
in the Appendix

If $C_v(T)$ doesn't vary much over ΔT , then

$$\Delta u \approx C_{v,avg} \int_{T_1}^{T_2} dT$$

or

$$\boxed{\Delta u \approx C_{v,avg} \Delta T} \left(\frac{\text{kJ}}{\text{kg}} \right)$$

(B) Δh

$$dh = C_p(T) dT$$

$$\Rightarrow \boxed{h_2 - h_1 = \int_{T_1}^{T_2} C_p(T) dT}$$

See Tables
A-17 through
A-25 in
Appendix

again,

$$\boxed{\Delta h \approx C_{p,avg} \Delta T}$$

Note :

- The Ideal gas tables A-17 to A-25 (SI) and A-17E to A-25E (English) use a "Zero datum"

$$\left. \begin{aligned} \bar{u} &\equiv 0 \frac{\text{kJ}}{\text{kmol}} \\ \bar{h} &\equiv 0 \frac{\text{kJ}}{\text{kmol}} \\ \bar{s}^0 &\equiv 0 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}} \end{aligned} \right\} \text{ at } T \equiv 0 \text{ Kelvin}$$

(the air table A-17 is a per mass basis but it doesn't matter)

- Using this reference datum, for State ①

$$u(T) - u_{\text{ref}}^0 \approx C_{v, \text{avg}} (T - T_{\text{ref}}^0)$$

$$\boxed{u(T) \approx C_{v, \text{avg}} T}$$

$$h(T) - h_{\text{ref}}^0 \approx C_{p, \text{avg}} (T - T_{\text{ref}}^0)$$

$$\boxed{h(T) \approx C_{p, \text{avg}} T}$$

- With the understanding of $u=0, h=0$ at $T=0$, these can be useful Ideal gas Relations later in the class.

Important Ideal Gas Relationships

Consider Enthalpy for an Ideal gas:

$$h = u + RT$$

Differentiating,

$$dh = du + RdT$$

Replacing dh and du by the specific heat relationships, we get

$$C_p dT = C_v dT + R dT$$

cancelling out dT ,

$$\left(\frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right)$$

$$C_p(T) = C_v(T) + R$$

Easy way to
Remember is that
 $C_p > C_v$

On a Molar Basis, multiply by the substance's
molecular mass, M (kg/kmol):

$$\underbrace{M C_p(T)}_{\bar{C}_p(T)} = \underbrace{M C_v(T)}_{\bar{C}_v(T)} + \underbrace{M R}_R$$

So

$$\bar{C}_p(T) = \bar{C}_v(T) + R$$

$$\left(\frac{\text{kJ}}{\text{kmol} \cdot \text{K}} \right)$$

Q: How does $C_p(T)$ & $C_v(T)$ for an Ideal gas relate to k ?

A:

$$k(T) \equiv \frac{C_p(T)}{C_v(T)}$$

From the relationship above,

$$C_p(T) = C_v(T) + R$$

$$\underbrace{\frac{C_p}{C_v}}_{k(T)} = 1 + \frac{R}{C_v}$$

so $k = 1 + \frac{R}{C_v}$

$$k - 1 = R/C_v$$

$$C_v(T) = \frac{R}{k(T) - 1}$$

Note: (1) $k = k(T)$ (Specific Heat Ratio) is NOT independent of temp.
(2) $k > 1$ for physically valid C_v

• It is easy to show that

$$C_p(T) = R \left[\frac{k}{k-1} \right]$$

If the temperature does not vary much,

then we can use $C_{p,avg}$ and $C_{v,avg}$ so that

$$k_{avg} = \frac{C_{p,avg}}{C_{v,avg}} = \text{constant average value}$$

then

$$C_{p,avg} = C_{v,avg} + R \quad (\text{mass})$$

$$\bar{C}_{p,avg} = \bar{C}_{v,avg} + R \quad (\text{molar})$$

and

$$C_{v,avg} = \frac{R}{k_{avg} - 1}$$
$$C_{p,avg} = R \left[\frac{k_{avg}}{k_{avg} - 1} \right]$$

(constant, avg. values)

Example 3

Ideal Gas: Tables vs. Avg. C_v

(19)

Consider a 27 m^3 rigid box containing 3 kg of air. The box is initially at 25°C , 1 atm . Heat is added to the box quasistatically, and the final temperature is 197°C . What is the change in the air Internal Energy, U , using

Solution

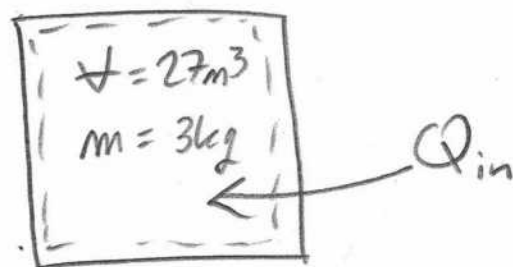
Given:

(a) $C_{v, \text{avg}}$
(b) Table A-17 (Exact)

State ① $\begin{cases} T_1 = 25^\circ\text{C} = 298 \text{ K} \\ P_1 = 1 \text{ atm} \end{cases}$

State ② $\begin{cases} T_2 = 197^\circ\text{C} = 470 \text{ K} \end{cases}$

Assume: Air behaves as an Ideal Gas.



Analysis

(a). The average temperature between State ① & ② is $T_{\text{avg}} = \frac{298 \text{ K} + 470 \text{ K}}{2}$

$$T_{\text{avg}} = 384 \text{ K}$$

From Table A-2(b),

$$C_{v, \text{avg}} \approx 0.726 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

• for an Ideal gas,

$$\begin{aligned} \Delta U &= C_{p, \text{avg}} \Delta T \\ &= \left(0.726 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right) (470 - 298 \text{ K}) \end{aligned}$$

$$\Delta u_{\text{avg}} = 124.872 \frac{\text{kJ}}{\text{kg}}$$

- The mass of air is $m = 3 \text{ kg}$, so

$$\Delta U \approx 124.872 \frac{\text{kJ}}{\text{kg}} \times 3 \text{ kg}$$

$$\Delta U_{\text{avg}} \approx 375 \text{ kJ}$$

(b) Using the Ideal Gas Table A-17,

$$@ T_1 = 298 \text{ K} \Rightarrow u_1 = 212.64 \frac{\text{kJ}}{\text{kg}}$$

$$@ T_2 = 470 \text{ K} \Rightarrow u_2 = 337.32 \frac{\text{kJ}}{\text{kg}}$$

Thus $\Delta u = u_2 - u_1 = 337.32 - 212.64 \frac{\text{kJ}}{\text{kg}}$

$$\Delta u = 124.68 \frac{\text{kJ}}{\text{kg}}$$

With $m = 3 \text{ kg}$,

$$\Delta U = \left(124.68 \frac{\text{kJ}}{\text{kg}} \right) (3 \text{ kg})$$

$$\Delta U_{\text{Table}} \approx 374 \text{ kJ}$$

Error : $\epsilon = \frac{|374 - 375|}{374} \times 100 \Rightarrow \epsilon \approx 0.2\%$

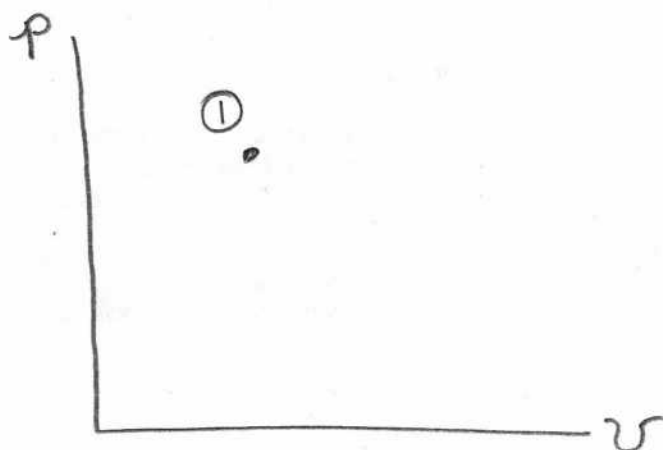


Lecture 13: Energy Transfer by Work and Heat Interactions

Properties are Independent of Process Path

- Recall from Lecture 4 that we said a thermodynamic Property of a System is related to a unique equilibrium State.

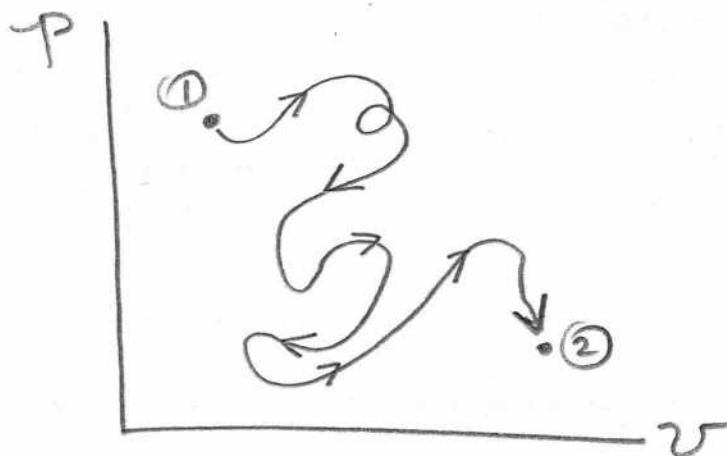
- Consider the State ① shown in the p - v diagram. Along with p_1 & v_1 , there are other properties associated with this state such as $E_{tot,1}$, U_1 , T_1 , PE_1 , KE_1 , etc.



- The System is at State ①, and there is no "History" or "Memory" of how it got there — this is an essential feature of a Property — The State does not depend upon the previous States that the System passed through.

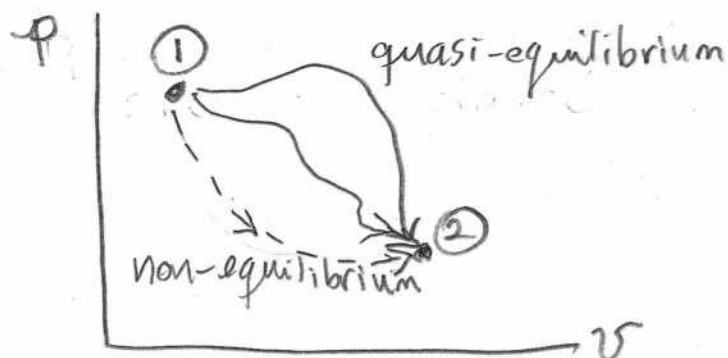
- Now consider two States of the System:

②



When the System is at State ② it also has no Memory of how it got there.

- Recall from Lecture 4 that a Process path indicates that the system had an interaction with the "outside world" (Work & Heat Transfer).
- The fact that when the system is in either State ① or ② it has no memory of how it arrived there means that the Properties of those States do not depend on the path or process that got them there!



(3)

- Since a given State and associated Properties are Path Independent, any change depends only on the Initial and Final States of the System.
- Infinitesimal changes in Properties, because they are path-independent, Properties are represented by perfect differentials:

$$dT, dE, dU, dp, dV, d\psi, dS, \text{etc.}$$

So that we can integrate along any path:

$$\cdot \int_{T_1}^{T_2} dT = T_2 - T_1$$

$$\cdot \int_{P_1}^{P_2} dp = P_2 - P_1$$

$$\cdot \int_{U_1}^{U_2} dU = U_2 - U_1$$

$$\cdot \int_{\psi_1}^{\psi_2} d\psi = \psi_2 - \psi_1$$

etc.

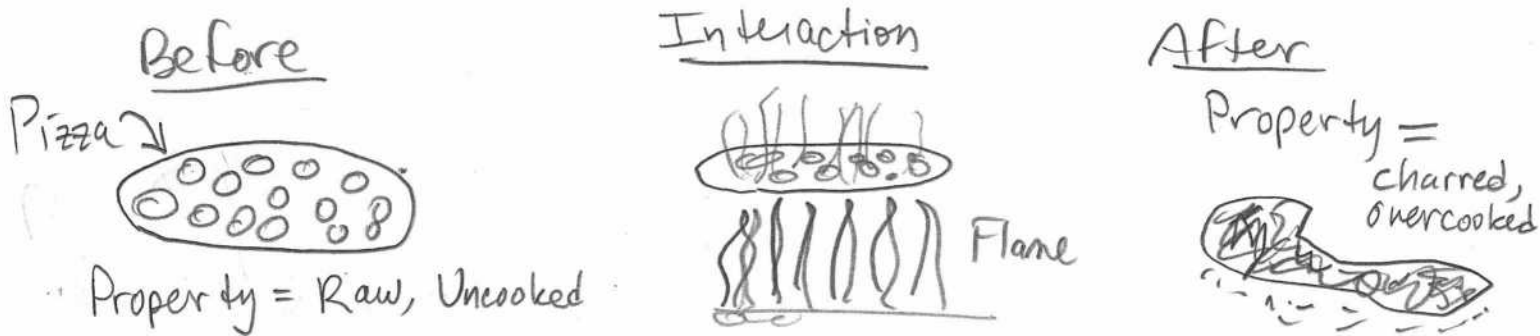
Q: What are the "things" that drive changes in a System's Properties,
ie., from State ① \rightarrow State ②?

- A: (1) Work Interactions with the System (4)
 (2) Thermal Interactions " " "
 (3) An Isolated System may undergo a Spontaneous Process.

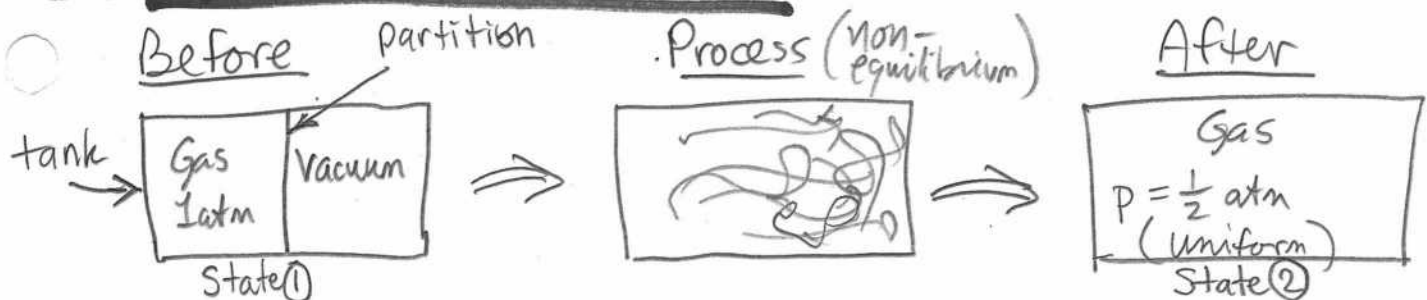
(1) Work Interaction



(2) Heat Interaction



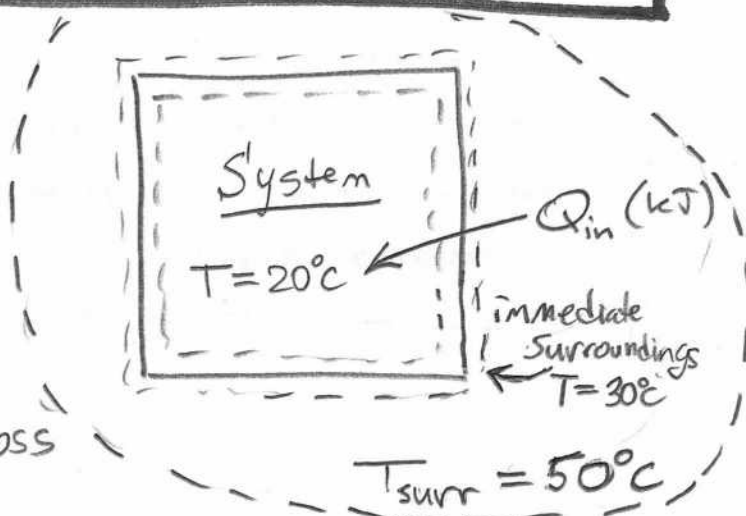
(3) Spontaneous Process



(A) Thermal Energy Transfer (Heat Transfer)

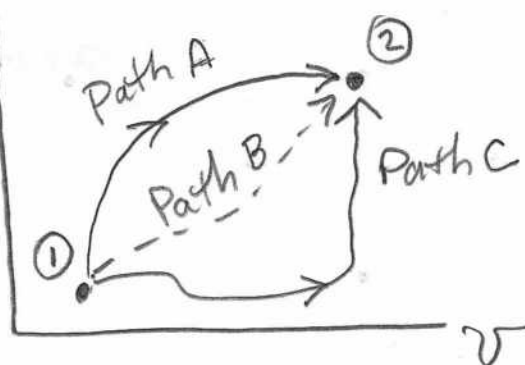
Heat "Heat" is random Thermal Energy that is transferred between two Systems (or a System and its Surroundings) because of a Temperature difference between the two Systems

- A System does not contain "Heat", it contains Energy.
- "Heat" is thermal Energy in transit across the System Boundary.



- "Heat" Q is Not a Property of a System
- The amount of Heat necessary to Change a Systems' Properties from State ① \rightarrow ② depends on the path

$$\Rightarrow Q_{1 \rightarrow 2}^{\text{Path A}} \neq Q_{1 \rightarrow 2}^{\text{Path B}} \neq Q_{1 \rightarrow 2}^{\text{Path C}}$$



- ⑥
- A Special notation is used to indicate an infinitesimal amount of Heat transfer:

$$\delta Q$$

⇒ This is called an "inexact differential" because the path is not specified.

- The total Amount of Heat Transferred along a particular process path ① → ② is

$$Q_{12} = \int_1^2 \delta Q$$

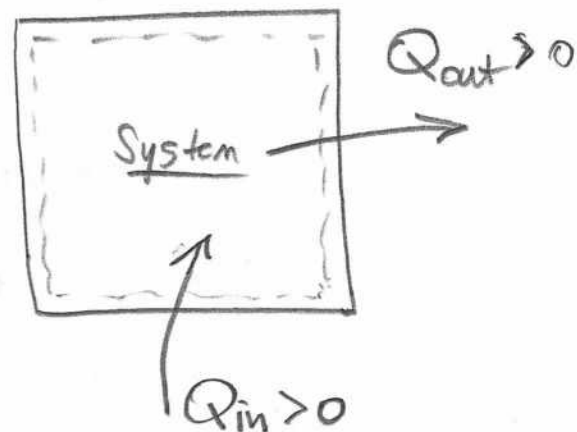
- It is meaningless to write $\int_1^2 \delta Q = Q_2 - Q_1$!! because this implies that Q is a property at states ① and ② which is incorrect.

- The rate of Heat Transfer is $\frac{dQ_{12}}{dt} = \dot{Q}_{12}$

Sign Convention

We always take $Q_{out} > 0$ and $Q_{in} > 0$ so that the Net Heat transfer in is

$$Q_{Net\ in} \equiv Q_{in} - Q_{out}$$

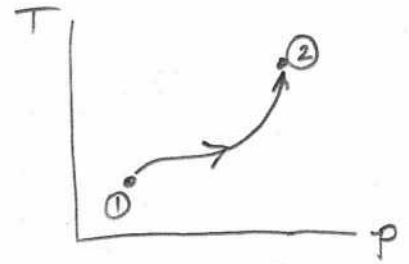


Path Dependent Heat Transfer

(7)

Note: (1) In order to use the equation

$$Q_{12} = \int_{\text{State 1}}^{\text{State 2}} \delta Q$$



We need to know how δQ varies as a function of intrinsic properties or external variables, in other words, we need to know the process path:

$$\delta Q = \text{function of } (T, p, \text{flame temperature, etc.})$$

note that since
 δQ is not a property, the function may depend on many things

(2) Until we encounter the Second Law of Thermodynamics, we will not be using δQ to compute anything.

(3) For now and much of the course, we will simply be using

$$Q_{\text{net, in}} \equiv Q_{\text{in}} - Q_{\text{out}}$$

because it is more practical for Engineering Thermodynamics.

(4) However, you should be aware of common ways that heat transfer, $Q_{\text{net, in}}$, may occur.

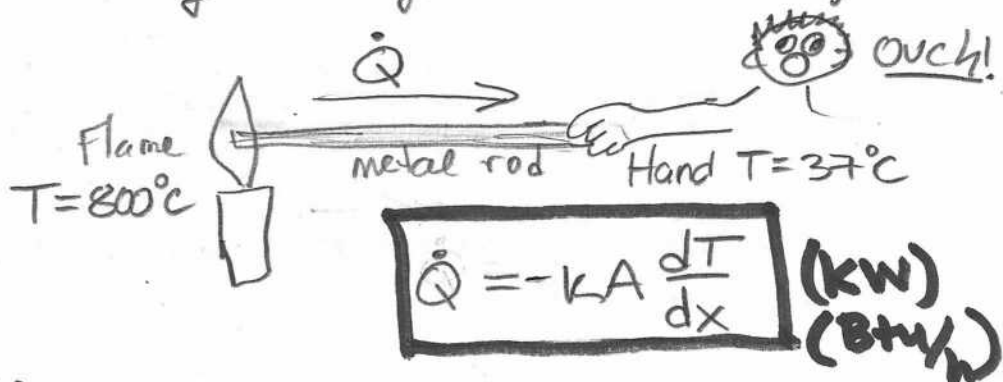
There are 3 common mechanisms for heat transfer

Mechanisms for Heat Transfer

Heat can be transferred via 3 mechanisms, each involves a temperature difference in which Heat "flows" from hot to cold spontaneously:

→ (1) Conduction

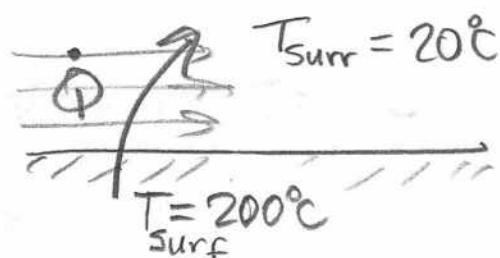
- Through direct contact with the System, Thermal Energy migrates from a body having higher molecular activity to a body having lower activity



(2) Convection

- Transfer of Heat between a solid surface of a System and the adjacent fluid that is in motion. This is a combination of conduction and fluid motion.

$$\dot{Q} = hA(T_{\text{surf}} - T_{\text{surr}})$$



(3) Radiation

- Transfer of Energy due to Electromagnetic waves (photons).

$$\dot{Q} = \epsilon \sigma A (T_{\text{surf}}^4 - T_{\text{surr}}^4)$$

Adiabatic Process

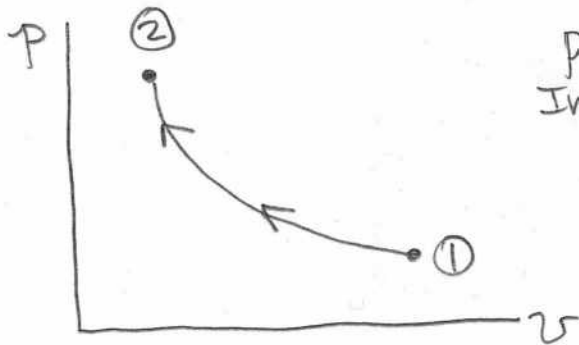
An Adiabatic Process is one in which there is NO HEAT TRANSFER into or out of the System:

Energy: $Q_{in} = Q_{out} = 0 \text{ kJ}$

Power: $\dot{Q}_{in} = \dot{Q}_{out} = 0 \text{ kW}$

Examples:

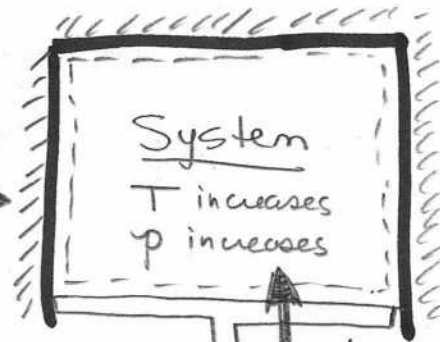
(1) Adiabatic Process



perfect
Insulation

$$Q_{in} = 0 \text{ J} \Rightarrow Q_{net, in} = 0 \text{ J} - 0 \text{ J} = 0 \text{ J}$$

$$Q_{out} = 0 \text{ J}$$



$T_{surr} = 20^\circ\text{C}$

However Work
can be performed
on or by the System

(2) Not Adiabatic Process

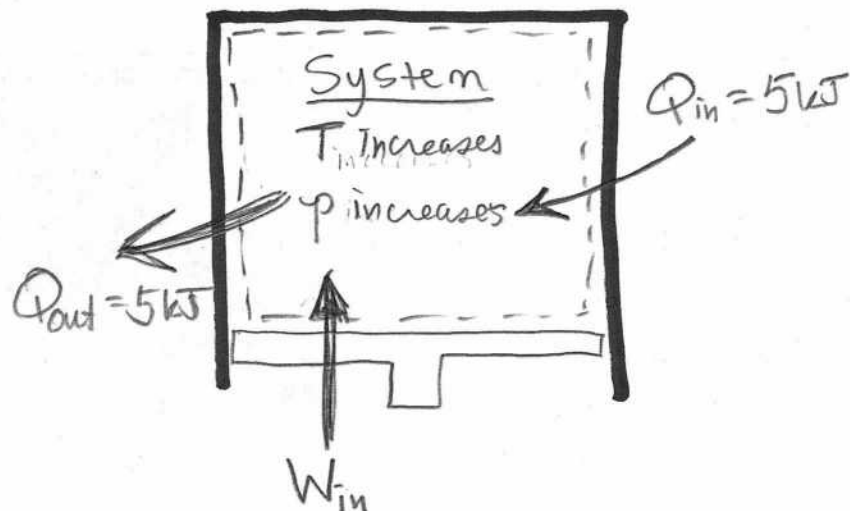
$$Q_{in} = 5 \text{ kJ}$$

$$Q_{out} = 5 \text{ kJ}$$

$$Q_{net, in} = Q_{in} - Q_{out}$$

$$Q_{net, in} = 5 \text{ kJ} - 5 \text{ kJ}$$

$$Q_{net, in} = 0 \text{ kJ}$$

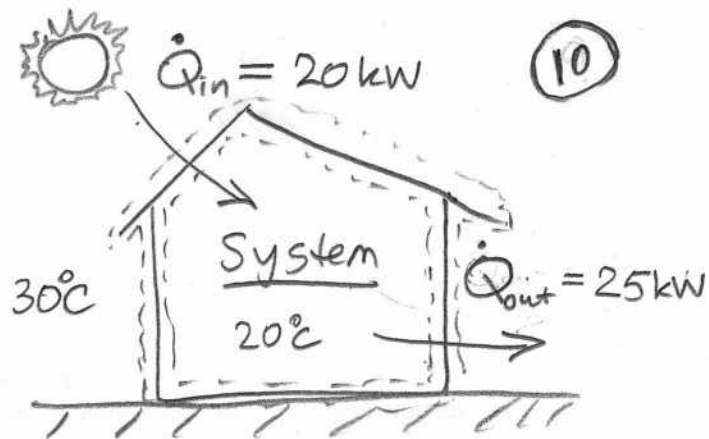


Example 1

On a hot day, Heat is transferred to a house at a rate of 20 kW.

The air conditioner has a maximum heat rejection rate of 25 kW.

What is the Net Heat transferred to the house when the air conditioner is turned on "maximum"?



Analysis

$\dot{Q}_{in} = 20 \text{ kW}$ into the system

$\dot{Q}_{out} = 25 \text{ kW}$ out of the system

$$\dot{Q}_{net,in} \equiv \dot{Q}_{in} - \dot{Q}_{out}$$

$$\dot{Q}_{net,in} = 20 \text{ kW} - 25 \text{ kW}$$

$$\dot{Q}_{net,in} = -5 \text{ kW}$$

Since $\dot{Q}_{net,in} < 0$, this means that the net transfer of Heat is out of the System

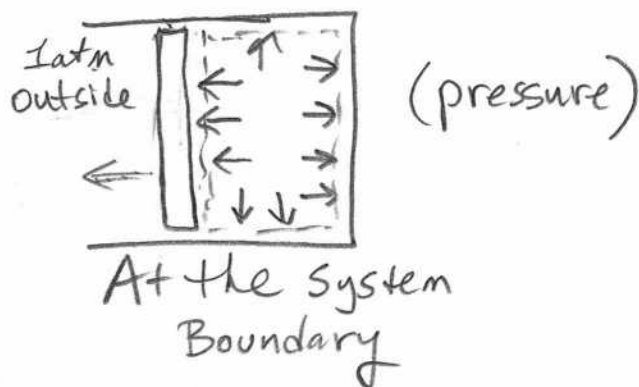
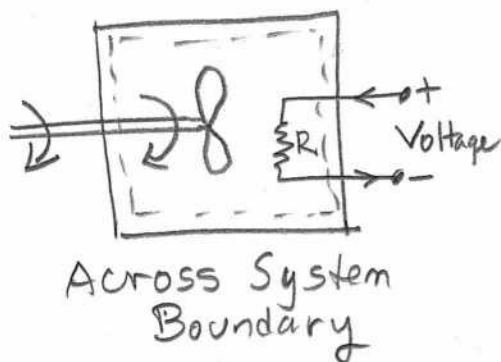
⑧ Energy Transfer by Work

⑪

Work

"Work" is the Energy transfer associated with a Force acting through a distance

- A System does not contain "Work",
It contains Energy.
- "Work" is an ordered transfer of Energy across or at the System Boundary

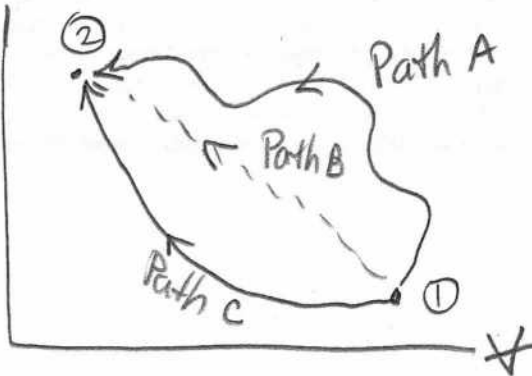


- "Work" W is Not a Property of a System

- The amount of work necessary to change a System's properties from State ① \rightarrow ② depends on the path

$$W_{12 \text{ path A}} \neq W_{12 \text{ path B}} \neq W_{12 \text{ path C}}$$

(Area under paths are not equal)



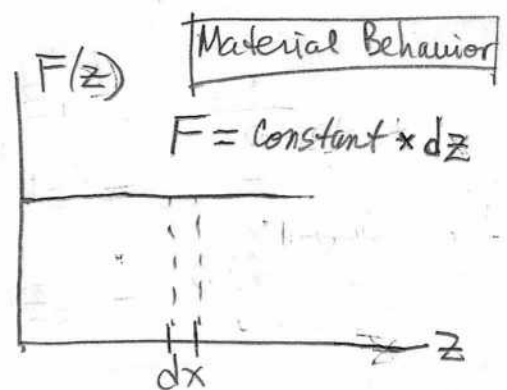
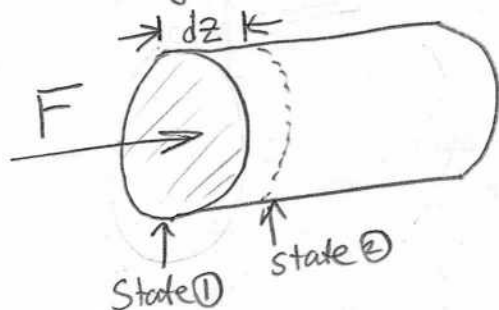
Path Dependent Work

(12)

- (1) In order to describe the work interaction in detail, (meaning how the work depends upon the process path in terms of thermodynamic properties or external "driving forces") we need a physically meaningful mathematical description of an infinitesimal portion of the process path:

$$\delta W_{12} = \text{function}(p, V, \text{current, Voltage, external applied torque, etc})$$

- (2) For example, say that we slightly compress cylinder of some material, and assume the cross-sectional area remains constant during the process.



- Step (1) Infinitesimal amount of Work input to the system ($W_{in} > 0$)

$$\delta W_{12} = \delta W_{in} \equiv F(z) dz \quad (\text{Force} \times \text{distance})$$

- Step (2) This material behaves different from Hooke's Law (elastic springs):

$$F(z) = \alpha \quad (\text{Constant})$$

- Step (3) Plug the material behavior to an applied force (the process path) into the expression for δW_{12} :

$$\delta W_{in} = \alpha dz$$

- Step (4) Integrate along the process path:

$$W_{in} = \int_1^2 \alpha dz = \alpha(z_2 - z_1)$$

- Since Work is path dependent, an infinitesimal amount of Energy transferred by Work in going from state ① → ② is

$$W_{12} = \int_1^2 \delta W$$

(Again, $\int \delta W = W_2 - W_1$ is a meaningless statement)

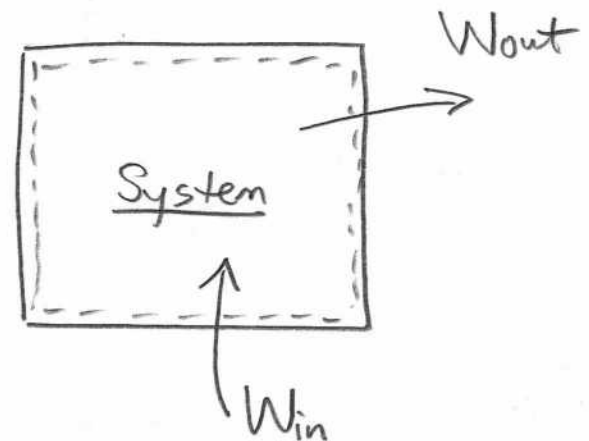
- The rate of Work Transfer is called Power (W)

$$\frac{dW_{12}}{dt} = \dot{W}_{12}$$

Sign Convention

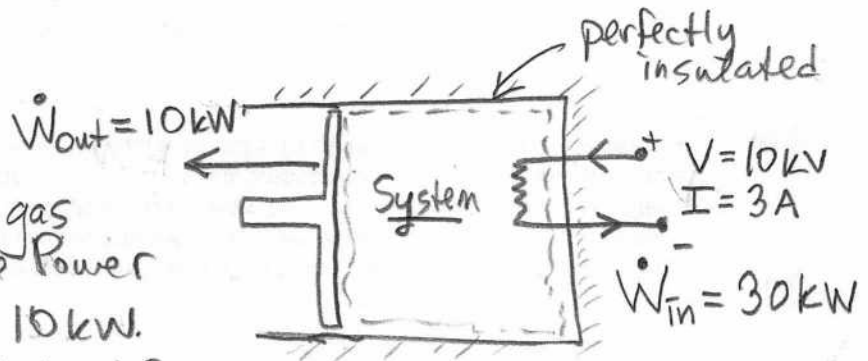
We always take $W_{out} > 0$ and $W_{in} > 0$ so that the Net Work out is:

$$W_{Net, out} \equiv W_{out} - W_{in}$$



Example 2

30 kW of Electrical Power is performed on the gas within a piston-cylinder. The Power output of the piston is 10 kW. What is the Net Power Output?



$$\dot{W}_{net, out} = 10 \text{ kW} - 30 \text{ kW} = \underline{\underline{-20 \text{ kW}}}$$

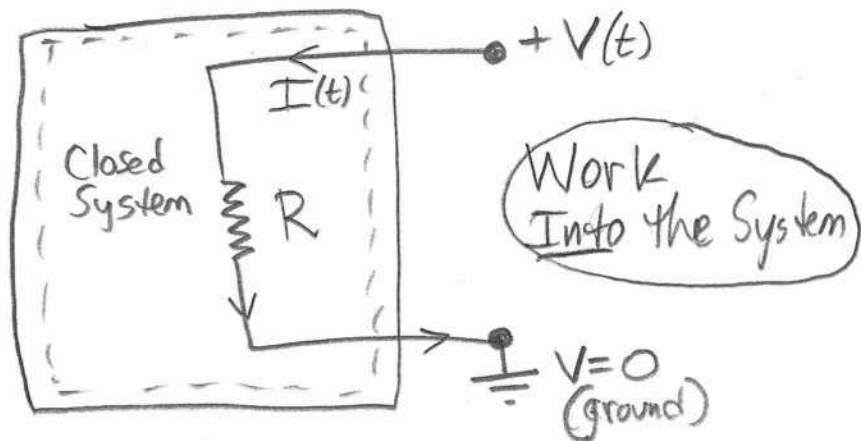
This 20 kW of net Work Input is increasing the System Enthalpy

Mechanisms for Work Interactions

(14)

(I) Electrical

- We normally think of a voltage applied to a resistor as dissipating Heat ($P_{\text{dissipate}} = I^2 R$) (Which is true)



- However, in this class we are going to consider this from an alternative viewpoint — as a Work (or Power) interaction into the System.
- Electrons are getting "pushed" through the wire by the Electric field (Force), so that

$$\delta W_{\text{in}}^{\text{elect}} = V(t) I(t) dt \quad (\text{kJ})$$

where $V(t)$ = Voltage difference in Volts

$I(t)$ = current in Amperes

dt = infinitesimal, exact time differential (sec)

- The Electrical Work necessary to change the System's State from ① \rightarrow ② is

$$W_{\text{in}}^{\text{elec}} = \int_1^2 V(t) I(t) dt \quad (\text{kJ})$$

To solve this integral, you need to know how both $V(t)$ and $I(t)$ depend on time — this defines the process path!

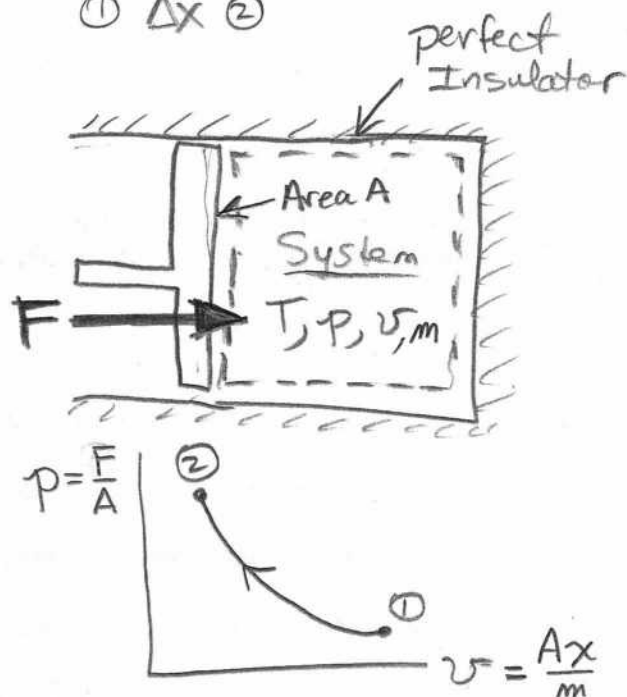
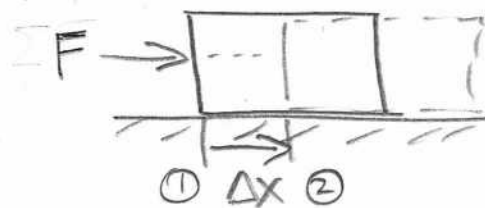
(2) Mechanical Work/Power

- The inexact differential amount of work is

$$\delta W_{12} = F(x) dx$$

The work transferred to the System from State ① \rightarrow ② is

$$W_{12} = \int_1^2 \delta W_{12} = \int_1^2 F(x) dx \quad (\text{kJ})$$



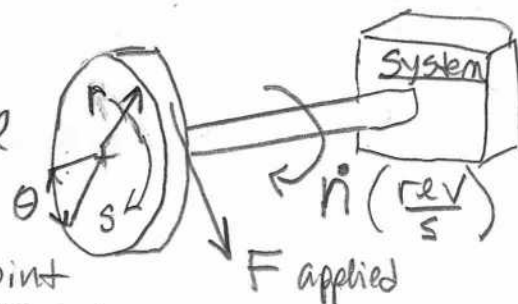
- There are two requirements for a work interaction between a System and its surroundings to exist:

(i) There must be a Force acting on the boundary

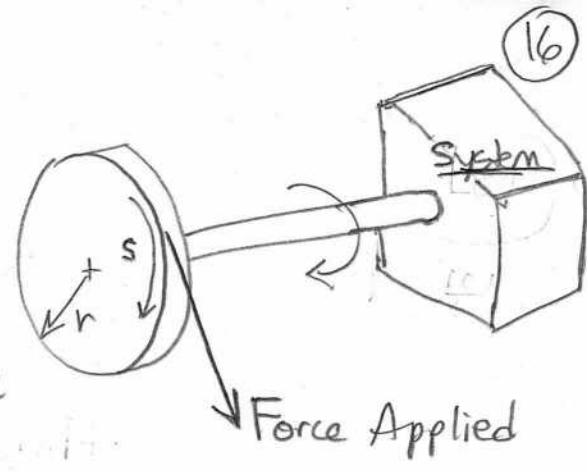
(ii) The boundary must move

(a) Shaft Work/Power

- F = Force applied to edge of wheel
- r = radius of wheel
- S = distance traveled by a point on the wheel edge $= 2\pi nr$
- n = number of revolutions of the wheel
- $\dot{n} = \frac{dn}{dt}$ = number of revolutions per unit time $(\frac{\text{rev}}{\text{sec}})$



- $\text{Torque} \equiv F \cdot r$ (N.m) or (ft.lbf)



- The path dependent work performed by the applied Torque into the System is:

$$W_{in}^{shaft} = \int_{\text{State ①}}^{\text{State ②}} \delta W^{shaft}$$

- The inexact differential (path dependent differential) amount of Work performed by the applied Force at the edge of the wheel is

$$\delta W_{in}^{shaft} = F ds$$

- Assuming a constant Applied Torque, T , $F = \frac{T}{r} \equiv \text{Constant}$

Also, $ds = d(2\pi nr) = 2\pi r dn$

$$\Rightarrow \delta W_{in}^{shaft} = \left(\frac{T}{r}\right)(2\pi r dn)$$

$$\delta W_{in}^{shaft} = 2\pi T dn$$

- The work performed on the System is

$$W_{in}^{shaft} = \int_{n_1=0}^{n_2} 2\pi T dn \rightarrow W_{in}^{shaft} = 2\pi T \bar{n} \quad (\text{kJ})$$

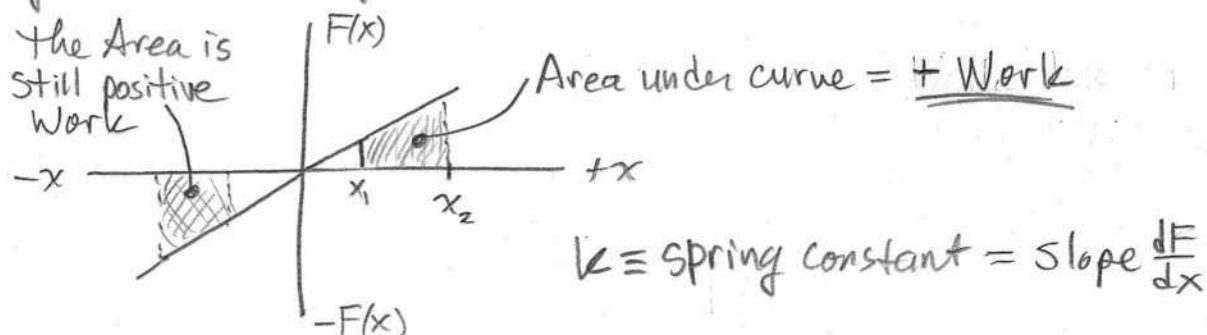
- The power input to the system is

$$\dot{W}_{in}^{shaft} = \frac{d}{dt} W_{in}^{shaft} = \frac{d}{dt} (2\pi T \bar{n}) \Rightarrow \dot{W}_{in}^{shaft} = 2\pi T \dot{\bar{n}} \quad (\text{W})$$

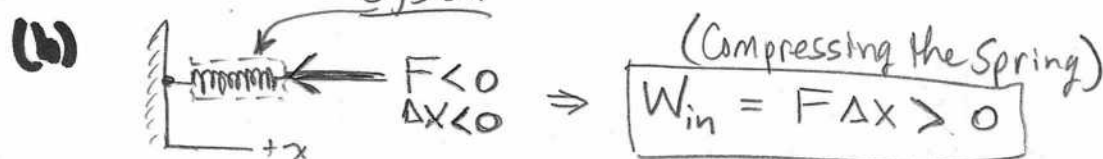
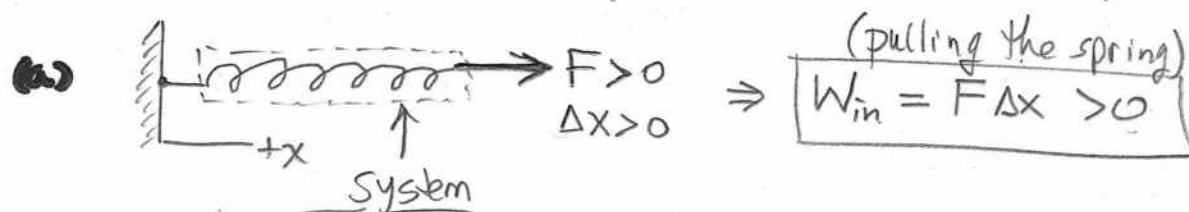
(b) Spring Work

(this perspective is different than using a free-body diagram in mechanics!)

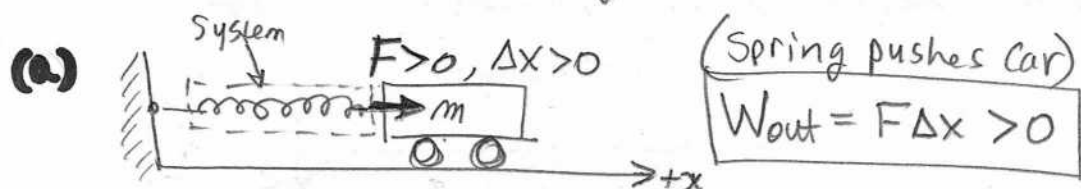
- A linearly elastic spring obeys Hooke's Law:



- Work performed on the spring is Work into the System



- Work performed by the Spring is Work out of the System



- Inexact differential Work performed on or by a Spring:

$$\delta W^{\text{Spring}} = F(x) dx$$

- Force exerted on or by the Spring: $F(x) = kx$

- Work performed on or by the spring:

$$W_{in/out} = \int_{\text{State 1}}^{\text{State 2}} \delta W^{\text{Spring}} = \int_{x_1}^{x_2} F(x) dx = \int_{x_1}^{x_2} kx^2 dx = \frac{1}{2} k(x_2^2 - x_1^2)$$

- Note that the Power exerted on or by the spring can be obtained by taking the time derivative:

$$\dot{W}_{in/out} = \text{Power} = \frac{dW^{\text{spring}}}{dt} = \frac{d}{dt} \left(\frac{1}{2} k (x_2^2 - x_1^2) \right) \quad (\text{Force} \times \text{Velocity})$$

$$\dot{W}_{in/out}^{\text{spring}} = F(x_2)V_2 - F(x_1)V_1 \quad (W)$$

System's
mechanical power
at State ②,
or x_2

System's mechanical power at
State ①, or x_1

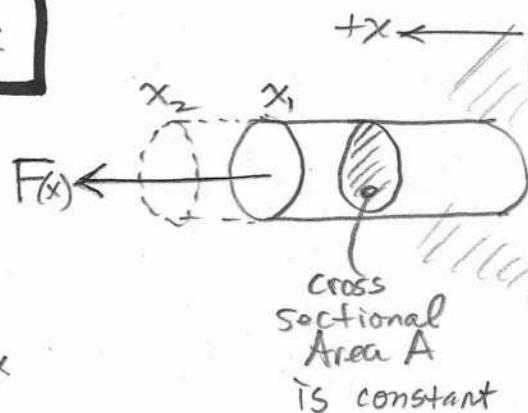
- Work performed on an elastic solid bar

- Solids are often modeled as linearly elastic springs
- The normal stress, $\sigma_n \equiv \frac{F_{\perp}(x)}{\text{Area}}$ so that

$$\delta W_{in}^{\text{bar}} = \sigma_n(x) A dx$$

- The work performed on the bar is then

$$W_{in}^{\text{bar}} = \int_{\text{state ①}}^{\text{state ②}} \delta W_{in}^{\text{bar}} = \int_{x_1}^{x_2} \sigma_n(x) A dx$$



Let $\sigma_n(x) \equiv \beta x$

Then $W_{in}^{\text{bar}} = A\beta \int_{x_1}^{x_2} x dx$

or

$$W_{in}^{\text{bar}} = \frac{1}{2} \beta A (x_2^2 - x_1^2) \quad (kJ)$$

$$\dot{W}_{in}^{\text{bar}} = A (F_2(x_2)V_2 - F_1(x_1)V_1) \quad (W)$$

(d) Boundary Work ("pdv" Work)

(19)

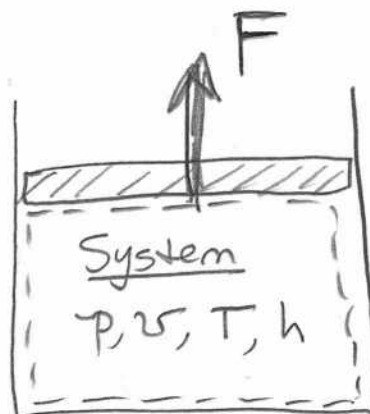
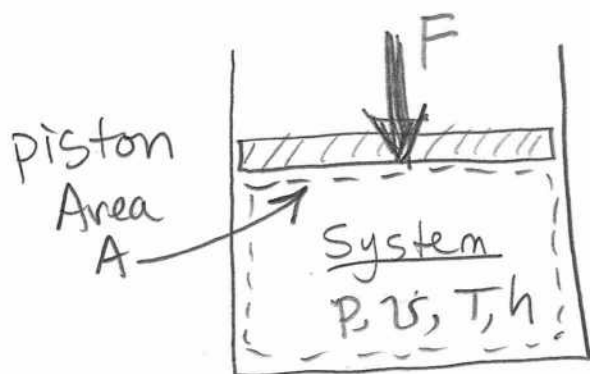
- Associated with one or more System Control Volume boundaries moving due to either:

(1) an Externally applied Force acting at the boundary surface

or

(2) the System pressure, averaged across the moving boundary

• Examples



• Inexact differential for "pdv" Work

- $P_{avg} = \frac{F_{avg}}{A}$

$\delta W^{pdv} = F_{avg} dx$

Note that for P_{avg} to Equal System P , we must have mechanical quasi-equilibrium!

- Substituting $F_{avg} = P_{avg} A$ into the inexact differential,

$$\delta W^{pd\forall} = (P_{avg} A) dx$$

But $A dx = d\forall$

So

$$\boxed{\delta W^{pd\forall} = p d\forall}$$

(the "avg" is dropped)
implying that the
process is quasiequilibrium.)

- The " $p d\forall$ " Boundary work is then

$$\boxed{W_{out}^{pd\forall} = + \int_1^2 p d\forall}$$

+ sign in front
for work output
to be + if
 $d\forall > 0$

$$\boxed{W_{in}^{pd\forall} = - \int_1^2 p d\forall}$$

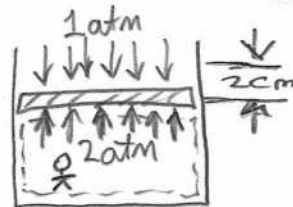
- sign in front for
work input to be
+ if $d\forall < 0$

- Again, the path must be specified

\Rightarrow is (1) $p = \text{constant}$?
(2) $p = \forall^{-n}$? etc.

\Rightarrow How does p relate to \forall ?!

Very Important Discussion:



(21)

Question: Look at the picture drawn for this problem. Notice that the Immediate surrounding air is "pushing down" on the massless piston? Why didn't we take this opposing force ($P_{\text{surr}} = 1\text{atm}$) into account!?

Answer: Because the question asked: What is the work done by the gas on the piston. The gas is our System, exerting a Force that acted through a 2cm distance \Rightarrow that is the work.

\Rightarrow The surrounding air has nothing to do with the Analysis of the thermodynamic System from an Energy viewpoint

\Rightarrow In other words, the Analysis of a system in thermo is not exactly like a Free-body diagram in Mechanics since we are not working with the sum of Forces on a Body

\Rightarrow Instead, we are dealing with Energy Changes of a System

Lecture 13 Examples

①

Basic Equations

(1) Heat transfer

$$Q_{\text{Net},in} = Q_{in} - Q_{out} \text{ (kJ)}$$

$$q_{\text{Net},in} = q_{in} - q_{out} \text{ (kJ/kg)}$$

$$Q_{1 \rightarrow 2} = \int_{\text{state 1}}^{\text{state 2}} \delta Q$$

Heat transfer rate is

$$\dot{Q}_{\text{Net},in} = \dot{Q}_{in} - \dot{Q}_{out}$$
$$\dot{q}_{\text{Net},in} = \dot{q}_{in} - \dot{q}_{out}$$

(2) Work

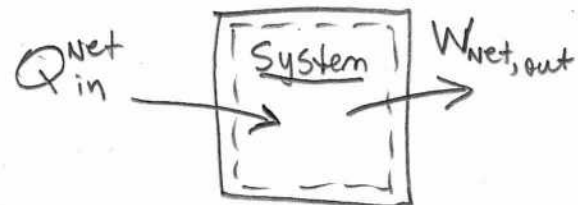
$$W_{\text{Net},out} = W_{in} - W_{out} \text{ (kJ)}$$

$$w_{\text{Net},out} = w_{in} - w_{out} \text{ (kJ/kg)}$$

$$W_{1 \rightarrow 2} = \int_{\text{state 1}}^{\text{state 2}} \delta W$$

Power is

$$\dot{W}_{\text{Net},out} = \dot{W}_{out} - \dot{W}_{in}$$
$$\dot{w}_{\text{Net},out} = \dot{w}_{out} - \dot{w}_{in}$$



Sign Convention

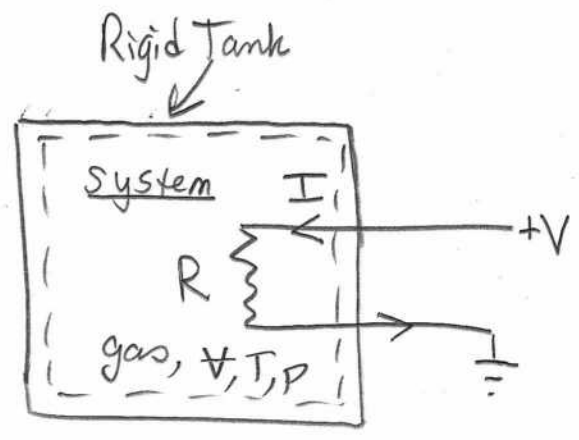
- All quantities Q_{in} , Q_{out} , W_{in} , W_{out} , etc. are positive by definition, no matter if they are going into or out of the System
- Net quantities as defined above can be (+) or (-) and help you determine the true direction of the transfer.

Example 1

Given, a closed, rigid box filled with a gas and a Resistor circuit into and out of the tank as shown.

Resistor value: $R = 100\text{m}\Omega$

At time $t=0$ a constant voltage of 3amps is applied to the circuit external to the tank.



Find: Determine the Work on the gas due to the circuit after 3 minutes of steady operation.

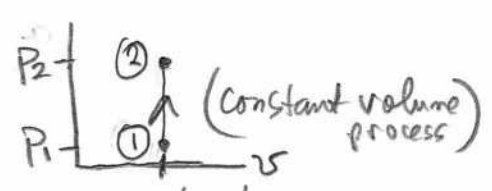
- Assume:
- (1) quasi-equilibrium process
 - (2) Ohm's Law is valid: $V = IR$

Analysis: By definition, $\delta W^{\text{elect}} = I(t)V(t)dt$

$$W_{\text{in}}^{\text{elec}} = \int_1^2 \delta W \quad (\text{kJ})$$

Substituting the expression for instantaneous Power into the expression for Work from state ① \rightarrow state ②,

$$W_{\text{in}}^{\text{elec}} = \int_1^2 I(t)V(t)dt$$



Since the applied voltage, $V(t)$, is not known, we use Ohm's Law ($V=IR$) to get

(3)

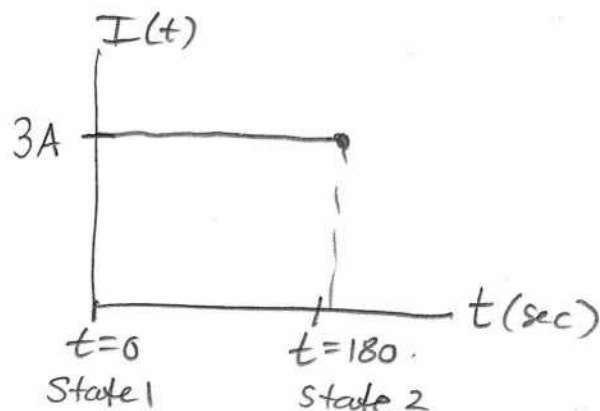
$$W_{in}^{elec} = \int_1^2 I(t)(I(t)R) dt = R \int_1^2 I^2(t) dt$$

We were told that the current was constant at 3A from $t=0$ to $t=3\text{min} \times \frac{60\text{s}}{\text{min}} = 180\text{sec}$, so

$$W_{elec} = I^2 R \int_1^2 dt$$

$$W_{in}^{elect} = I^2 R (t_2 - t_1)$$

Substituting values,



$$W_{elec} = (3A)^2 \left(\frac{100}{1000} \Omega \right) (180\text{s} - 0\text{s})$$

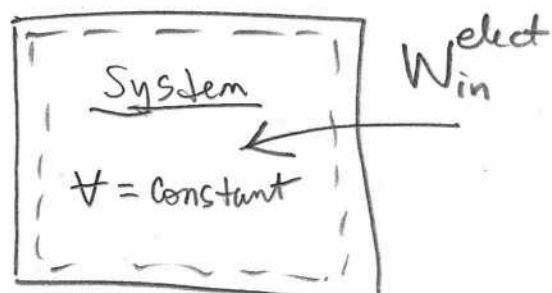
$$W_{in}^{elec} = 162 \text{ J}$$

Note: This is the work done on the gas by the external circuit, even though we used Ohm's Law to express IV in terms of $I^2 R$

Also:

$$W_{net,out}^{elect} = W_{out}^{elect} - W_{in}^{elect}$$

The circuit did not perform work output



$$\Rightarrow W_{net,out}^{elect} = -W_{in}^{elect} = -162 \text{ J} \Rightarrow \text{The Net work output of the gas (System) done on the circuit!}$$

Example 2

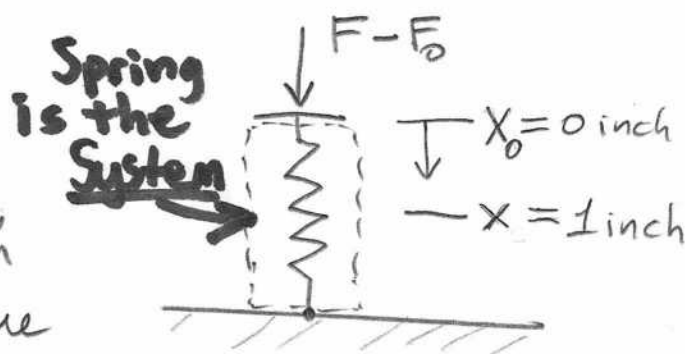
Given: The force required to compress a spring a distance x is given by $F - F_0 = kx$ where $k = 200 \text{ lb}_f/\text{in}$ and F_0 is the "pre-load".

Find: The work required to compress the spring a distance of 1 inch starting at its free length where $F_0 = 0 \text{ lb}_f$.

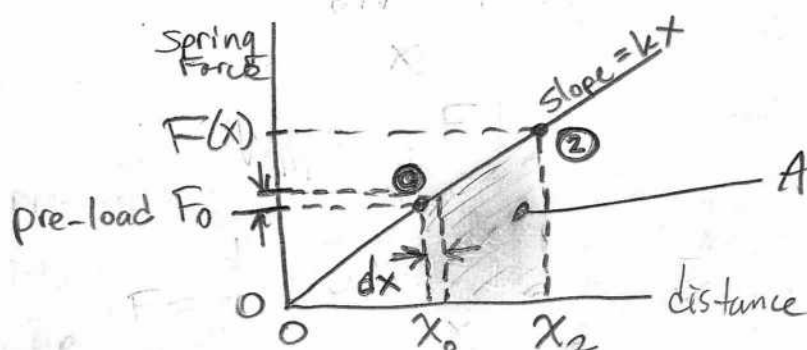
(Express in both $\text{lb}_f \cdot \text{ft}$ and Btu)

Analysis

In this case we have the spring as the system with a force acting on it. There is a "pre-load" force F_0 . We are not interested in the work performed on the spring due to the pre-load, F_0 , but only that due to F .



⇒ The Force - distance plot looks like this:



All we want to compute is the work from state ① → state ② which is this Area

So that the infinitesimal (inexact, path dependent) differential work is

$$\boxed{\delta W_{1 \rightarrow 2} = (F(x) - F_0) dx}$$

(5)

Substituting this inexact differential work into the equation for total work integrated along a specified path:

$$W_{0 \rightarrow 2} = \int_0^2 \delta W_{0 \rightarrow 2} = \int_0^2 (F(x) - F_0) dx$$

But we know that the relationship between Force and distance for a linear spring is (we are now explicitly defining the process path!)

$$\boxed{F(x) - F_0 = kx} \quad (kx = \text{user specified process path})$$

Substituting this into the above equation,

$$W_{0 \rightarrow 2}^{\text{spring}} = \int_{x_0}^{x_2} kx dx = k \int_{x_0}^{x_2} x dx$$

Integration gives

$$\boxed{W_{0 \rightarrow 2}^{\text{spring}} = W_{\text{in}}^{\text{spring}} = \frac{1}{2} k (x_2^2 - x_0^2)}$$

(Notice that I did not place the superscript "spring" onto the W-symbol until we expressed the Force in terms of a specific process path — Here, a linear spring — subtle!)

Substituting values,

$$\begin{aligned} W_{\text{in}}^{\text{spring}} &= \frac{1}{2} \times (200 \frac{\text{lbs}}{\text{in}}) [(1 \text{ in})^2 - (0 \text{ in})^2] \\ &= (100 \frac{\text{lbs}}{\text{in}}) \left(\frac{12 \text{ in}}{1 \text{ ft}} \right) \left[\left(1 \text{ in} \times \frac{1 \text{ ft}}{12 \text{ in}} \right)^2 \right] \end{aligned}$$

⑥

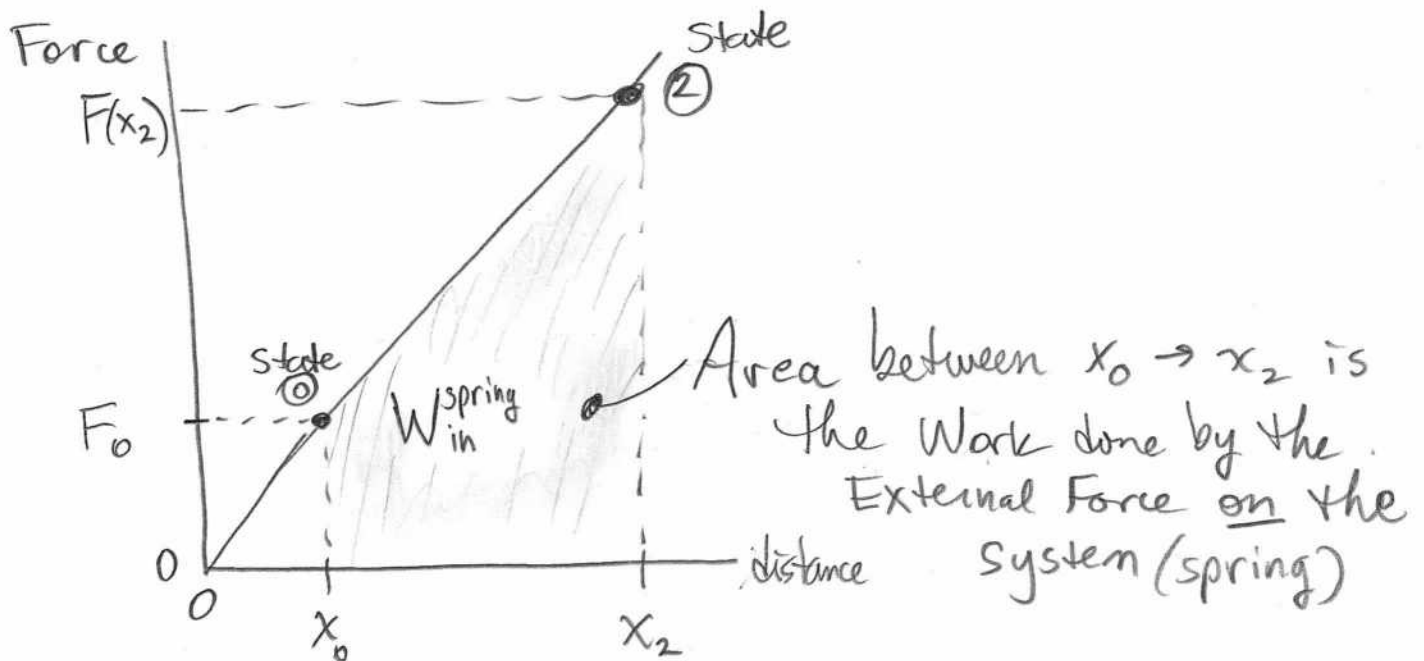
$$W_{in}^{spring} = \left(1200 \frac{lb_f}{ft}\right) (0.006944 ft^2)$$

$$W_{in}^{spring} = 8.33 lb_f \cdot ft$$

Expressed in Btu's, from the textbook back cover,

$$W_{in}^{spring} = 8.33 lb_f \cdot ft \times \frac{1 Btu}{778.169 lb_f \cdot ft}$$

$$W_{in}^{spring} = 0.0107 Btu$$



From the System's perspective, $W_{Net,out}^{spring} = W_{out}^{spring} - W_{in}^{spring}$

so

$$W_{Net,out}^{spring} = -8.33 lb_f \cdot ft = 0.0107 Btu$$

Example 3 Heat Conduction

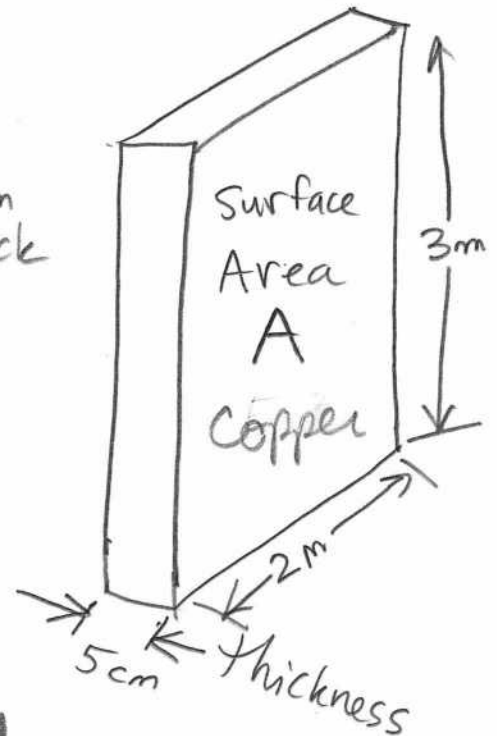
Given:

- Consider the Copper wall shown:

The wall is 2m wide x 3m high x 5cm thick

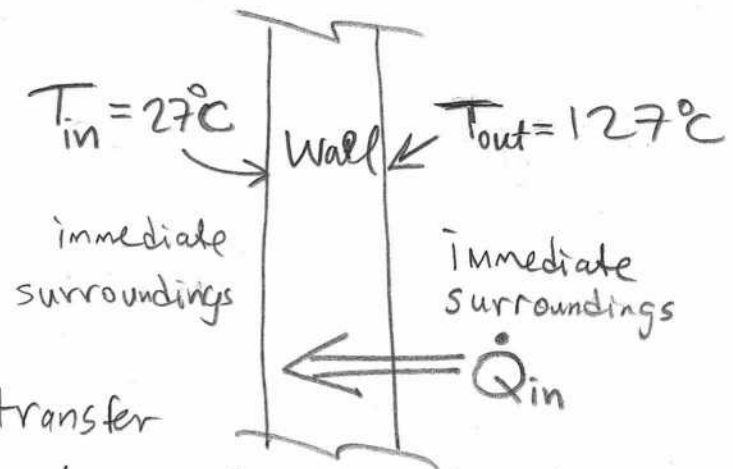
- Let the temperature at the wall's outside immediate surroundings be

$T_{out} = 127^\circ\text{C}$ while the inside is at $T_{in} = 27^\circ\text{C}$



Find:

- Under steady-state heat-conduction through the wall, what is the amount of thermal energy, Q (kJ), transferred through the wall after 1 minute has elapsed?



- Assume:
 - (1) Steady state heat transfer
 - (2) The temperature distribution inside the wall is linear, i.e., $T(x) = a + bx$. This is a reasonable approximation.
 - (3) The edges of the wall are perfectly insulated.
 - (4) The average thermal conductivity, k , of copper is $k_{avg} \approx 397 \frac{\text{W}}{\text{m-K}}$ (Wikipedia)

Analysis

8

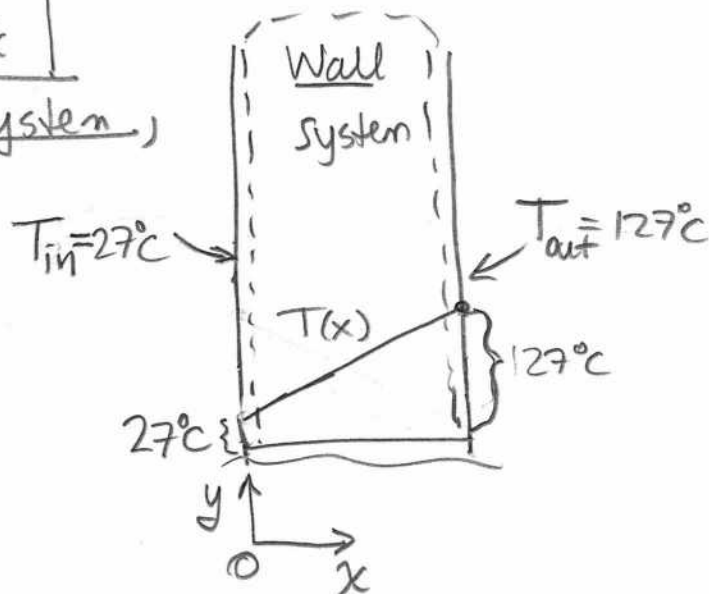
- From page 7 of Lecture 13, the equation for heat conduction (kW) is

$$\dot{Q} = -kA \frac{dT}{dx}$$

- Taking the Wall as the System, the temperature gradient is

$$\frac{dT}{dx} = \left(\frac{127^\circ\text{C} - 27^\circ\text{C}}{5\text{cm} - 0\text{cm}} \right) \times \left(\frac{100\text{cm}}{\text{m}} \right)$$

$$\frac{dT}{dx} = 2000 \frac{\text{K}}{\text{m}}$$



- The amount of heat transferred is (kJ)

$$Q = \int_1^2 \delta Q$$

where the integration limits will be determined shortly.

- The inexact differential for the heat transferred during an infinitesimal time, dt , is

$$\delta Q = \dot{Q} dt$$

$$\delta Q = -kA \left(\frac{dT}{dx} \right) dt \quad (\text{kJ})$$

- Substituting this expression into the integral, we get

$$Q_{1 \rightarrow 2} = \int_1^2 -kA \left(\frac{dT}{dx} \right) dt = -kA \frac{dT}{dx} \int_{t=0}^{t=1\text{min}} dt$$

Performing the time integration from $0 \rightarrow 1$ minute,

$$Q_{1 \rightarrow 2, \text{in}} = -kA \left(\frac{dT}{dx} \right) (t_2 - t_1) \quad (\text{kJ})$$

(9)

plugging in values,

$$Q_{1 \rightarrow 2, \text{in}} = -(397 \frac{\text{W}}{\text{m} \cdot \text{K}}) (\overset{\text{Area}}{3\text{m} \cdot 5\text{m}}) (2000 \frac{\text{K}}{\text{m}}) (1 \text{ min} \times \frac{60 \text{ sec}}{\text{min}})$$

$$Q_{\text{in}} = -7.146 \times 10^8 \text{ J}$$

$$Q_{\text{in}} = -714,600 \text{ kJ after 1 minute}$$

Note 1: The negative sign simply means that from the System's perspective, the Heat flow is opposite from the temperature gradient

\Rightarrow Just like Voltage and Current, pressure and mass flow, the heat transfer is driven by the temperature gradient across the wall from Hot to cold

Note 2: we can calculate the heat transferred per unit mass:

From Table A-3b, Copper density at 100°C is
 $\rho = 8,900 \text{ kg/m}^3$

The Volume of the wall is $V = 3\text{m} \times 5\text{m} \times \frac{5}{100}\text{m} = 0.75\text{m}^3$

The mass of the wall is $m = 0.75\text{m}^3 \times 8900 \text{ kg/m}^3 = 6675 \text{ kg}$

$$\Rightarrow q_{\text{in}} = \frac{714,600}{6675 \text{ kg}} \Rightarrow q_{\text{in}} = 107.0 \frac{\text{kJ}}{\text{kg}} \text{ after 1 minute}$$

Example 4 Moving "pdV" Boundary Work

Given:

Consider the piston-cylinder containing 0.002 kg of Xenon gas. The piston is pushed downward from a height (at the piston face) of 10 cm to 2 cm. Heat is transferred out of the cylinder walls so that the process remains at constant pressure (this statement specifies the process "path"!).

Find: Assuming that the gas is Ideal, $p = 1 \text{ atm}$, and the initial

temperature is 1227°C

Determine

(1) The Work performed on the gas

(2) The final temperature of the gas

Assume:

(1) Ideal gas

(2) quasi-equilibrium process

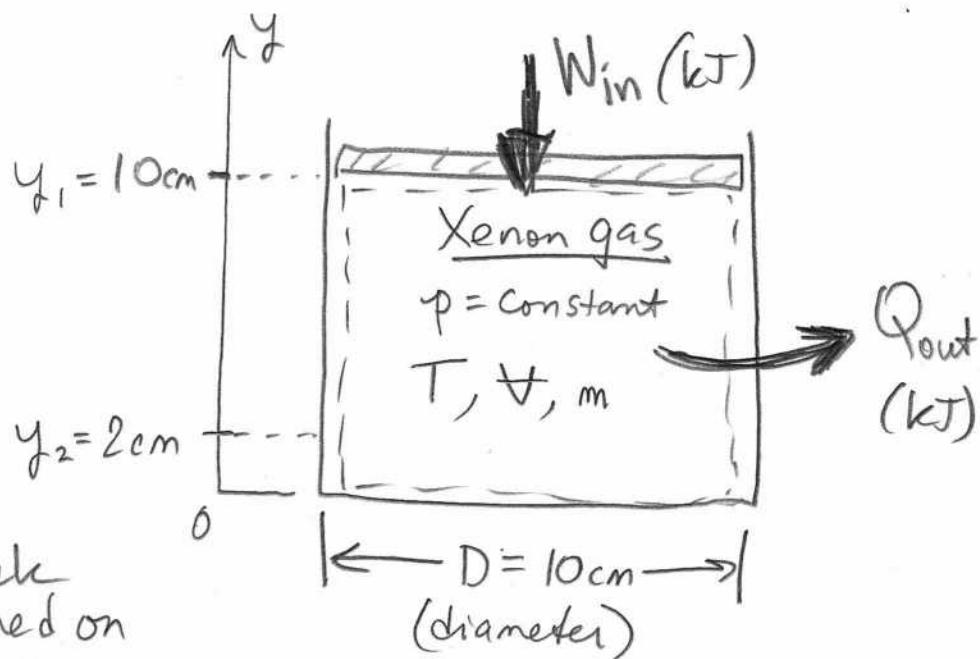
Analysis

For quasiequilibrium moving boundary work input the inexact differential is

(1)

$$\boxed{\delta W_{in}^{pdV} = -p dV} \quad (\text{kJ})$$

(the (-) sign is for work input to keep δW (+))



The mechanical work performed from state ① → ② is ⑪

$$W_{1 \rightarrow 2} = \int_1^2 \delta W$$

Substituting δW_{in}^{pdv} into the integral from State ① → ②,

$$W_{in}^{pdv} = - \int_1^2 p dV$$

Now we need to find a relationship between p and V in order to integrate this thing!

The problem stated that $p = \text{constant} = 1 \text{ atm}$ during the process, so

$$W_{in}^{pdv} = -p \int_{V_1}^{V_2} dV$$

$$W_{in}^{pdv} = -p(V_2 - V_1)$$

So that I don't have to compute V 's, I take V_1 outside.

$$W_{in}^{pdv} = -pV_1 \left[\frac{V_2}{V_1} - 1 \right]$$

$$\frac{V_2}{V_1} = \frac{\frac{\pi D_2^2}{4} \times y_2}{\frac{\pi D_1^2}{4} \times y_1} = \frac{y_2}{y_1} \quad \text{since } D_1 = D_2$$

$$W_{in}^{pdv} = -pV_1 \left[\frac{y_2}{y_1} - 1 \right]$$

Using the Ideal gas E.O.S. ,

$$pV = RT$$

$$p \frac{V}{m} = RT$$

$$\boxed{pV = mRT}$$

Substituting gives

$$\boxed{W_{in}^{pdV} = -mRT_1 \left[\frac{V_2}{V_1} - 1 \right]}$$

From Table A-1 for Xenon, $R = 0.06332 \frac{kJ}{kg \cdot K}$

So

$$W_{in}^{pdV} = -(0.002 kg) \left(0.06332 \frac{kJ}{kg \cdot K} \right) (1227^\circ C + 273) \left[\frac{2 cm}{10 cm} - 1 \right]$$

$$W_{in}^{pdV} = -0.15197$$

$$\boxed{W_{in}^{pdV} = 152 J}$$

(2) Since the process is at constant pressure, we use the Ideal gas E.O.S. :

$$P_1 = P_2$$

$$\frac{RT_1}{V_1} = \frac{RT_2}{V_2}$$

$$T_2 = T_1 \left(\frac{V_2}{V_1} \right) = T_1 \left(\frac{V_2}{V_1} \right) = T_1 \left(\frac{\frac{\pi D_2^2}{4} \times y_2}{\frac{\pi D_1^2}{4} \times y_1} \right)$$

$$T_2 = T_1 \left(\frac{y_2}{y_1} \right)$$

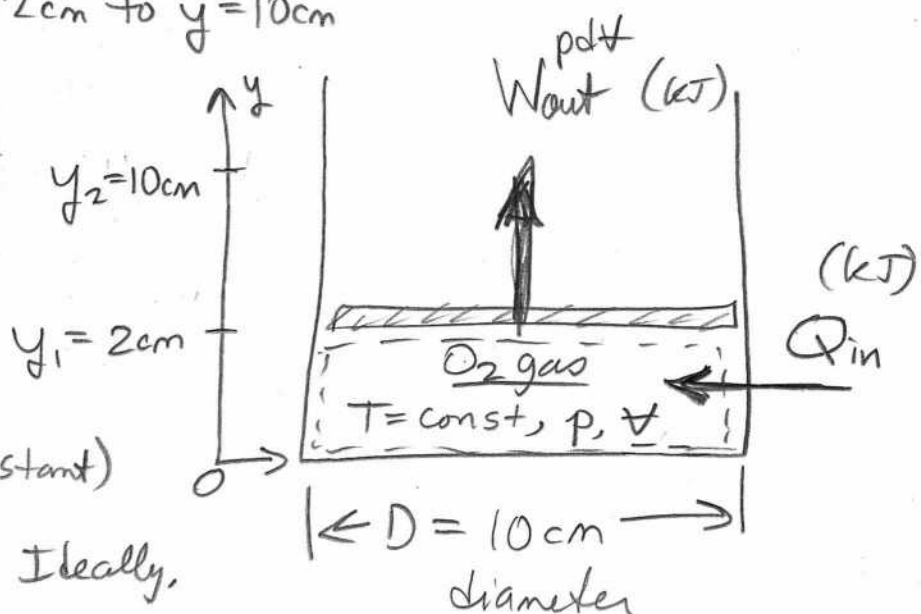
$$T_2 = (1227^\circ C + 273) \left(\frac{2 cm}{10 cm} \right) = 300 K \Rightarrow \boxed{T_2 = 27^\circ C}$$

Example 5 Moving "pdV" Boundary Work

Given: Consider the piston-cylinder containing 0.01 kg O_2 gas initially at $150^\circ C$, 506.6 kPa. The piston is allowed to move upwards from $y = 2\text{ cm}$ to $y = 10\text{ cm}$

such that the O_2 gas performs work on the outside environment.

The process is isothermal ($T = \text{constant}$) and the gas behaves ideally.



Find: (1) The moving boundary work performed by the gas on the external environment
(2) The final O_2 gas pressure.

Assume (1) quasi-equilibrium process
(2) Isothermal process
(3) O_2 behaves as an Ideal gas

Analysis

(1) For quasiequilibrium moving boundary work output, the inexact differential is

$$\delta W_{\text{out}}^{pdV} = +p dV$$

the mechanical work performed from state ① \rightarrow ② is (14)

$$W_{1 \rightarrow 2} = \int_1^2 \delta W$$

Substituting $\delta W_{\text{out}}^{\text{pdv}}$ into the integral from state ① \rightarrow ②,

$$W_{\text{out}}^{\text{pdv}} = \int_1^2 p dV$$

Now we must find a relationship between p and V in order to integrate this thing!

The problem states that the process path is isothermal, or $T_1 = T_2 = 150^\circ\text{C}$. Thus, using the Ideal gas EOS,

$$pV = RT$$
$$p = \frac{RT}{V} = \frac{mRT}{V}$$

we have

$$W_{\text{out}}^{\text{pdv}} = \int_1^2 \left(\frac{mRT}{V} \right) dV$$

$$W_{\text{out}}^{\text{pdv}} = mRT \int_{V_1}^{V_2} \frac{dV}{V}$$

The solution is $\left[\int_{x_1}^{x_2} \frac{dx}{x} = \ln(x_2) - \ln(x_1) = \ln\left(\frac{x_2}{x_1}\right) \right]$

$$W_{\text{out}}^{\text{pdv}} = mRT \ln \left[\frac{V_2}{V_1} \right]$$

This can be simplified since

(15)

$$\frac{V_2}{V_1} = \frac{\frac{\pi D^2}{4} y_2}{\frac{\pi D^2}{4} y_1} \quad \text{since } D_1 = D_2$$

$$\Rightarrow \boxed{\frac{V_2}{V_1} = \frac{y_2}{y_1}}$$

$$\Rightarrow \boxed{W_{\text{out}}^{\text{pdV}} = mRT \ln \left[\frac{y_2}{y_1} \right]}$$

Substituting values, ($R = 0.2598 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$ from Table 1 for D_2)

$$W_{\text{out}}^{\text{pdV}} = (0.01 \text{ kg}) (0.2598 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}) (150^\circ\text{C} + 273) \ln \left[\frac{10 \text{ cm}}{2 \text{ cm}} \right]$$

$$\boxed{W_{\text{out}}^{\text{pdV}} = 1.769 \text{ kJ}}$$

(2) For an Ideal gas undergoing an isothermal process,

$$T_1 = T_2$$

or

$$\frac{P_1 V_1}{R} = \frac{P_2 V_2}{R}$$

$$P_2 = P_1 \left(\frac{V_1}{V_2} \right) = P_1 \left(\frac{V_1}{V_2} \right) = P_1 \left(\frac{\frac{\pi D_1^2}{4} y_1}{\frac{\pi D_2^2}{4} y_2} \right)$$
$$\boxed{P_2 = P_1 \left(y_1 / y_2 \right)}$$

$$P_2 = (506.6 \text{ kPa}) \left(\frac{2 \text{ cm}}{10 \text{ cm}} \right) \Rightarrow$$

$$\boxed{P_2 = 101.32 \text{ kPa} \approx 1 \text{ atm}}$$

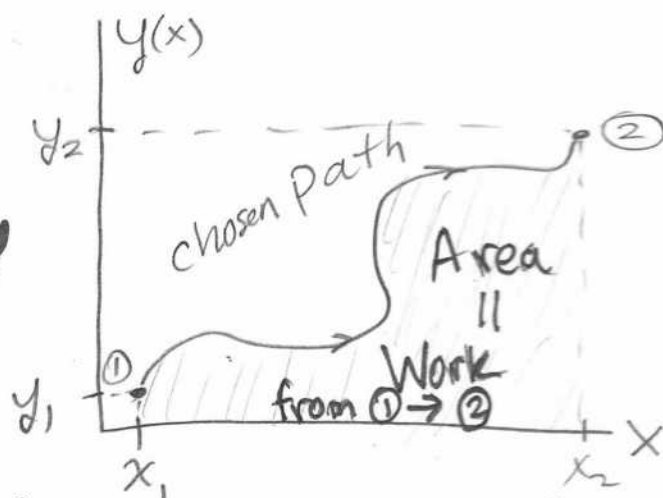
Lecture 14 Polytropic Processes for Gases, vapors, saturated mixtures

- In lecture 13 it was obvious that once you obtained the integral (in general terms)

$$W_{1 \rightarrow 2} = \int_{\text{state 1}}^{\text{state 2}} y dx$$

for any of the forms of work (electrical, mechanical, etc.) then you needed to Specify the relationship between x and y

- An infinite number of choices for the functional form of $y=f(x)$ can be dreamed of!



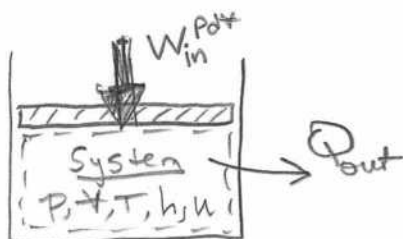
⇒ Remember : Work (and Heat Transfer Q) is "path dependent"!

⇒ W (and Q) depend on your choice of $y=f(x)$

⇒ W (and Q) are path dependent Interactions with the System that DRIVE changes in States (Properties).

"pdV" Boundary Work using a special Process path: $pV^n = \text{Constant}$ ("Polytropic")

- In many engineering and physics problems we encounter the need to use a particular functional relationship between pressure and volume in order to calculate the boundary work ("pdV" work) integral



$$W_{pdV} = \pm \int_1^2 p dV$$

(+) \Rightarrow Expansion W_{pdV}^{out}
 (-) \Rightarrow Compression W_{pdV}^{in}

Valid for solids, liquids, gases, vapors, saturated mixtures

- There is a special relationship between pressure and volume, valid for gases, vapors, and saturated mixtures called a "polytropic Process".

Polytropic Process (gases, vapors, saturated mixtures)

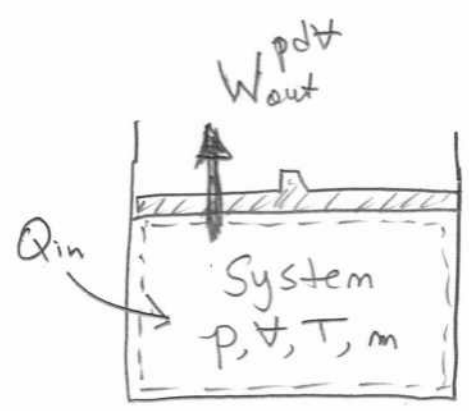
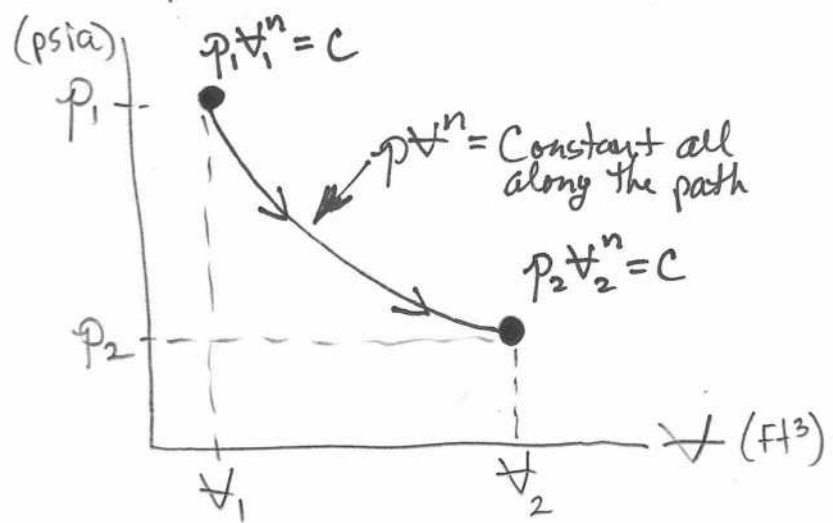
A quasiequilibrium Process between states that relates the system (Closed or Open) pressure and volume according to

$$pV^n = \text{Constant "C"}$$

where n is any number: $-\infty \leq n \leq +\infty$
 and is called the polytropic exponent (or "index")

General Expressions for "pdv" Work Using a Polytropic Process Path

- Consider a closed piston-cylinder system with the p-v diagram showing an expansion process:



- From the basic definition of Polytropic process, we can immediately find a relationship between p and v connecting States ① and ②:

$p v^n = \text{Constant all along the process path, including the initial and final states, ① and ②}$

$$p_2 v_2^n = C = p_1 v_1^n$$

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2} \right)^n$$

$$\frac{p_2}{p_1} = \left[\frac{v_1}{v_2} \right]^n$$

(Valid for gases, vapors, saturated mixtures)

- From the general expression for "pdV" boundary work output, (4)

$$W_{\text{out}}^{\text{pdV}} = + \int_{\text{state 1}}^{\text{state 2}} p dV$$

we substitute in the polytropic process $p(V) = C V^{-n}$:

$$W_{\text{out}}^{\text{pdV}} = + \int_{V_1}^{V_2} C V^{-n} dV = C \int_{V_1}^{V_2} V^{-n} dV$$

Integrating from V_1 to V_2 ,

$$W_{\text{out}}^{\text{pdV}} = \frac{C}{(-n+1)} \left[V^{-n+1} \right]_{V_1}^{V_2} = \frac{C}{1-n} \left[V_2^{1-n} - V_1^{1-n} \right]$$

Q: How do we find the constant C ?

A: Use the definition of polytropic process: $C = p_1 V_1^n = p_2 V_2^n$

$$W_{\text{out}}^{\text{pdV}} = \frac{C V_2^{1-n} - C V_1^{1-n}}{1-n} = \frac{p_2 V_2^n V_2^{1-n} - p_1 V_1^n V_2^{1-n}}{1-n}$$

simplifying exponents,

$$W_{\text{out}}^{\text{pdV}} = \frac{p_2 V_2^{1-n+n} - p_1 V_1^{1-n+n}}{1-n}$$

$$W_{\text{out}}^{\text{pdV}} = \frac{p_2 V_2 - p_1 V_1}{1-n}$$

By our sign convention, $W_{\text{out}}^{\text{pdV}} \geq 0$ and $W_{\text{in}}^{\text{pdV}} \geq 0$

so Work output (expansion) or work input (compression) is distinguished by either a (+) or (-) sign:

$$W^{\text{pdV}} = \frac{\pm 1}{1-n} [p_2 V_2 - p_1 V_1] \geq 0$$

(+) \Rightarrow expansion, Work out
 (-) \Rightarrow compression, Work in

Example 1 "pdv" Work using a polytropic process for a Saturated Mixture

(5)

1 kg of H_2O is initially a saturated mixture at $150^\circ C$ and 476.16 kPa with a quality $x = 50\%$ inside a piston-cylinder assembly. Work is performed by the piston on the system using a polytropic process in which the index is $n = 0.585$. The final state is a saturated mixture at $300^\circ C$.

Find: W_{in}^{pdv} for this process.

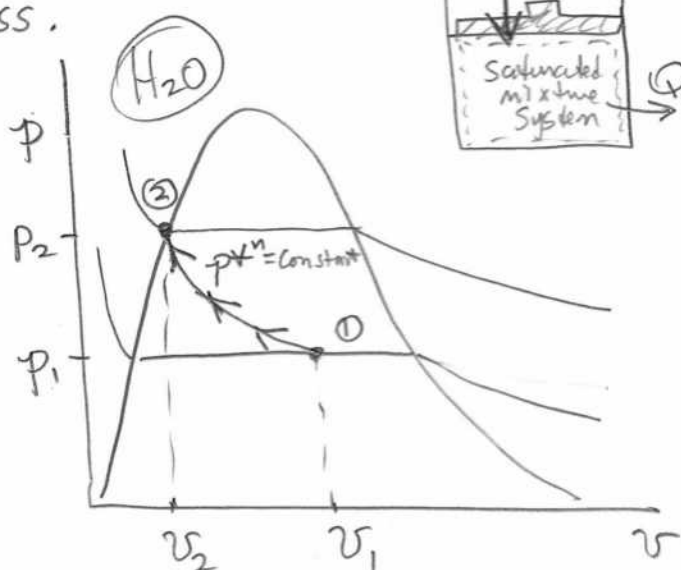
Assume: Quasiequilibrium process.

Analysis: Draw a picture!

State ① { saturated mixture
 $T_1 = 150^\circ C$
 $p_1 = 476.16 \text{ kPa}$
 $x_1 = 50\% = 0.5$

$$n = 0.585$$

State ② { Saturated liquid
 $T_2 = 300^\circ C$



- In general, the "pdv" work performed on a system for either liquid, gases, vapors, or saturated mixtures is given by

$$W_{in}^{pdv} = \frac{1}{1-n} [p_2 v_2 - p_1 v_1]$$

- We need to find p_2 , v_2 , and v_1 to compute the necessary work input for this process.
- Let's start with state ① using $x = 0.5$

$$v_1 = v_{f@150^\circ C} + x v_{fg@150^\circ C}$$

(6)

From Table A-4, at $T_{\text{sat}} = 300^\circ\text{C}$,

$$v_{f@150^\circ\text{C}} = 0.001091 \text{ m}^3/\text{kg}$$

$$v_{fg@150^\circ\text{C}} = 0.39248 \text{ m}^3/\text{kg} - 0.001091 \text{ m}^3/\text{kg} = 0.391389 \text{ m}^3/\text{kg}$$

$$\text{So } v_1 = 0.001091 \text{ m}^3/\text{kg} + (0.5)(0.391389 \text{ m}^3/\text{kg})$$

$$\boxed{v_1 = 0.196785 \text{ m}^3/\text{kg}}$$

Also, from Table A-4, @ $T_{\text{sat}} = 300^\circ\text{C}$,

$$\boxed{P_1 = P_{\text{sat}@300^\circ\text{C}} = 476.16 \text{ kPa}}$$

For state ②, we need to find P_2 , which, for a saturated liquid is just

$$\boxed{P_2 = P_{\text{sat}@300^\circ\text{C}} = 8587.9 \text{ kPa}}$$

$$\boxed{v_2 = v_{\text{sat}@300^\circ\text{C}} = 0.001404 \text{ m}^3/\text{kg}}$$

The "pdV" Work input necessary to accomplish the process from ① → ② is then

$$W_{\text{in}}^{\text{pdV}} = \frac{-m}{1-n} (P_2 v_2 - P_1 v_1)$$

$$W_{\text{in}}^{\text{pdV}} = \frac{-1 \text{ kg}}{(1-0.585)} \left[(8587.9 \text{ kPa})(0.001404 \text{ m}^3/\text{kg}) - (476.16 \text{ kPa})(0.196785 \text{ m}^3/\text{kg}) \right]$$

$$W_{\text{in}}^{\text{pdV}} = (-2.4096) \left[12.0574 \text{ kPa} \cdot \text{m}^3 - 93.7011 \text{ kPa} \cdot \text{m}^3 \right]$$

(kPa · m³ = $\frac{\text{kN}}{\text{m}^2} \cdot \text{m}^3 = \text{kN} \cdot \text{m} = \text{kJ}$)

$$W_{\text{in}}^{\text{pdV}} = (-2.4096)(-81.6437 \text{ kJ})$$

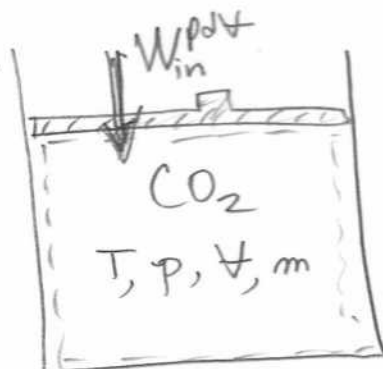
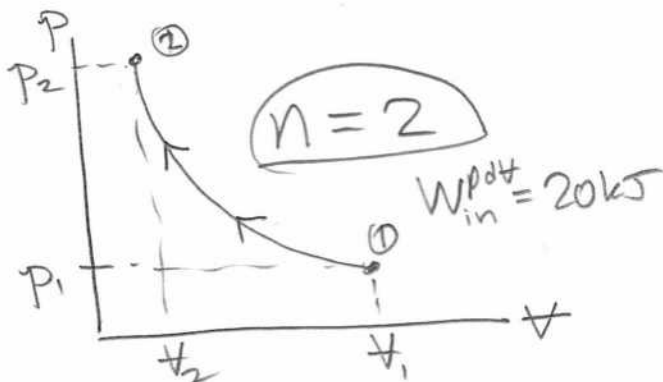
$$\boxed{W_{\text{in}}^{\text{pdV}} = 196.7 \text{ kJ}}$$

Example 2 CO_2 Polytropic process, $n=2$

(7)

Carbon dioxide is contained within a piston-cylinder device initially at 100 kPa within a volume of 0.3 m^3 . The piston compresses the CO_2 until the volume is 0.1 m^3 and the work input performed on the gas is 20 kJ. What is the final pressure, assuming that the compression process is polytropic having an index $n=2$?

Given:



Find: P_2 (kPa)

Assume: (1) Polytropic process with $n=2$
(2) quasiequilibrium process

Analysis

State ①

$$P_1 = 100 \text{ kPa}$$

$$v_1 = 0.3 \text{ m}^3$$

State ②

$$P_2 = ?$$

$$v_2 = 0.1 \text{ m}^3$$

$$\text{and } W_{in}^{pdv} = 20 \text{ kJ}$$

From the general expression for W^{pdv} work for polytropic process,

$$W_{in}^{pdv} = \frac{-1}{1-n} [P_2 v_2 - P_1 v_1]$$

re-arranging,

$$P_2 = \frac{(n-1)W_{in}^{pdv} + P_1 v_1}{v_2}$$

So

$$P_2 = \frac{(2-1)(20 \text{ kJ}) + (100 \text{ kPa})(0.3 \text{ m}^3)}{0.1 \text{ m}^3} \Rightarrow$$

$$P_2 = 500 \text{ kPa}$$

Special cases for n

- While the value of n for a polytropic process can be $-\infty \leq n \leq +\infty$,

there are certain special cases of interest that are used a lot in thermodynamic analysis.

- Special cases = $\begin{cases} n=0 \\ n=1 \\ n=\infty \end{cases}$ $W^{pd\gamma} = \frac{\pm 1}{1-n} [p_2 \gamma_2 - p_1 \gamma_1]$

Case 1 $n=0$ (isobaric polytropic process)

- In this case, $W^{pd\gamma} = \frac{\pm 1}{1-0} [p_2 \gamma_2 - p_1 \gamma_1]$, or

$$W^{pd\gamma} = \pm [p_2 \gamma_2 - p_1 \gamma_1]$$

Q: But why is this so important thermodynamically ... what does it mean?

A: By definition, a polytropic process has a process path defined by $p\gamma^n = \text{constant}$ all along the path

or, as we found above, $\frac{p_2}{p_1} = \left[\frac{\gamma_1}{\gamma_2} \right]^n$

for $n=0$ $\frac{p_2}{p_1} = \left[\frac{\gamma_1}{\gamma_2} \right]^0 = 1$

meaning $p_1 = p_2 \equiv p_0$ Constant all along the path

then $W^{pd\gamma} = \pm [p_0 \gamma_2 - p_0 \gamma_1]$

"isobaric"

$$W^{pd\gamma} = \pm p_0 [\gamma_2 - \gamma_1] \geq 0$$

$n=0$ is a constant pressure polytropic process.

Case 2 $n=1$

9

- When we plug $n=1$ into the equation for polytropic "pdV" work we immediately see a problem:

$$W^{pdV} = \frac{\pm 1}{1-1} [P_2 V_2 - P_1 V_1] = \pm \infty$$

Which is nonsense.

Q: What is going wrong and how do we compute W^{pdV} for $n=1$?

A: The problem is with the integration. We have to go back to the original integral:

$$\text{for } n=1, \quad p(V) = C V^{-1}$$

$$\text{so} \quad W_{\text{out}}^{pdV} = \int_1^2 p(V) dV = C \int_{V_1}^{V_2} \frac{dV}{V}$$

$$\text{then} \quad W_{\text{out}}^{pdV} = C \left[\ln(V) \right]_{V_1}^{V_2} = C \left[\ln(V_2) - \ln(V_1) \right]$$

$$\boxed{W_{\text{out}}^{pdV} = C \ln\left(\frac{V_2}{V_1}\right)}$$

The constant C is determined from $C = P_1 V_1 = P_2 V_2$, so we get two equally valid expressions:

$$W^{pdV} = \begin{cases} \pm P_1 V_1 \ln(V_2/V_1) \geq 0 \\ \pm P_2 V_2 \ln(V_2/V_1) \geq 0 \end{cases}$$

(+) \Rightarrow expansion, work output

(-) \Rightarrow compression, work input

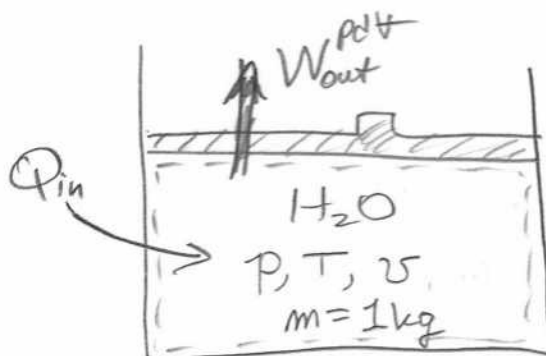
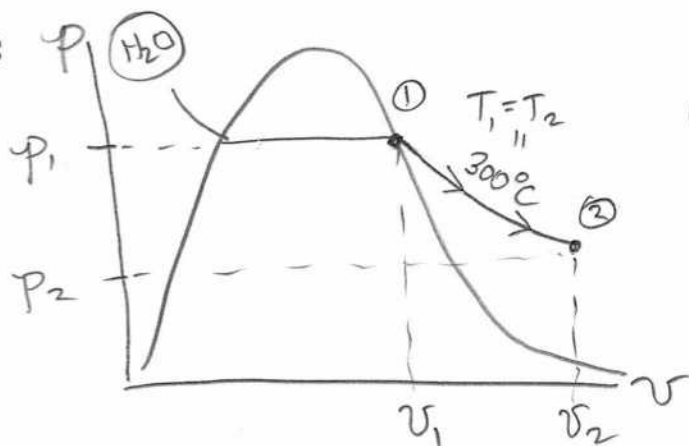
$n=1$

Warning!!
Not Isothermal
in general unless
it is for an
Ideal Gas!!

Example 3 Isothermal Expansion of Superheated Water Vapor (10)

1 kg of Water vapor is expanded isothermally from a saturated liquid state of 300°C to a final pressure of 2 MPa in a superheated vapor state. What is the boundary "pdv" work output?

Given:



State ①

$$T_1 = 300^\circ\text{C}$$

$$P_1 = P_{\text{sat}@300^\circ\text{C}} = 8587.9 \text{ kPa} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{Table A-4}$$

$$v_1 = v_{g@300^\circ\text{C}} = 0.001404 \frac{\text{m}^3}{\text{kg}}$$

State ②

$$T_2 = 300^\circ\text{C}$$

$$P_2 = 2000 \text{ kPa}$$

$$v_2 = 0.12551 \frac{\text{m}^3}{\text{kg}} \quad (\text{Table A-6})$$

Find: $W_{\text{out}}^{\text{pdv}}$ (kJ)

Assume: Quasi-equilibrium process

Analysis

First, demonstrate that a polytropic process for $n=1$, which is "isothermal" only for an Ideal gas, is not valid for vapor states.

$$W_{\text{out}}^{\text{pdv}} = p_1 v_1 \ln\left(\frac{v_2}{v_1}\right) = p_2 v_2 \ln\left(\frac{v_2}{v_1}\right) \quad n=1$$

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The value of $\ln\left(\frac{v_2}{v_1}\right) = \ln\left(\frac{v_2}{v_1}\right) = \ln\left(\frac{0.12551 \text{ m}^3/\text{kg}}{0.001404 \text{ m}^3/\text{kg}}\right)$

$$\ln\left(\frac{v_2}{v_1}\right) = 4.493$$

$$p_1 v_1 = m p_1 v_1 = (1 \text{ kg})(8587.9 \text{ kPa})(0.001404 \text{ m}^3/\text{kg})$$

$$p_1 v_1 = 12.05 \text{ kJ}$$

$$p_2 v_2 = m p_2 v_2 = (1 \text{ kg})(2000 \text{ kPa})(0.12551 \text{ m}^3/\text{kg})$$

$$p_2 v_2 = 251.0 \text{ kJ}$$

$n=1$ is NOT isothermal process
except for an Ideal Gas !!!

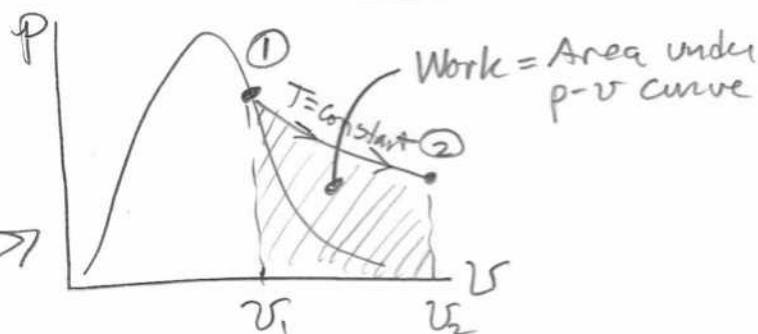
$$W_{\text{out}}^{p d v, n=1}$$

$$= 54.2 \text{ kJ and/or } 1127.8 \text{ kJ} !!!$$

You can't have two values for work output!

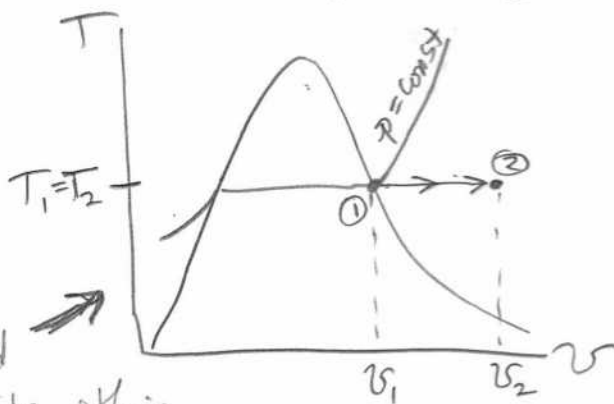
Q: What do we do?

A: We are stuck without a computer numerical solution.



Q: Why?

A: Although $W^{p d v} = \int_1^2 p dv$, we do not have an analytic equation for the isothermal process path for a superheated vapor. The tables provide this equation of state and we need to integrate numerically.



Note: you cannot assume that finding the index will give you the isothermal path! $n = \frac{\ln(p_2/p_1)}{\ln(v_2/v_1)}$
For these states ① and ②, $n \approx -0.324$
But it is not necessarily an isothermal path!

Example 4

Isobaric / Iso-thermal process for a saturated mixture (within the "Dome")

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20 grams of R-134a refrigerant, as a saturated mixture, is contained within a piston-cylinder assembly. The initial quality and pressure within the piston-cylinder assembly is 0.56 and 1.0171 MPa. The refrigerant is allowed to expand at a constant temperature of $T_0 = 40^\circ\text{C}$ until it becomes a saturated vapor at 40°C (Note that heat Q must be added to the refrigerant during the phase change process so that T remains constant while volume expansion occurs simultaneously). What is the "pdV" boundary work output?

Given: State ① $\left\{ \begin{array}{l} x_1 = 0.56 \\ P_1 = 1.0171 \text{ MPa} \\ T_1 = 40^\circ\text{C} = 313 \text{ K} \end{array} \right.$
Saturated mixture

State ② $\left\{ \begin{array}{l} T_2 = 40^\circ\text{C} = 313 \text{ K} \\ P_2 = 1.0171 \text{ MPa} \end{array} \right.$
Saturated vapor
(Since this is a constant T-process)

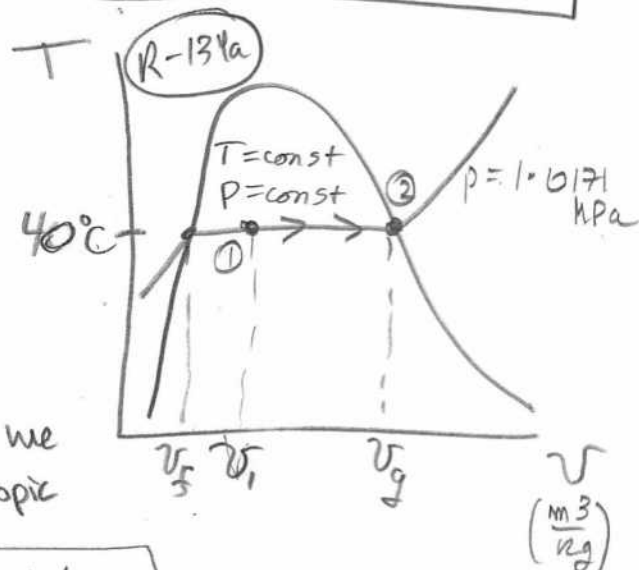
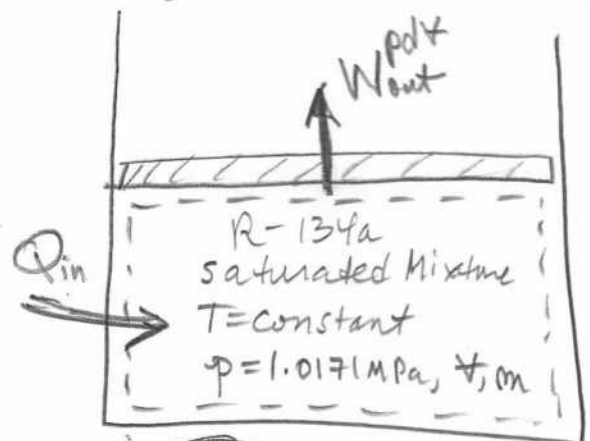
Find: $W_{\text{out}}^{\text{pdV}}$ (J)

Assume: Quasi-equilibrium process

Analysis: The process path under the vapor dome is both isobaric and isothermal.

Lets see what happens when we (erroneously) apply the $n=1$ polytropic expression to the process ① \rightarrow ②

$$W_{\text{out}}^{\text{pdV}} = P_1 v_1 \ln\left(\frac{v_2}{v_1}\right) = P_2 v_2 \ln\left(\frac{v_2}{v_1}\right)$$



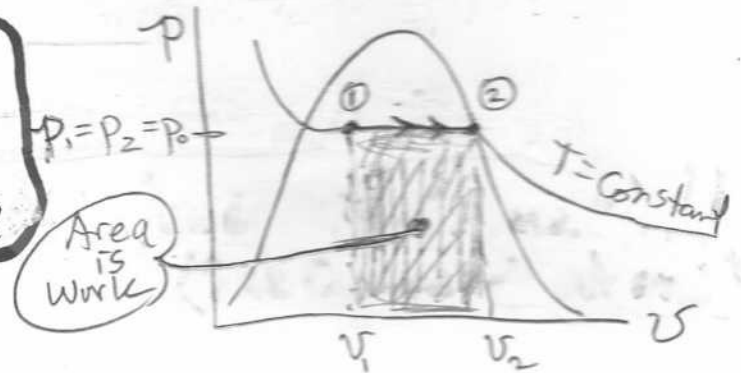
Q: Do you notice something wrong?

A: Yes! The Work output has two values!! ??

$$W_{out}^{pdv} = p_0 \underbrace{V_1}_{\text{not same!}} \ln\left(\frac{V_2}{V_1}\right) = p_0 \underbrace{V_2}_{\text{same}} \ln\left(\frac{V_2}{V_1}\right)$$

Clearly, the Volume changed and work output was performed:

This expression for $n=1$ is NOT VALID for an isothermal process unless the system is an Ideal Gas!



Q: What do we do?

A: Go back to $pV^n = \text{constant}$.

Q: What value of n will work for this process?

A: $n=0$, so that $pV^0 = p = \text{constant}$ during this isothermal/isobaric phase-change process!

With $n=0$ in the other expression for W^{pdv} (see pg.6)

$$W_{out}^{pdv} = \left(\frac{1}{1-0}\right)(p_2 V_2 - p_1 V_1)$$

$$W_{out}^{pdv} = p_0 (V_2 - V_1)$$

Now we need to find V_2 and V_1 :

$$V_1 = m V_f$$

$$V_2 = m (V_f + x_1 V_{fg})$$

From table A-11 at 40°C ,

$$v_f = 0.0008720 \frac{\text{m}^3}{\text{kg}}, \quad v_g = 0.019952 \frac{\text{m}^3}{\text{kg}}$$

$$\Rightarrow v_{fg} = 0.01908 \frac{\text{m}^3}{\text{kg}}$$

$$\Rightarrow V_1 = (0.02 \text{ kg}) \left(0.0008720 + (0.56)(0.01908 \frac{\text{m}^3}{\text{kg}}) \right)$$

$$V_1 = 2.31136 \times 10^{-4} \text{ m}^3$$

$$V_2 = m v_g = (0.02 \text{ kg}) \left(0.019952 \frac{\text{m}^3}{\text{kg}} \right)$$

$$V_2 = 3.9904 \times 10^{-4} \text{ m}^3$$

Thus,

$$W_{\text{out}}^{\text{pd}} = (1017.1 \text{ kPa}) (3.9904 \times 10^{-4} - 2.31136 \times 10^{-4} \text{ m}^3)$$

$$W_{\text{out}}^{\text{pd}} = (1017.1 \text{ kPa}) (1.67904 \times 10^{-4} \text{ m}^3)$$

$$W_{\text{out}}^{\text{pd}} = 0.17077 \text{ kJ}$$

$$W_{\text{out}}^{\text{pd}} = 170.8 \text{ J}$$

Moral of Story: Don't just grab for equations or you'll be a goner! You have to draw a picture and THINK.

polytropic $n=1$ is NOT Isothermal except for an Ideal Gas!!

• Case 3 $n = \pm \infty$

(15)

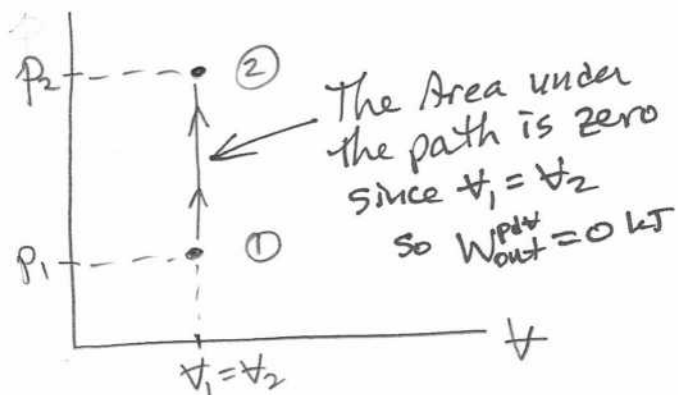
Let $p_1 > p_2$ so that we have an expansion process:

$$W_{out}^{pdV} = \frac{+1}{1-n} [p_2 V_2 - p_1 V_1]$$

Now take the limit of this expression as $n \rightarrow +\infty$ (or $-\infty$)

$$\lim_{n \rightarrow \infty} (W_{out}^{pdV}) = \lim_{n \rightarrow \infty} \left[\frac{1}{1-n} [p_2 V_2 - p_1 V_1] \right]$$

$$= 0 \text{ kJ}$$



- The Work output is zero for $n = \pm \infty$, which means that the area under the process path is zero.
- This can only be true if $V_1 = V_2$

⇒ Constant volume process is called "Isochoric" and $W^{pdV} = 0 \text{ kJ}$

Summary of Polytropic process "pdv"

Work Equations for gases, vapors, and Saturated mixtures

$n \neq 1$	$W^{pdv} = \frac{\pm 1}{1-n} [P_2 v_2 - P_1 v_1] \geq 0$	General
$n = 0$	$W^{pdv} = \pm P_0 [v_2 - v_1] \geq 0$	Isobaric
$n = 1$	$W^{pdv} = \begin{cases} \pm P_1 v_1 \ln \left[\frac{v_2}{v_1} \right] \\ \pm P_2 v_2 \ln \left[\frac{v_2}{v_1} \right] \end{cases}$	<u>NOT</u> Isothermal except for an Ideal gas!
$n = \pm \infty$	Isochoric \rightarrow constant $\underset{\text{Volume}}{}$, $W^{pdv} = 0$	

$$\frac{P_2}{P_1} = \left[\frac{v_1}{v_2} \right]^n$$

General Definition for polytropic process

(+) \Rightarrow expansion, W_{out}^{pdv}

(-) \Rightarrow compression, W_{in}^{pdv}

Ideal Gas Relationships for T, p, v using a Polytropic Process

- Introducing $pV = mRT$ into the Equation $pV^n = \text{Constant}$ provides simple relations between p, V (or v), and T .
- From page 3, for either liquids or gases (Ideal or real),

$$pV^n = \text{Const} \Rightarrow \boxed{\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^n}$$

between only two states ① and ②

- Restricting ourselves to Ideal Gases only,

$$\boxed{pV = mRT}$$

where $m = \text{constant}$ for Closed System Analysis.

- Substituting The Ideal gas EOS into the polytropic relation above at States ① & ②,

$$V_1 = \frac{mRT_1}{p_1} \quad \text{and} \quad V_2 = \frac{mRT_2}{p_2}$$

\Rightarrow

$$\frac{p_2}{p_1} = \left[\frac{\cancel{mRT_1}}{p_1} \times \frac{p_2}{\cancel{mRT_2}} \right]^n$$

$$\frac{p_2}{p_1} = \left[\frac{T_2}{T_1} \right]^n \left[\frac{p_2}{p_1} \right]^n$$

re-arranging,

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$$\frac{P_2}{P_1} \times \frac{P_1^n}{P_2^n} = \left(\frac{T_2}{T_1}\right)^n$$

$$\frac{P_2^{1-n}}{P_1^{1-n}} = \left(\frac{T_2}{T_1}\right)^n$$

$$\left(\frac{P_2}{P_1}\right)^{1-n} = \left(\frac{T_2}{T_1}\right)^n \Rightarrow$$

From this you can take the n^{th} root to obtain T_2/T_1 , or you take the $(1-n)^{\text{th}}$ root to get P_2/P_1 :

$$(a) \frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{n}{n-1}}$$

$$(b) \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

Use the form that is most convenient for your problem

Polytropic Relations for Ideal Gas

$$\frac{T_2}{T_1} = \left[\frac{P_2}{P_1}\right]^{\frac{n-1}{n}} = \left[\frac{V_1}{V_2}\right]^{n-1}$$

VERY

Important!

[$(V_2/V_1)^{n-1}$ follows from substituting $(\frac{V_2}{V_1})^n = \frac{P_2}{P_1}$ into the expression $(\frac{P_2}{P_1})^{\frac{n-1}{n}} = \frac{T_2}{T_1}$ above]

These relationships will be VERY IMPORTANT for both Thermo I and II, especially when $n \equiv k$, the ratio of specific heats \Rightarrow the polytropic process becomes a very special one called "isentropic" process

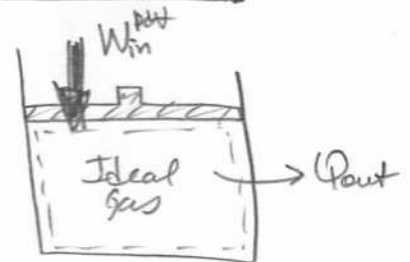
"pdV" Boundary Work using a Polytropic Process for an Ideal Gas

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- Ideal Gas E.O.S. for closed system:

$$pV = mRT$$

- In the general case, valid for gases (real or Ideal), vapors, and saturated mixtures,



$$W^{pdv} = \frac{\pm 1}{1-n} [p_2 V_2 - p_1 V_1] \geq 0 \quad n \neq 1$$

- Restricting ourselves to Ideal Gases only,

$$p_2 V_2 = mRT_2$$

$$p_1 V_1 = mRT_1$$

so that

$$W^{pdv} = \frac{\pm 1}{1-n} [mRT_2 - mRT_1]$$

$$W^{pdv} = \frac{\pm mR}{1-n} [T_2 - T_1] \geq 0 \quad \begin{matrix} n \neq 1 \\ \text{Ideal Gases} \\ \text{only} \end{matrix}$$

- These two equations are valid for Ideal Gases, along with the polytropic Relations for Ideal Gases only:

$$\frac{T_2}{T_1} = \left[\frac{p_2}{p_1} \right]^{\frac{n-1}{n}} = \left[\frac{V_1}{V_2} \right]^{n-1} \quad -\infty \leq n \leq +\infty$$

Special cases for n valid for Ideal Gases

(20)

Case 1 $n=0$ (Isobaric)

- for an Ideal gas, $W^{pd\psi} = \pm \frac{mR}{1-n} [T_2 - T_1]$
- With $n=0$, $W^{pd\psi} = \pm \frac{mR}{1-0} [T_2 - T_1]$

$$W^{pd\psi} = \pm mR [T_2 - T_1] \quad n=0$$

- From the definition of polytropic process, for $n=0$,

$$p_2 v_2^0 = \text{Constant} = p_1 v_1^0$$

$$\Rightarrow p_2 = p_1 = p_0 = \text{constant}$$

- Substituting this into the Ideal gas EOS,

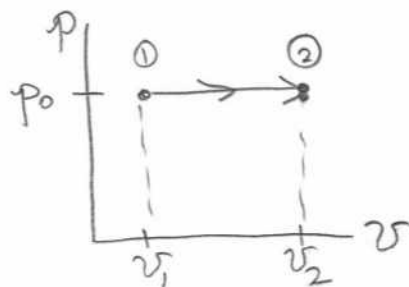
$$T_2 = \frac{p_0 v_2}{mR} \quad \text{and} \quad T_1 = \frac{p_0 v_1}{mR}$$

- Substituting these into the expression for $W^{pd\psi}$ with $n=0$,

$$W^{pd\psi} = \pm mR \left[\frac{p_0 v_2}{mR} - \frac{p_0 v_1}{mR} \right]$$

$$W^{pd\psi} = \pm \frac{mR p_0}{mR} [v_2 - v_1]$$

$$W^{pd\psi} = \pm p_0 [v_2 - v_1] \quad n=0$$



\Rightarrow There are 2 useful expressions for an isobaric (polytropic) process valid for Ideal gases only

Case 2 $n=1$ (Isothermal for Ideal Gas only) (21)

- The "pdv" work with $n=1$ is found by integration:

$$p(v) = C v^{-1}$$
$$W_{\text{out}}^{\text{pdv}} = \int_1^2 p(v) dv = C \int_{v_1}^{v_2} \frac{dv}{v} = C \ln[v_2/v_1]$$

so that the work has two expressions:

$$W^{\text{pdv}} = \begin{cases} \pm p_1 v_1 \ln(v_2/v_1) \\ \pm p_2 v_2 \ln(v_2/v_1) \end{cases} \quad n=1$$

- From the polytropic relations for an Ideal gas,

$$\frac{T_2}{T_1} = \left[p_2/p_1 \right]^{\frac{1-1}{1}} = \left[v_1/v_2 \right]^{1-1}$$

$$\frac{T_2}{T_1} = \left[p_2/p_1 \right]^0 = \left[v_1/v_2 \right]^0 = 1$$

$$\Rightarrow T_2 = T_1 = T_0 = \text{Constant}$$

- Using the Ideal gas EOS, $p_1 v_1 = m R T_0$, and $p_2 v_2 = m R T_0$, substituting into the work expression,

$$W^{\text{pdv}} = \pm m R T_0 \ln(v_2/v_1) \quad n=1, \text{ Ideal gas only}$$

- Substituting the Ideal gas EOS for $T = \text{constant} \equiv T_0$, $v_2 = \frac{m R T_0}{p_2}$ and $v_1 = \frac{m R T_0}{p_1}$

$$\Rightarrow v_2/v_1 = p_1/p_2$$

so

$$W^{\text{pdv}} = \pm m R T_0 \ln(p_1/p_2) \quad n=1, \text{ Ideal gas only}$$

Summary of polytropic process " $p dV$ " Work valid for Ideal Gases

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$n \neq 1$	$W^{pdV} = \frac{\pm 1}{1-n} [p_2 V_2 - p_1 V_1] \geq 0$	General
$n = 0$	$W^{pdV} = \pm mR [T_2 - T_1] \geq 0$ or $W^{pdV} = \pm p_0 [V_2 - V_1] \geq 0$	Isobaric
$n = 1$	$W^{pdV} = \pm mRT_0 \ln\left(\frac{V_2}{V_1}\right)$ or $W^{pdV} = \pm mRT_0 \ln\left(\frac{p_1}{p_2}\right)$	Isothermal
$n = \pm \infty$	Isochoric \rightarrow constant volume $W^{pdV} = 0$	

Polytropic Relations for Ideal gases:

$$\boxed{\frac{T_2}{T_1} = \left[\frac{p_2}{p_1}\right]^{\frac{n-1}{n}} = \left[\frac{V_1}{V_2}\right]^{n-1}}$$

Example 5 Polytropic Compression, Ideal Gas

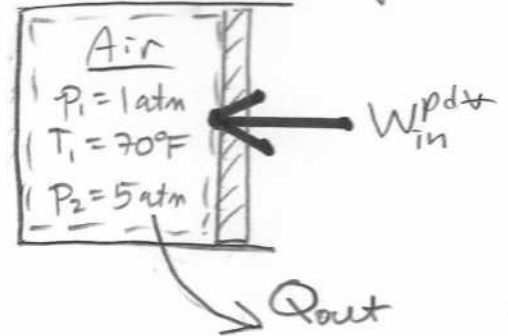
Air undergoes a polytropic compression in a piston-cylinder assembly from $p_1 = 1 \text{ atm}$, $T_1 = 70^\circ\text{F}$ to $p_2 = 5 \text{ atm}$. Employing the Ideal gas E.O.S., determine the " $p dV$ " work per-unit-mass, if the polytropic index is $n = 1.3$. (Note that heat transfer out of the cylinder must occur... you will learn this when you use the 1st Law Closed Energy Balance)

Given

State ① $\begin{cases} p_1 = 1 \text{ atm} \\ T_1 = 70^\circ\text{F} = 530\text{R} \end{cases}$

State ② $\begin{cases} p_2 = 5 \text{ atm} \end{cases}$

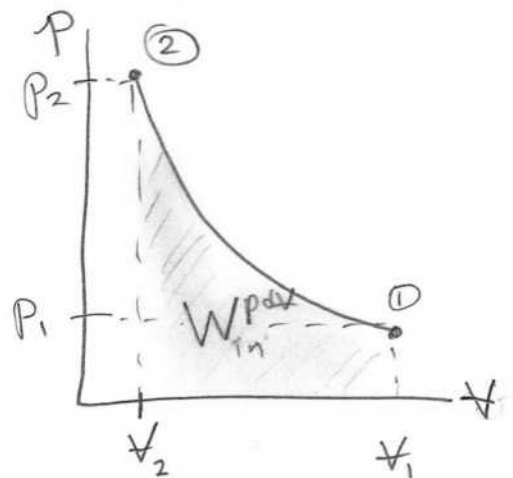
$n = 1.3$



Find: $W_{in}^{pdV} \left(\frac{\text{Btu}}{\text{lb}_m} \right)$

Assume: (1) quasi-equilibrium process
(2) Ideal gas behavior

Analysis



The work can be evaluated from the expression given on page 7 of Lecture 14:

$$W_{in}^{pdV} = -\frac{mR}{1-n} [T_2 - T_1] = -\frac{mR}{1-1.3} [T_2 - T_1]$$

$$W_{in}^{pdV} = \frac{mR}{0.3} [T_2 - T_1]$$

We don't know the temperature of State ②. However, for a polytropic process,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}}$$

thus,

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} = (530 R) \left(\frac{5 \text{ atm}}{1 \text{ atm}} \right)^{\frac{1.3-1}{1.3}}$$

$$= (530 R) (5)^{0.23076 \dots}$$

$$= (530 R) (1.44977 \dots)$$

$$\boxed{T_2 = 768 R}$$

The Work-per-unit mass is

$$w_{in}^{pd} = \frac{W_{in}^{pd}}{m} = \frac{R}{0.3} [T_2 - T_1]$$

From Table A-1E for air, $R = 0.06855 \frac{\text{Btu}}{\text{lb}_m \cdot R}$ so that

$$w_{in}^{pd} = \frac{0.06855 \frac{\text{Btu}}{\text{lb}_m \cdot R}}{0.3} [768 R - 530 R]$$

$$\boxed{w_{in}^{pd} = 54.38 \frac{\text{Btu}}{\text{lb}_m}}$$

Ideal Gas

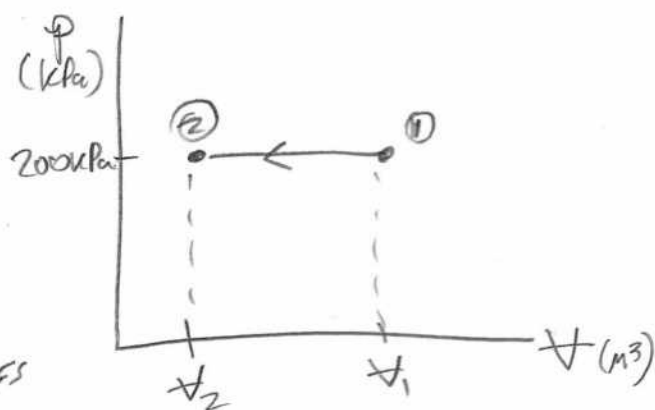
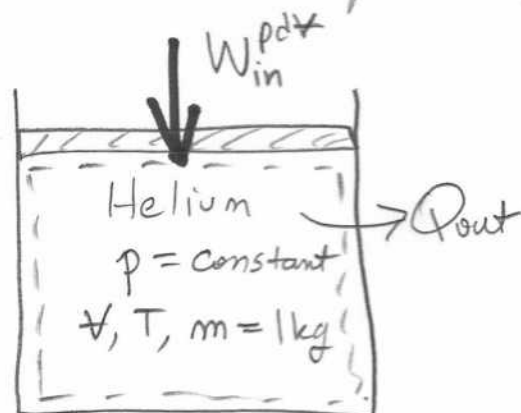
Example 6 Isobaric process (Polytropic with $n=0$)

The Volume of 1 kg of helium in a piston-cylinder device is initially 5 m^3 . Now helium is compressed to 3 m^3 while its pressure is maintained constant at 200 kPa . Determine the initial and final temperatures of helium as well as the work required to compress it, in kJ .

Given: $m = 1 \text{ kg}$ helium
 $p = \text{constant} = 200 \text{ kPa}$

State ① $\{ V_1 = 5 \text{ m}^3$

State ② $\{ V_2 = 3 \text{ m}^3$



Find: (a) T_1, T_2 (K)

(b) W_{in}^{pd} (kJ)

Assume (1) Quasi-equilibrium process
 (2) Ideal gas behavior

Analysis ($R = 2.0769 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$ for Helium from Table A-1)

(a) For a constant pressure process, the Ideal gas EOS gives

$$T_1 = \frac{p_1 V_1}{R} = \frac{p_1 V_1}{R m} = \frac{(200 \text{ kPa})(5 \text{ m}^3)}{(2.0769 \frac{\text{kJ}}{\text{kg} \cdot \text{K}})(1 \text{ kg})}$$

$$T_1 = 481.5 \text{ K}$$

Since $p_1 = p_2$, the Ideal gas EOS gives

$$P_1 = P_2$$

$$\frac{mRT_1}{V_1} = \frac{mRT_2}{V_2}$$

$$T_2 = T_1 \left(\frac{V_2}{V_1} \right) = (481.5 \text{ K}) \left(\frac{3 \text{ m}^3}{5 \text{ m}^3} \right)$$

$$T_2 = 288.9 \text{ K}$$

(b) "pdV" moving boundary work input for an Ideal gas is
(see page 7 of Lecture notes)

$$W_{in}^{pdV} = -\frac{mR}{1-n} [T_2 - T_1]$$

For a constant pressure process, the polytropic exponent is $n=0$

$$\Rightarrow W_{in}^{pdV} = -mR [T_2 - T_1]$$

$$W_{in}^{pdV} = -(1 \text{ kg}) (2.0769 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}) (288.9 \text{ K} - 481.5 \text{ K})$$

$$W_{in}^{pdV} = 400 \text{ kJ}$$

Ideal Gas

(27)

Example 7 Isothermal Process (Polytropic, $n=1$)

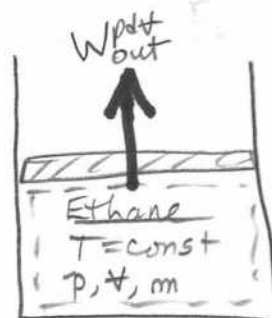
0.01 lb_m of Ethane gas (C₂H₆) expands in a piston-cylinder assembly, performing work on the environment during an isothermal process (Polytropic, $n=1$). The design specifications require that the work output should be 462 lb_f·ft and an initial pressure of 80 psia. The temperature of the Ethane must remain at 70°F at all times during the process. What is the final pressure at the end of the process?

Given: Ethane gas (C₂H₆)

State ① $\begin{cases} m = 0.01 \text{ lb}_m \\ T_1 = 70^\circ\text{F} = 529.67\text{R} \\ p_1 = 80 \text{ psia} \end{cases}$

$W_{out}^{pdv} = 462 \text{ lb}_f \cdot \text{ft}$

State ② $\begin{cases} m = 0.01 \text{ lb}_m \\ T_2 = 70^\circ\text{F} = 529.67\text{R} \end{cases}$



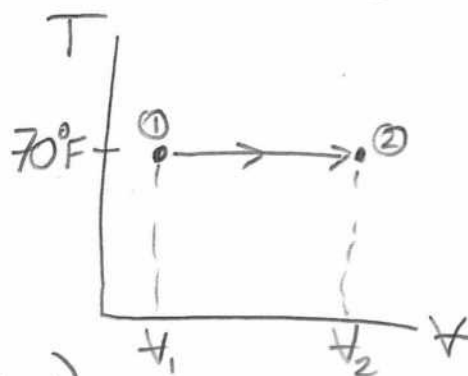
Find: p_2

Assume: (1) quasiequilibrium process

(2) Ideal gas behavior

Analysis

For an Ideal gas undergoing an isothermal process (Polytropic, $n=1$)



$$W_{out}^{pdv} = m R T_0 \ln\left(\frac{v_2}{v_1}\right) = m R T_0 \ln\left[\frac{m R T_2}{p_2} \times \frac{p_1}{m R T_1}\right] \quad (T_2 = T_1 = T_0)$$

$$W_{out}^{pdv} = m R T_0 \ln\left[p_1/p_2\right]$$

Solving for p_2 ,

$$\ln\left(\frac{p_1}{p_2}\right) = \frac{W_{out}^{pdv}}{mRT_0}$$

$$\frac{p_1}{p_2} = e^{\frac{W_{out}^{pdv}}{mRT_0}}$$

$$\Rightarrow \boxed{p_2 = p_1 e^{-\frac{W_{out}^{pdv}}{mRT_0}}}$$

From Table A-1E for Ethane, $R_{\text{ethane}} = 0.06616 \frac{\text{Btu}}{\text{lb}_m \text{R}}$

Converting the work to Btu units,

$$W_{out}^{pdv} = 462 \text{ lb}_f \cdot \text{ft} \times \frac{1 \text{ Btu}}{778.169 \text{ lb}_f \cdot \text{ft}} = 0.5937 \text{ Btu}$$

Then

$$\frac{W_{out}^{pdv}}{mRT_0} = \frac{0.5937 \text{ Btu}}{(0.01 \text{ lb}_m)(0.06616 \frac{\text{Btu}}{\text{lb}_m \text{R}})(529.67 \text{ R})}$$
$$= 1.6942$$

$$\Rightarrow p_2 = (80 \text{ psia})(e^{-1.6942})$$

$$p_2 = (80 \text{ psia})(0.183746)$$

$$\boxed{p_2 = 14.7 \text{ psia} \approx 1 \text{ atm}}$$

Lecture 15

The 1st Law of Thermodynamics for Closed Systems

(1)

- The 1st Law of thermodynamics is an expression of the Law of Conservation of Energy:

Energy can neither be created nor destroyed during a Process — it can only change forms

- While this last definition is correct, we need to express it in a more detailed way in order to solve thermodynamics problems.
- Recall from Lecture 2 where I noted that Newton's 2nd Law connected the subject of Kinematics to that of Dynamics:

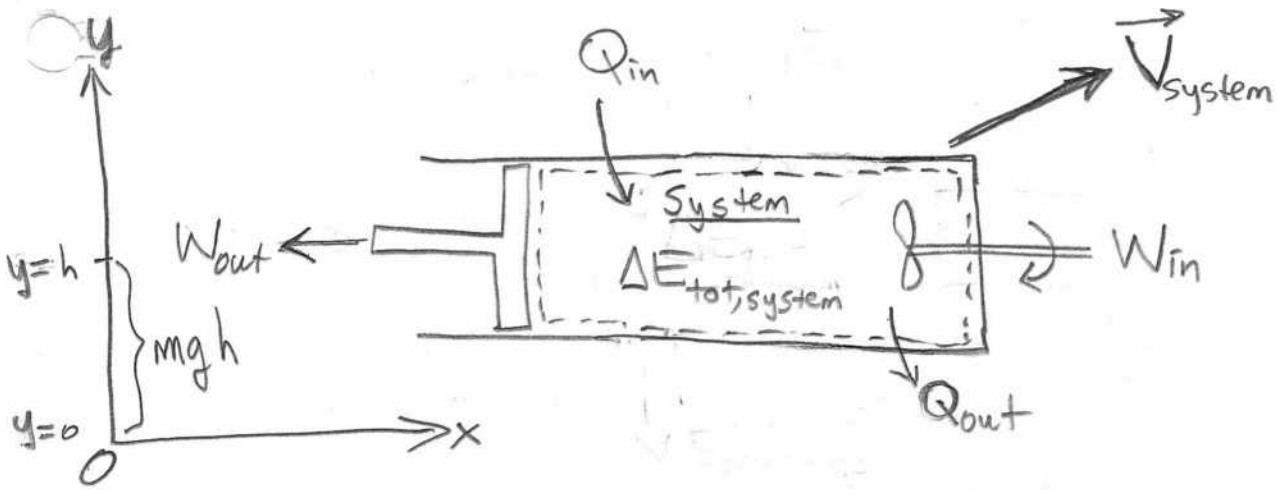
$$\Sigma \vec{F} = m\vec{a}$$

- We want a similar connection, thermodynamically speaking, between Energy Transfers across or at the Boundary (Interactions) and changes in the State of the System's Total Energy (Properties)

1st Law of Thermo

The Net Change in the Total Energy of a System during a Process is equal to the difference between the Total Energy entering and leaving the System during the Process.

Expressed As the "Energy Balance Equation"



$$\left[\begin{array}{l} \text{Net Change in the} \\ \text{Total Energy of} \\ \text{the System, Expressed} \\ \text{in terms of Properties} \end{array} \right] = \left[\begin{array}{l} \text{Total Energy} \\ \text{Transferred Into} \\ \text{the System} \end{array} \right] - \left[\begin{array}{l} \text{Total Energy} \\ \text{Transferred Out} \\ \text{of the System} \end{array} \right]$$

or

$$\Delta E_{Tot, system} = E_{in} - E_{out}$$

(RHS)

(LHS) = Net change in System Energy
[Properties]
[U, KE, P.E.]

Net Energy transfer
across or performed at
the System boundaries
[Interactions due
to Work, Heat, and Mass Flow]

• For Energy, this is the conceptual connection between

Net Change in
System Energy State
Expressed in terms of
Path Independent Properties

= [Net Energy transfer
into System caused
by path dependent
Interactions with
outside Universe]

(3)

- Now we want to Express this Energy Balance Eqn. in more detail :

LHS = $\Delta E_{\text{Total system}}$ (Energy State/Property Change)

Recall from Lecture 6 that the Total Energy Change of a System is expressed

$$\Delta E_{\text{tot, system}} = \Delta U_{\text{sys}} + \Delta E_{\text{mech}}$$

where $\Delta E_{\text{mech}} = \frac{1}{2} m (V_2^2 - V_1^2) + mg(z_2 - z_1)$

or $\Delta e_{\text{mech}} = \frac{1}{2} (V_2^2 - V_1^2) + g(z_2 - z_1)$

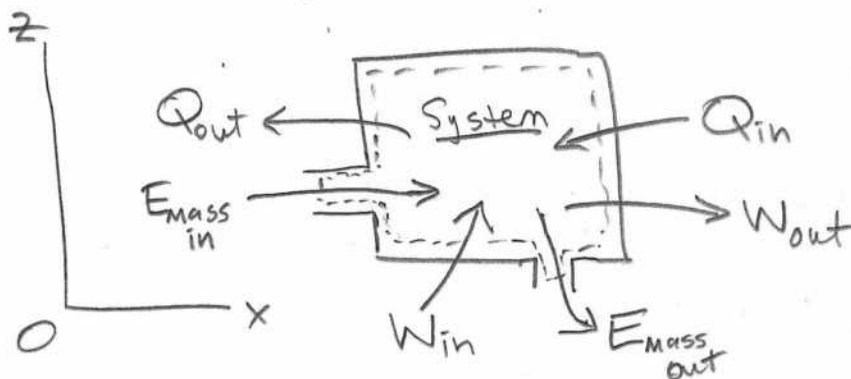
RHS = $E_{\text{in}} - E_{\text{out}}$ (Interaction of System with Environment)

- Types of interactions for ME 315/515 :

(1) Heat Transfer Q (kJ) or \dot{Q} (kW)

(2) Work Transfer W (kJ) or \dot{W} (kW)

(3) Mass Flow, E_{mass} (kJ) or \dot{E}_{mass} (kW)
[We will deal with mass flow later in ch.5]



The Closed System Energy Balance Egn

(4)

$$\Delta E_{sys} = E_{in} - E_{out} \quad (kJ)$$

$$\Delta U_{sys} + \Delta KE_{sys} + \Delta PE_{sys} = Q_{net,in} - W_{net,out}$$

$$\Delta U_{sys} + \Delta KE_{sys} + \Delta PE_{sys} = (Q_{in} - Q_{out}) - [W_{net,out}^{pd\&} + W_{net,out}^{other}]$$

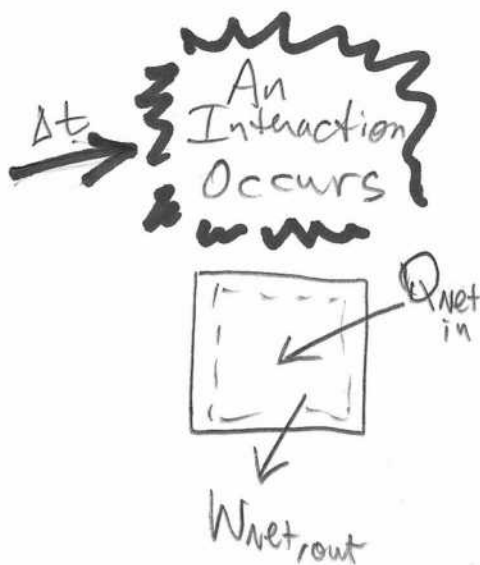
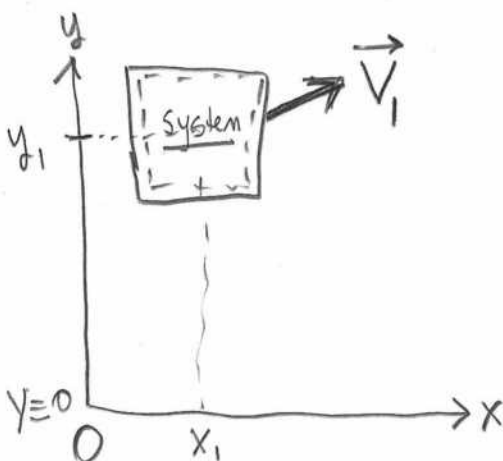
where

$W^{other} \Rightarrow$ Any work interaction that crosses the system boundaries, such as

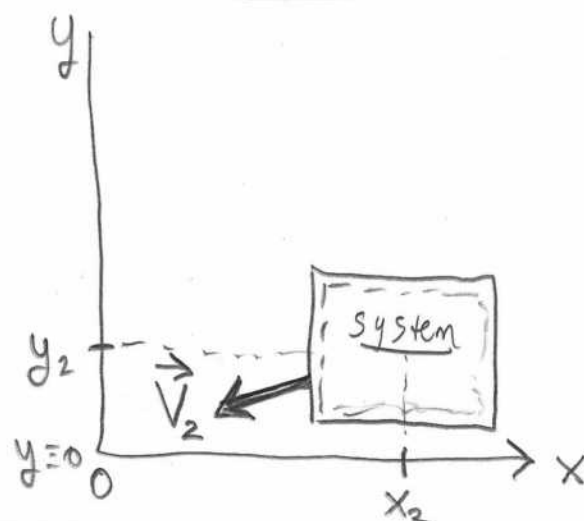
- (1) Electrical work
- (2) paddle wheel work etc.

$$\Delta U_{sys} + \Delta KE_{sys} + \Delta PE_{sys} = (Q_{in} - Q_{out}) - [(W_{out}^{pd\&} - W_{in}^{pd\&}) + (W_{out}^{other} - W_{in}^{other})]$$

State ①



State ②

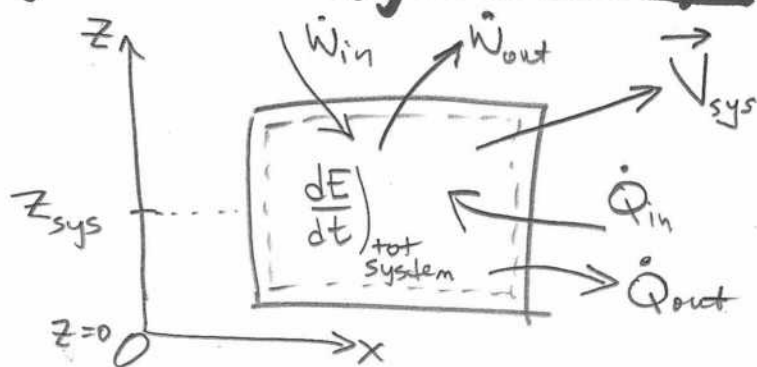


(5)

Rate Form of Closed System Energy Balance Egn

- The rate form is

$$\left(\frac{dE}{dt} \right)_{\text{sys}} = \dot{E}_{\text{in}} - \dot{E}_{\text{out}}$$



In more detail, the LHS and RHS are:

$$\frac{dU_{\text{sys}}}{dt} + \frac{d}{dt}(KE)_{\text{sys}} + \frac{d}{dt}(PE)_{\text{sys}} = \dot{Q}_{\text{in,net}} - \dot{W}_{\text{out,net}}$$

$$\dot{U}_{\text{sys}} + \frac{1}{2}m \frac{d}{dt}(V_{\text{sys}}^2) + mg \frac{dz}{dt} = \dot{Q}_{\text{in,net}} - \dot{W}_{\text{out,net}}$$

$$\dot{U}_{\text{sys}} + m V_{\text{sys}} \dot{V}_{\text{sys}} + mg V_{z,\text{sys}} = \dot{Q}_{\text{in,net}} - \dot{W}_{\text{out,net}} \quad (W)$$

Per-Unit mass Basis

Both forms of the Energy Balance Egn. can be expressed per-unit-mass by dividing by the total System mass m :

$$\text{Change form: } (u_2 - u_1)_{\text{sys}} + \frac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1) = q_{\text{in,net}} - w_{\text{out,net}}$$

$$\text{Rate form: } \dot{u}_{\text{sys}} + V_{\text{sys}} \dot{V}_{\text{sys}} + g V_{z,\text{sys}} = \dot{q}_{\text{in,net}} - \dot{w}_{\text{out,net}}$$

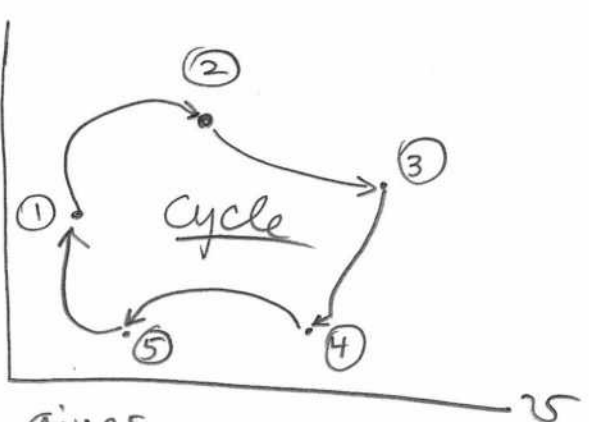
System Undergoing a Cycle

⑥

For any system undergoing a cycle of states where

$$① \rightarrow ② \rightarrow ③ \rightarrow \dots \rightarrow ①$$

The final state is the initial state, then the "change" form gives



$$\Delta E_{\text{total, sys}} = E_{\text{in}} - E_{\text{out}}$$

so

$$E_{\text{in}} = E_{\text{out}}$$

$$Q_{\text{in}} + W_{\text{in}} = Q_{\text{out}} + W_{\text{out}}$$

or

$$Q_{\text{in}} - Q_{\text{out}} = W_{\text{out}} - W_{\text{in}}$$

$$Q_{\text{net, in}} = W_{\text{net, out}} \quad (\text{kJ})$$

on a per-unit mass basis,

$$q_{\text{net, in}} = w_{\text{net, out}} \quad \left(\frac{\text{kJ}}{\text{kg}} \right)$$

On a Rate Basis,

$$\dot{Q}_{\text{net, in}} = \dot{W}_{\text{net, out}} \quad (\text{kW})$$

$$\dot{q}_{\text{net, in}} = \dot{w}_{\text{net, out}} \quad \left(\frac{\text{kW}}{\text{kg}} \right)$$

These
are
very
important!

Example 1 Ideal Gas Closed System Energy Balance

1 kg of oxygen is heated from 25°C to 300°C. Determine the amount of heat transfer required when the process is performed (a) $V = \text{constant}$ (isochoric), (b) $p = \text{constant}$ (isobaric)

Given: $m = 1 \text{ kg } O_2$

State ① $\begin{cases} T_1 = 25^\circ\text{C} \\ = 298 \text{ K} \end{cases}$

State ② $\begin{cases} T_2 = 300^\circ\text{C} \\ = 573 \text{ K} \end{cases}$

Find: Q_{in} for (a) isochoric process
(b) isobaric process

Assume:

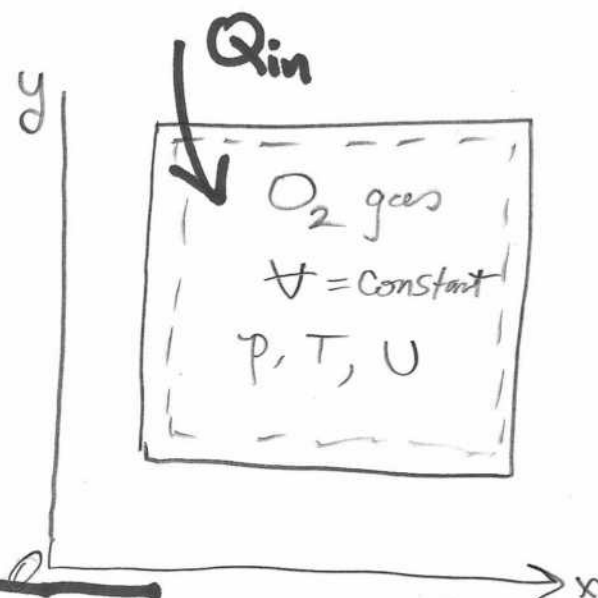
- Ideal gas behavior
- Quasi-equilibrium process
- $\Delta KE_{sys} = 0, \Delta PE_{sys} = 0$

Analysis

(a) The First Law Energy Balance, Closed System is

$$\Delta E_{sys} = E_{in} - E_{out} \quad (\text{kJ})$$

This can be expressed in more detail as



$$\Delta U_{sys} + \Delta KE_{sys} + \Delta PE_{sys} = Q_{Net,in} - W_{Net,out}$$

Expanding in even more detail,

⑧

$$\Delta U_{sys} + \Delta KE_{sys} + \Delta PE_{sys} = [Q_{in} - Q_{out}] - \left[W_{Net,out}^{pd\&} + W_{Net,out}^{other} \right]$$

moving
"pd&"
Boundary
Work

Work that
Crosses system
Boundary
(paddle wheel
electrical, etc)

And in even more detail:

$$\Delta U_{sys} + \Delta KE_{sys} + \Delta PE_{sys} = [Q_{in} - Q_{out}] - \left[(W_{out}^{pd\&} - W_{in}^{pd\&}) + (W_{out}^{other} - W_{in}^{other}) \right]$$

Now we simplify this Equation:

- $\Delta KE = 0$
- $\Delta PE = 0$
- $W_{out}^{other} = W_{in}^{other} = W_{out}^{pd\&} = W_{in}^{pd\&} = 0$
- $Q_{out} = 0$

$$\cancel{\Delta U_{sys}} + \cancel{\Delta KE_{sys}} + \cancel{\Delta PE_{sys}} = [\cancel{Q_{in}} - \cancel{Q_{out}}] - \left[(\cancel{W_{out}^{pd\&}} - \cancel{W_{in}^{pd\&}}) + (\cancel{W_{out}^{other}} - \cancel{W_{in}^{other}}) \right]$$

$$\boxed{\Delta U_{sys} = Q_{in}}$$

Q: How do we express this equation in terms of T_1 and T_2 ?

A: Specific Heat!

- Since this is a process at constant volume, we use the ^⑨ definition of specific heat at constant volume:

$$C_v(T, v) \equiv \left(\frac{\partial u}{\partial T} \right)_v$$

- For an Ideal gas, $C_v = C_v(T)$ only, so

$$C_v(T) = \left(\frac{du}{dT} \right)_v$$

- We can integrate this between states ① & ②:

$$\Delta u = \int_{T_1}^{T_2} C_v(T) dT$$

- Without any further information about $C_v(T)$ dependency on T , we can

- (1) Use the Ideal gas Table A-19 to get the exact values of

$$\int_{T_1}^{T_2} C_v(T) dT$$

or

- (2) Make an approximation using an average C_v : $\Delta u \approx C_{v, \text{avg}} \Delta T$

\Rightarrow Lets use $C_{v, \text{avg}}$: From Table A-2b, for O_2 ,

$$C_{v, \text{avg}} = 0.692 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \text{ at } T_{\text{avg}} = \frac{25 + 300}{2} = 162.5^\circ\text{C} (436\text{K})$$

\Rightarrow

$$\Delta U_{\text{sys}} = m \Delta u_{\text{sys}} = Q_{\text{in}}$$

$$m C_{v, \text{avg}} \Delta T = Q_{\text{in}}$$

substituting values,

$$Q_{in} = (1 \text{ kg}) \left(0.692 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right) (300 - 25^\circ\text{C})$$

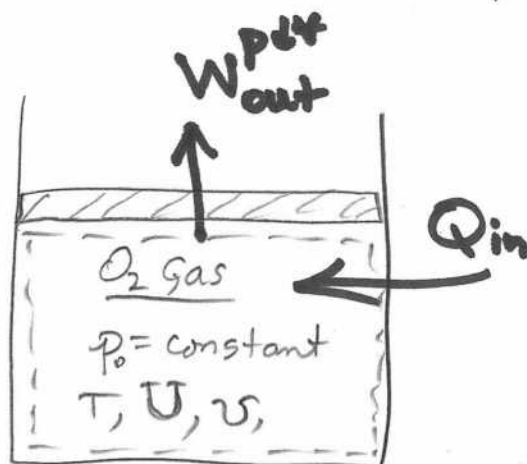
$$Q_{in} = 190.3 \text{ kJ}$$

Constant Volume process

(b) In this case, we add heat Q_{in} and the pressure must remain constant \Rightarrow the Volume must increase in order for $p_0 = \text{constant}$!

Closed System Energy Balance:

$$\Delta U_{sys} + \cancel{\Delta KE_{sys}} + \cancel{\Delta PE_{sys}} = [Q_{in} - \cancel{Q_{out}}] - [W_{Net, out}^{pd} + \cancel{W_{out}^{other}}]$$



$$\Delta U_{sys} = Q_{in} - W_{Net, out}^{pd}$$

$$\Delta U_{sys} = Q_{in} - [W_{out}^{pd} - \cancel{W_{in}^{pd}}]$$

$$\Delta U_{sys} = Q_{in} - W_{out}^{pd}$$

Now, From Lecture 14, page 8 for an Ideal gas undergoing a (polytropic, $n=0$) constant pressure process,

$$W_{out}^{pd} = p_0 (v_2 - v_1) \quad (\text{kJ})$$

$$W_{out}^{pd} = m p_0 (v_2 - v_1)$$

$$\boxed{W_{out}^{pd} = m p_0 v_1 - m p_0 v_2}$$

(11)

Substituting this into the Energy Balance Equation,

$$\Delta U_{sys} = Q_{in} - [m p_0 v_2 - m p_0 v_1]$$

$$U_{2,sys} - U_{1,sys} = Q_{in} - m p_0 v_2 + m p_0 v_1$$

re-arranging,

$$[U_{2,sys} + m p_0 v_2] - [U_{1,sys} + m p_0 v_1] = Q_{in}$$

$$m [u_{2,sys} + p_0 v_2] - m [u_{1,sys} + p_0 v_1] = Q_{in}$$

$$m h_2 - m h_1 = Q_{in}$$

$$m (h_2 - h_1) = Q_{in}$$

$$\boxed{m \Delta h = Q_{in}}$$

For a constant pressure process, we relate u and T using the specific heat at constant pressure:

$$C_p(T, v) \equiv \left(\frac{\partial h}{\partial T} \right)_p$$

For an Ideal gas, $C_p = C_p(T)$ so

$$C_p = \left(\frac{dh}{dT} \right)_p$$

$$dh = C_p dT$$

Integrating,

$$\Delta h = \int_{T_1}^{T_2} C_p(T) dT$$

(12)

Again, we will not use the Ideal gas tables to obtain the exact value of the integral, but instead use Table A-3b for O_2 to get $C_{p,avg}$:

$$C_{p,avg} = 0.952 \frac{kJ}{kg \cdot K} \text{ for } T_{avg} = \frac{300 + 25}{2} = 162.5^\circ C$$

$$\Delta h = C_{p,avg} \Delta T$$

Substituting this into the Energy Balance Equation,

$$m C_{p,avg} \Delta T = Q_{in}$$

Substituting values,

$$Q_{in} = (1 kg) (0.952 \frac{kJ}{kg \cdot K}) (300 - 25^\circ C)$$

$$Q_{in} = 261.8 kJ$$

Note: $Q_{in}^{\Delta V = \text{const}} = 190.3 kJ < Q^{\Delta P = \text{const}}$

This makes sense because more Energy (thermal) input was required to raise the piston, thus performing Work output (see Example 4, Lecture 14 part 2)

Example 2**Determining Heat Transfer for Ideal Gas Polytropic process**

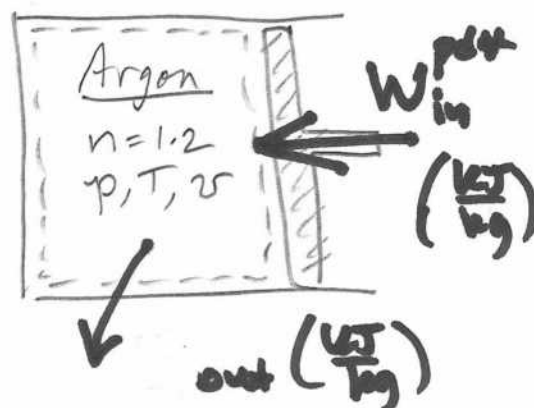
Argon is compressed in a polytropic process with $n=1.2$ from 120 kPa, 30°C to 1200 kPa in a piston-cylinder device.

Determine the Heat transferred, per-unit-mass, during this process

Given: $n=1.2$, general polytropic process
 $pV^n = \text{constant}$

State ① $\begin{cases} p_1 = 120 \text{ kPa} \\ T_1 = 30^\circ\text{C} = 303 \text{ K} \end{cases}$

State ② $\begin{cases} p_2 = 1200 \text{ kPa} \end{cases}$



Assume: (1) Quasi-equilibrium process (3) $\Delta KE_{\text{sys}} = 0$
 (2) Ideal gas behavior $\Delta PE_{\text{sys}} = 0$


Find: q_{in} (or it could be out, we don't know)

Analysis: The 1st Law of Thermodynamics for a Closed System is

$$\Delta E_{\text{sys}} = E_{\text{in}} - E_{\text{out}} \quad (\text{kJ})$$

Expanding into more detail, and simplifying as we go,

$$\Delta U_{\text{sys}} + \cancel{\Delta KE_{\text{sys}}} + \cancel{\Delta PE_{\text{sys}}} = Q_{\text{net, in}} - [W_{\text{net, out}}^{\text{pdv}} + W_{\text{net, out}}^{\text{other}}]$$


$$\Delta U_{sys} = Q_{net,in} - [W_{out}^{pd} - W_{in}^{pd}] = Q_{net,in} + W_{in}^{pd}$$

solving for the Net Heat transfer

$$Q_{net,in} = \Delta U_{sys} - W_{in}^{pd}$$

dividing by the mass of the system,

$$Q_{net,in} = \Delta u_{sys} - w_{in}^{pd} \quad \left(\frac{kJ}{kg} \right)$$

Note that since we don't know if the Heat Transfer is "in" or "out", then I have kept it in general form (Net) and at the end the sign will tell me what the direction is.

(a) Find an expression for w_{out}^{pd}

From Lecture 14, page 7 for an Ideal gas with $n \neq 1$,

$$w_{in}^{pd} = \frac{-mR}{(1-n)} [T_2 - T_1]$$

We don't know T_2 !!

\Rightarrow Use the polytropic expressions relating T, P, v given on page 10, Lecture 14:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \quad (\text{Since we know } P_1 \text{ \& } P_2)$$

Thus, $W_{in}^{pdv} = \frac{-mRT_1}{(1-n)} \left[\frac{T_2}{T_1} - 1 \right]$

Substituting for T_2/T_1

$$W_{in}^{pdv} = \frac{-mRT_1}{(1-n)} \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

dividing by the mass, m ,

$$w_{in}^{pdv} = \frac{-RT_1}{(1-n)} \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

you will see
this type of
Equation in
supersonic flows
⇒ Fluid Mech.

Substituting in known values ($R = 0.2081 \frac{\text{kJ}}{\text{kg-K}}$, Table A-1)
argon

$$w_{in}^{pdv} = \frac{(0.2081 \frac{\text{kJ}}{\text{kg-K}})(303\text{K})}{(1-1.2)} \left[\left(\frac{1200\text{kPa}}{120\text{kPa}} \right)^{\frac{1.2-1}{1.2}} - 1 \right]$$

$$w_{in}^{pdv} = -(-315.271 \frac{\text{kJ}}{\text{kg}}) \left((10)^{0.1666...} - 1 \right)$$

$$w_{in}^{pdv} = (315.271 \frac{\text{kJ}}{\text{kg}}) (1.46779 - 1)$$

$$w_{in}^{pdv} = 147.5 \frac{\text{kJ}}{\text{kg}}$$

(b) Express ΔU_{sys} in terms of ΔT

Even though neither Volume or pressure is constant,
we have ΔU_{sys} in the energy balance equation, so

$$\Delta U_{sys} = \int_{T_1}^{T_2} C_v(T) dT \text{ for an Ideal gas.}$$

- Since C_v is a Property (Intensive) of the System, it is independent of path, so it is ok that we use it in this situation where $\forall \neq \text{const}$, $p \neq \text{const}$.

(Note that if we had Δh_{sys} in the energy balance equation, then we would use $\Delta h_{\text{sys}} = C_p \Delta T$ instead.)

- Using an average value of C_v , $\Delta U_{\text{sys}} = C_{v,\text{avg}} \Delta T$ so

$$q_{\text{net,in}} = C_v(T_2 - T_1) - 147.5 \frac{\text{kJ}}{\text{kg}}$$

- We need to find $T_2 \Rightarrow$ Use the polytropic relations !!

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}}$$

$$T_2 = T_1 \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} = (303\text{K}) \left(\frac{1200\text{kPa}}{120\text{kPa}} \right)^{\frac{1.2-1}{1.2}}$$

$$T_2 = 444.7\text{K}$$

Substituting into the Energy Balance Equation, using

$$C_{v,\text{avg}} = 0.3122 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \text{ (Table A-2a) ,}$$

$$q_{\text{net,in}} = (0.3122 \frac{\text{kJ}}{\text{kg} \cdot \text{K}})(444.7\text{K} - 303\text{K}) - 147.5 \frac{\text{kJ}}{\text{kg}}$$

$$q_{\text{net,in}} = 44.2 \frac{\text{kJ}}{\text{kg}} - 147.5 \frac{\text{kJ}}{\text{kg}} = -103.3 \frac{\text{kJ}}{\text{kg}} = (q_{\text{in}} - q_{\text{out}})$$

$$\Rightarrow q_{\text{out}} = 103.3 \frac{\text{kJ}}{\text{kg}}$$

by sign convention

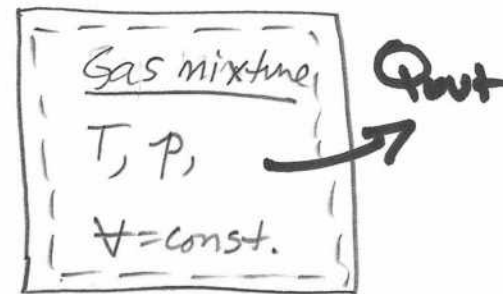
Example 3 Heat transfer from Rigid Tank filled with gas mixture

A rigid tank contains a gas mixture with a specific heat $C_v = 0.748 \frac{\text{kJ}}{\text{kg-K}}$. The mixture is cooled from 200 kPa and 200°C until its pressure is 100 kPa. Determine the heat transfer during this process in $\frac{\text{kJ}}{\text{kg}}$.

Given: State ① $\begin{cases} P_1 = 200 \text{ kPa} \\ T_1 = 200^\circ\text{C} = 473 \text{ K} \end{cases}$

State ② $\begin{cases} P_2 = 100 \text{ kPa} \end{cases}$

$$C_v = 0.748 \frac{\text{kJ}}{\text{kg-K}}$$



Assume: (1) Ideal Gas behavior
(2) $\Delta PE_{\text{sys}} = 0$, $\Delta KE_{\text{sys}} = 0$

Find: $q_{\text{out}} \left(\frac{\text{kJ}}{\text{kg}} \right)$

Analysis: The 1st Law Energy Balance for a Closed System is

$$\Delta E_{\text{sys}} = E_{\text{in}} - E_{\text{out}}$$

$$\Delta U_{\text{sys}} + \cancel{\Delta KE_{\text{sys}}} + \cancel{\Delta PE_{\text{sys}}} = (\cancel{Q_{\text{in}}} - Q_{\text{out}}) - (\cancel{W_{\text{pdy, net, out}}} + \cancel{W_{\text{other, net, out}}})$$

$$\Delta U_{\text{sys}} = -Q_{\text{out}}$$

Dividing by the (unknown) mass:

$$\Delta U_{\text{sys}} = -q_{\text{out}}$$

Solving for q_{out} and using $\Delta U = C_v \Delta T$ for an Ideal Gas,

$$q_{\text{out}} = -\Delta U_{\text{sys}} = -C_v(T_2 - T_1)$$

$$\boxed{q_{\text{out}} = C_v(T_1 - T_2)} \quad \left(\frac{\text{kJ}}{\text{kg}}\right)$$

• We need to find T_2

Use the Ideal Gas EOS for $V = \text{const}$!

$$pV = mRT \Rightarrow V = \frac{mRT}{p}$$

$$V_2 = V_1$$

$$\frac{mRT_2}{p_2} = \frac{mRT_1}{p_1}$$

$$\frac{T_2}{p_2} = \frac{T_1}{p_1} \Rightarrow \boxed{T_2 = T_1 \left(\frac{p_2}{p_1} \right)}$$

$$T_2 = (200^\circ\text{C} + 273^\circ\text{C}) \left(\frac{100 \text{ kPa}}{200 \text{ kPa}} \right) = (473 \text{ K}) \left(\frac{1}{2} \right)$$

$$\boxed{T_2 = 236.5 \text{ K}}$$

$$\Rightarrow q_{\text{out}} = \left(0.718 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right) (473 \text{ K} - 236.5)$$

$$\boxed{q_{\text{out}} \approx 177 \frac{\text{kJ}}{\text{kg}}}$$

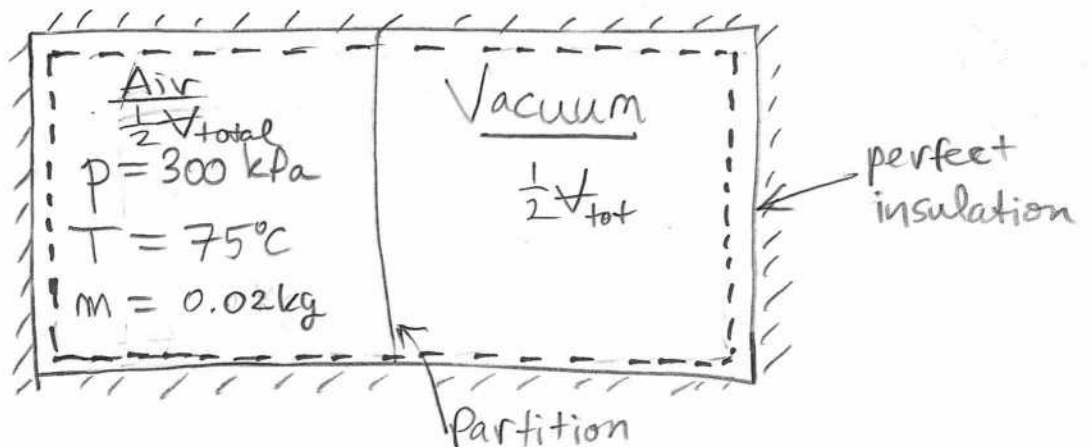
Lecture 15 Part 2 | Examples, cont.

①

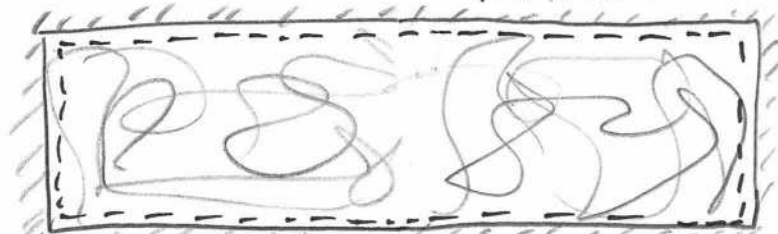
Example 4 A perfectly insulated rigid tank has its Volume divided into two equal parts. One half of the tank is filled with 0.02 kg of air at 300 kPa, 75°C while the other half is a perfect vacuum. The total Volume of the tank is 34 L. Assume the air behaves Ideally. At $t_i = 0$, the partition is removed and the air expands unrestrained (Non-equilibrium process) to fill the entire volume. A "long time later" the air achieves total thermodynamic equilibrium within the entire tank. Determine (a) U_2 , (b) T_2 , (c) P_2 . Note: This is one of Joules' famous experiments.

Given: 0.02 kg of air inside $\frac{1}{2}$ of a 30 L perfectly insulated tank at 300 kPa, 75°C.

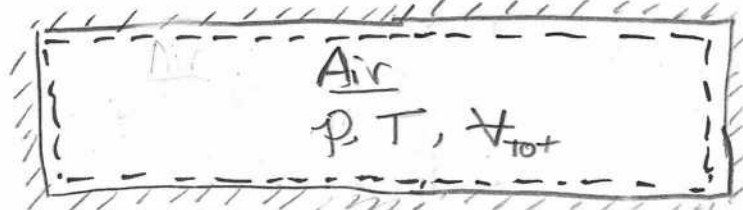
$t = 0$
State ①



$t > 0$
nonequilibrium
condition



"long time later"
 $t \gg 0$
State ②



Find : (a) U_2
 (b) T_2
 (c) P_2

Assume : (1) Air always behaves as an Ideal gas
 (2) Adiabatic process
 (3) $\Delta KE_{sys} = 0$, $\Delta PE_{sys} = 0$

Analysis

(a) Take the Entire Tank volume as the system, then

$$\Delta E_{sys} = E_{in} - E_{out}$$

$$\Delta U_{sys} + \cancel{\Delta KE_{sys}} + \cancel{\Delta PE_{sys}} = \cancel{Q_{net,in}} - \cancel{W_{net,out}}$$

\downarrow \downarrow \downarrow \downarrow
 0 0 Adiabatic process No work performed on or by the tank during the process

$$\Delta U_{sys} = 0$$

$$U_2 = U_1$$

This is very general, independent of gas type, "Real" or "Ideal"... in fact, it is independent of phase, also!

Using table A-17, at $T = 75^\circ\text{C} + 273 = 348\text{K}$,

$$U_1 = 248.58 \text{ kJ (by interpolation)}$$

so $U_2 = U_1 = 248.58 \text{ kJ}$ (Air, assumed to be an Ideal Gas)

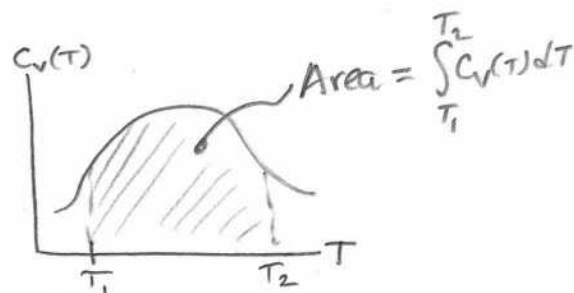
(b). The Internal Energy change of an Ideal Gas is

$$U_2 - U_1 = \int_{T_1}^{T_2} C_V(T) dT$$

regardless of the process (nonequilibrium, quasiequilibrium, constant pressure, constant volume, etc.)

- Since $U_1 = U_2$, then

$$\int_{T_1}^{T_2} C_V(T) dT = 0$$



- We know that $C_V(T) \neq 0$ between T_1 and T_2 , so the integral equation above implies that

$$\boxed{T_2 = T_1} \quad \left(\begin{array}{l} \text{Ideal} \\ \text{Gas} \end{array} \right)$$

This is a general result, independent of the gas type. However, it is restricted to an Ideal Gas.

- For this problem,

$$\boxed{T_2 = T_1 = 75^\circ\text{C}}$$

(c) We know that $V_{\text{gas}_1} = \frac{1}{2} V_{\text{total}}$ and $V_{\text{gas}_2} = V_{\text{total}}$

From the Ideal Gas EOS,

$$p_1 = \frac{mRT_1}{V_{\text{gas}_1}} = \frac{mRT_1}{\frac{1}{2} V_{\text{tot}}} \Rightarrow$$

$$\boxed{V_{\text{tot}} = \frac{2mRT_1}{p_1}}$$

But $p_2 = \frac{mRT_2}{V_{\text{gas}_2}} = \frac{mRT_2}{V_{\text{tot}}} \Rightarrow$

$$\boxed{V_{\text{tot}} = \frac{mRT_2}{p_2}}$$

(4)

Equating these two expressions,

$$V_{\text{tot}} = \frac{2mRT_1}{P_1} = \frac{mRT_2}{P_2}$$

we have

$$\frac{2T_1}{P_1} = \frac{T_2}{P_2}$$

$$P_2 = \frac{1}{2} P_1 \left(\frac{T_2}{T_1} \right)$$

In part (b) we found that $T_1 = T_2$ for an Ideal Gas,

so

$$\boxed{P_2 = \frac{1}{2} P_1} \quad \left(\begin{array}{l} \text{Ideal} \\ \text{Gas} \end{array} \right)$$

Again this is independent of gas composition, but is restricted to behaving as Ideal.

- In general for the tank being partitioned such that the gas occupies fraction f of the total Volume, then you can show that

$$\boxed{P_2 = f P_1}$$

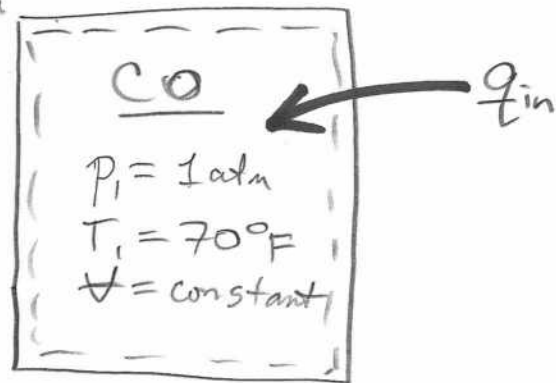
- In this problem,

$$\boxed{P_2 = 150 \text{ kPa}}$$

Example 5

A closed, rigid tank is filled with carbon monoxide at 70°F, 1 atm. Heat is transferred into the tank and the pressure increases to 3 atm. What is the amount of heat transferred in $\frac{\text{Btu}}{\text{lbm}}$? The specific heats are not to be considered constant, but the CO gas behaves ideally.

Given: State ① $\left\{ \begin{array}{l} P_1 = 1 \text{ atm} = 14.696 \text{ psia} \\ T_1 = 70^\circ\text{F} \end{array} \right.$
 State ② $\left\{ \begin{array}{l} P_2 = 3 \text{ atm} = 44.088 \text{ psia} \end{array} \right.$



Assume: (1) Ideal gas EOS.
 (2) $\Delta KE_{\text{sys}} = 0$, $\Delta PE_{\text{sys}} = 0$

Find: $q_{\text{in}} \left(\frac{\text{Btu}}{\text{lbm}} \right)$

Analysis The Closed System Energy Balance in "Change form" is

$$\Delta E_{\text{sys}} = E_{\text{in}} - E_{\text{out}}$$

$$\Delta U_{\text{sys}} + \cancel{\Delta KE_{\text{sys}}} + \cancel{\Delta PE_{\text{sys}}} = \cancel{Q_{\text{net, in}}} - \cancel{W_{\text{net, out}}}$$

$$\Delta U_{\text{sys}} = (Q_{\text{in}} - \cancel{Q_{\text{out}}})$$

$$\Delta U_{\text{sys}} = Q_{\text{in}}$$

per unit mass basis: $u_2 - u_1 = q_{\text{in}}$

(6)

- We now need to find $u_1(T_1)$ and $u_2(T_2)$. We know that

$$u_2(T_2) - u_2(T_1) = \int_{T_1=70^\circ\text{F}}^{T_2=?} C_v(T) dT$$

where we are told to use the Ideal gas table A-21E for CO.

⇒ We need to find T_2

⇒ Use the fact that $V = \text{constant}$ and the gas behaves ideally:

$$p_1 V_1 = m R T_1 \quad \text{and} \quad p_2 V_2 = m R T_2$$

so that

$$\boxed{V_1 = V_2}$$

Key Concept

$$\frac{m R T_1}{p_1} = \frac{m R T_2}{p_2}$$

$$T_2 = T_1 \left(\frac{p_2}{p_1} \right) = (70^\circ\text{F} + 459.67) \left(\frac{3 \text{ atm}}{1 \text{ atm}} \right)$$

$$T_2 = (529.67 \text{ R})(3)$$

$$\boxed{T_2 = 1589 \text{ R}}$$

- Now we can find $\int_{T_1}^{T_2} C_v(T) dT$ using Table A-21E:

$$@ 70^\circ\text{F} \approx 530 \text{ R} \Rightarrow \boxed{\bar{u}_1 = 2628.3 \frac{\text{Btu}}{\text{lbmol}}}$$

$$@ 1589 \text{ R} \Rightarrow \boxed{\bar{u}_2 = 8242.7 \frac{\text{Btu}}{\text{lbmol}}}$$

[using linear interpolation]

(7)

- We need to convert $\frac{\text{Btu}}{\text{lbmol}}$ into $\frac{\text{Btu}}{\text{lb}_m}$ using the Molecular masses given in Table A-1E:

$$M_{\text{CO}} = 28 \frac{\text{lbmol}}{\text{lbmass}}$$

$$\Rightarrow U_1 = 2628.3 \frac{\text{Btu}}{\text{lbmol}} \times \frac{1}{28 \frac{\text{lbmol}}{\text{lb}_m}} \Rightarrow U_1 = 93.87 \frac{\text{Btu}}{\text{lb}_m}$$

$$U_2 = 8242.7 \frac{\text{Btu}}{\text{lbmol}} \times \frac{1}{28 \frac{\text{lbmol}}{\text{lb}_m}} \Rightarrow U_2 = 294.38 \frac{\text{Btu}}{\text{lb}_m}$$

Thus, from the Energy Balance,

$$q_{\text{in}} = U_2 - U_1$$

$$q_{\text{in}} = 294.38 \frac{\text{Btu}}{\text{lb}_m} - 93.87 \frac{\text{Btu}}{\text{lb}_m}$$

$$q_{\text{in}} = 200.51 \frac{\text{Btu}}{\text{lb}_m}$$

Example 6

A 30 cm diameter piston-cylinder device with stops

Initially contains 0.3 kg of steam at 1.0 MPa and 400°C.

The location of the stops corresponds to 60% of the initial volume. Now the steam is cooled. Determine the compression work and heat transferred if the final state is:

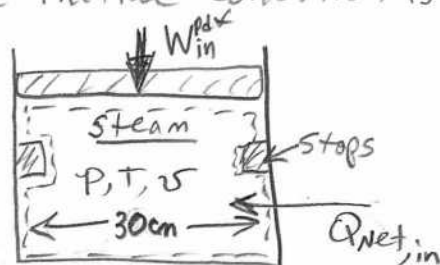
(a) 1.0 MPa, 250°C, find height of piston above the stops.

(b) 500 kPa and knowing the piston rests upon the stops.

Given: For part (a), (b), and (c) the initial condition is

State ①
superheated H_2O

$$\begin{cases} m = 0.3 \text{ kg steam} \\ P_1 = 1.0 \text{ MPa} \\ T_1 = 400^\circ\text{C} \\ V_{\text{stop}} = 0.6 V_1 \end{cases}$$



Assume: • quasi equilibrium process

• $\Delta KE_{\text{sys}} = 0$, $\Delta PE_{\text{sys}} = 0$

Note: (We cannot assume Ideal gas behavior. This is a non-Ideal gas - see pg. 137

Find: (a) $W_{\text{in}}^{pd} \cancel{\cancel{}}$ and $Q_{\text{net,in}}$ @ state ② $\left\{ \begin{array}{l} P_2 = 1.0 \text{ MPa} \\ T_2 = 250^\circ\text{C} \end{array} \right.$

(b) $W_{\text{in}}^{pd} \cancel{\cancel{}}$, $Q_{\text{net,in}}$, and T_2 @ state ② $\left\{ \begin{array}{l} P_2 = 500 \text{ kPa} \end{array} \right.$

Fig. 3-47 of text!

Analysis

(a) The Closed System Energy Balance in "Change Form" is

$$\Delta E_{\text{sys}} = E_{\text{in}} - E_{\text{out}}$$

$$\Delta U_{\text{sys}} + \cancel{\Delta KE_{\text{sys}}} + \cancel{\Delta PE_{\text{sys}}} = Q_{\text{net,in}} - [W_{\text{net,out}}^{pd} \cancel{\cancel{}} + W_{\text{net,out}}^{\text{other}} \cancel{\cancel{}}]$$

$$\Delta U_{\text{sys}} = Q_{\text{net},in} - (W_{\text{pd},out} - W_{\text{pd},in})$$

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$$\Delta U_{\text{sys}} = Q_{\text{net},in} + W_{\text{pd},in}$$

$$Q_{\text{net},in} = \Delta U_{\text{sys}} - W_{\text{pd},in}$$

**Closed System
Energy Balance**

The steam under these conditions behaves as a "Real" gas, so you cannot use the Ideal gas expression for W_{pd} given in Lecture 14 Part I, pg. 6 or the polytropic relations on pages 9 & 10.

⇒ You must use the general expression for a gas (or liquid) given on page 6 of Lecture 14, Part I.

Q: What is n ?

A: In this part, $p = \text{constant} \Rightarrow p V^n = \text{const}$
 $\Rightarrow \boxed{n=0}$

$$\Rightarrow W_{\text{pd},in} = \frac{-1}{(1-n)} [p_2 V_2 - p_1 V_1] \quad (\text{kJ})$$

$$W_{\text{pd},in} = \frac{-1}{(1-0)} p_0 [V_2 - V_1]$$

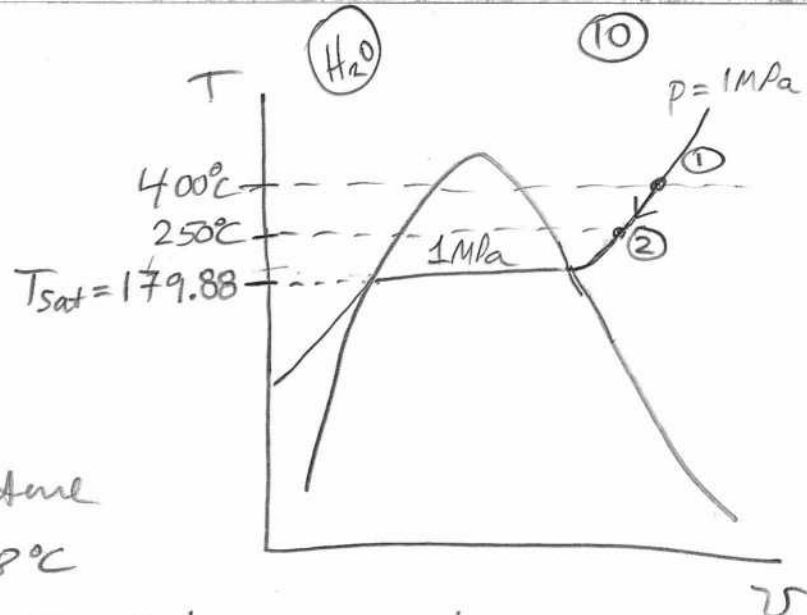
$$W_{\text{pd},in} = p_0 [V_1 - V_2]$$

Cast in terms of specific volume so we can use the Tables, $\boxed{W_{\text{pd},in} = m p_0 [v_1 - v_2]} \quad (\text{kJ})$

Q: What tables do we use? What is the phase of State ②? (State ① is given as a super-heated vapor)

A:

The phase of State ② is determined by using the given info: $P_2 = 1 \text{ MPa}$, $T_2 = 250^\circ\text{C}$. We first find



the saturated vapor temperature for $P_{\text{sat}} = 1 \text{ MPa} \Rightarrow T_{\text{sat}} = 179.88^\circ\text{C}$

from Table A-5. From the T - v diagram above, we see that State ② must be a superheated vapor, but still a "Real" gas.

\Rightarrow Use Table A-6 for State ②

From Table A-6, for both states,

$$\text{State ①} \begin{cases} P_1 = 1 \text{ MPa} \\ T_1 = 400^\circ\text{C} \end{cases} \left\{ v_1 = 0.30661 \frac{\text{m}^3}{\text{kg}} \right.$$

$$\text{State ②} \begin{cases} P_2 = 1 \text{ MPa} \\ T_2 = 250^\circ\text{C} \end{cases} \left\{ v_2 = 0.23275 \frac{\text{m}^3}{\text{kg}} \right.$$

$$\text{So, } W_{\text{in}}^{\text{pdv}} = (0.3 \text{ kg})(1000 \text{ kPa})(0.30661 \frac{\text{m}^3}{\text{kg}} - 0.23275 \frac{\text{m}^3}{\text{kg}})$$

$$W_{\text{in}}^{\text{pdv}} = 22.16 \text{ kJ}$$

$$\Delta U_{\text{sys}} = m \Delta u_{\text{sys}} = m(u_2 - u_1)$$

$$\text{From Table A-6, } \begin{aligned} \text{state ①} &\Rightarrow u_1 = 2957.9 \frac{\text{kJ}}{\text{kg}} \\ \text{state ②} &\Rightarrow u_2 = 2710.4 \frac{\text{kJ}}{\text{kg}} \end{aligned}$$

(continued on next page)

So $\Delta U_{\text{sys}} = (0.3 \text{ kg}) (2710.4 - 2957.9 \frac{\text{kJ}}{\text{kg}})$

(11)

$$\Delta U_{\text{sys}} = -74.25 \text{ kJ}$$

Using the Energy Balance expression,

$$Q_{\text{net, in}} = -74.25 \text{ kJ} - 22.16 \text{ kJ}$$

$$Q_{\text{net, in}} = -96.41 \text{ kJ}$$

$$(Q_{\text{in}} - Q_{\text{out}}) = -96.41 \text{ kJ} \Rightarrow$$

$$Q_{\text{out}} = 96.41 \text{ kJ}$$

must be zero
according to our
sign convention!

Height of Piston above the Stops :

The ratio of volumes is $\frac{V_2}{V_1} = \frac{v_2}{v_1} = \frac{0.23275 \text{ m}^3/\text{kg}}{0.30661 \text{ m}^3/\text{kg}}$

or $\frac{V_2}{V_1} = 0.7591 \Rightarrow V_2 \approx 76\%$ the height of V_1 , so
the piston does not hit the
stop.

The initial volume is $V_1 = m v_1$ so that the height,
 y_1 is $V_1 = \frac{\pi D_1^2}{4} y_1 = m v_1$

$$y_1 = \frac{4 m v_1}{\pi D^2}$$

$$y_1 = \frac{(4)(0.3 \text{ kg})(0.30661 \text{ m}^3/\text{kg})}{\pi (30 \text{ cm} \times \frac{1 \text{ m}}{100 \text{ cm}})^2} \Rightarrow$$

$$y_1 = 1.301 \text{ m}$$

$$y_2 = (0.7591) y_1 = (0.7591)(1.301 \text{ m}) \Rightarrow$$

$$y_2 = 0.987 \text{ m}$$

(b) State ② $\{ p_2 = 500 \text{ kPa} \}$

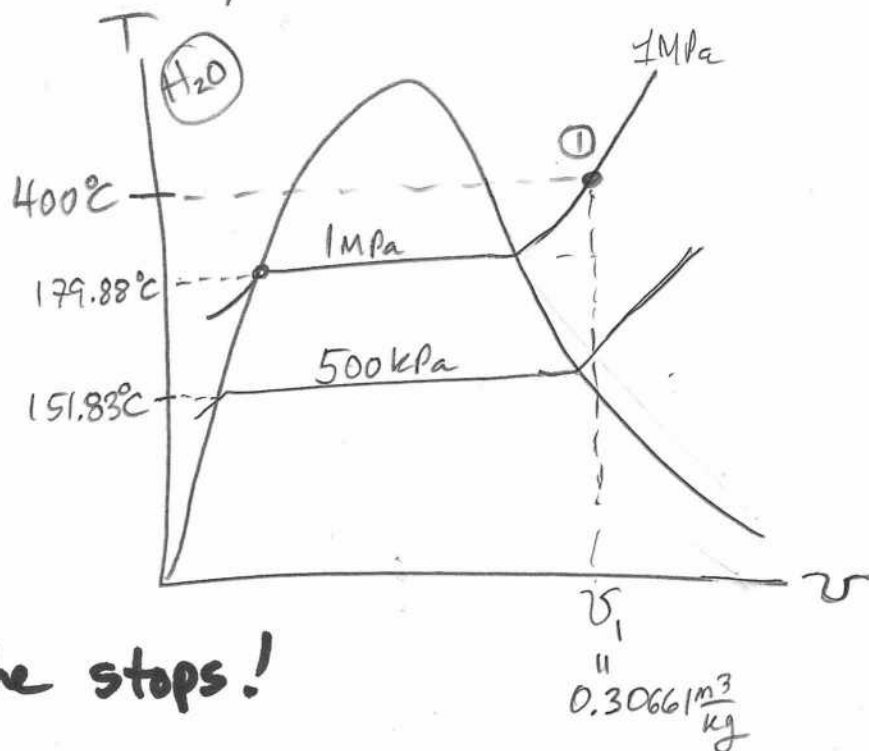
(12)

Q: What is the phase of State ②?

A: Re-draw the T-v diagram:

Since $p_2 = 500 \text{ kPa}$, then
a line of constant
pressure must be
below that of 1 MPa .

Since we don't know
the temperature, we must
use other given info



\Rightarrow piston rests on the stops!

In this case, $v_{\text{stop}} = 0.6 v_1$

$$\Rightarrow v_{\text{stop}} = 0.6 v_1 = (0.6)(0.30661 \frac{\text{m}^3}{\text{kg}})$$

$$v_{\text{stop}} = 0.183966 \frac{\text{m}^3}{\text{kg}}$$

Now we must ask, on which table is there a
unique value of v_{stop} in which $p = 500 \text{ kPa}$?

Table A-6: For 0.5 MPa , the lowest value of v is
 $0.37483 \frac{\text{m}^3}{\text{kg}}$ at $T_{\text{sat}} = 151.83^\circ\text{C}$, so the phase
cannot be superheated vapor!

Table A-5: For 500 kPa , $v_f < v_{\text{stop}} < v_g$

\Rightarrow phase is a saturated mixture!

The work input is computed from

$$W_{in}^{pdv} = -\frac{1}{(1-n)} [p_2 v_2 - p_1 v_1] = -\frac{n}{(1-n)} [p v_2 - p v_1]$$

But what is n !?

Go Back to $p v^n = \text{constant}$!

$$p_1 v_1^n = p_2 v_2^n$$

$$\left(\frac{v_1}{v_2}\right)^n = \frac{p_2}{p_1}$$

$$n \ln\left(\frac{v_1}{v_2}\right) = \ln\left(\frac{p_2}{p_1}\right)$$

$$n = \frac{\ln(p_2/p_1)}{\ln(v_1/v_2)}$$

We know that $\frac{v_1}{v_2} = \frac{v_1}{v_{\text{stop}}} = \frac{1}{0.6} = 1.66\bar{6}$

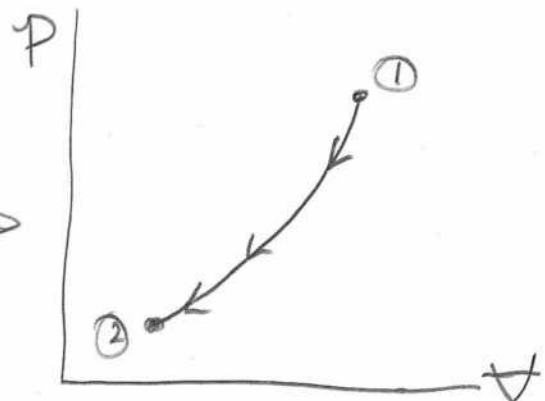
So

$$n = \frac{\ln\left[\frac{500 \text{ kPa}}{1000 \text{ kPa}}\right]}{\ln[1.666\bar{6}]} = \frac{\ln[1/2]}{\ln[1.666\bar{6}]} = \frac{-0.693147\ldots}{0.5108256\ldots}$$

$$n = -1.3569$$

$$p v^{1.3569} = \text{const}$$

$$p \propto v^{1.3569}$$



Substituting in values,

$$W_{in}^{pdv} = \frac{-1 (0.3 \text{ kg})}{[1 - (-1.3569)]} \left[(500 \text{ kPa}) \left(0.183966 \frac{\text{m}^3}{\text{kg}} \right) - (1000 \text{ kPa}) \left(0.30661 \frac{\text{m}^3}{\text{kg}} \right) \right]$$

$$W_{in}^{pdv} = \left(\frac{-0.3 \text{ kg}}{2.3569} \right) (91.983 \text{ kJ} - 306.61 \text{ kJ})$$

$$W_{in}^{pdv} = 27.32 \text{ kJ}$$

Now, $\Delta U_{sys} = m \Delta u_{sys} = m(u_2 - u_1)_{sys}$

From Table A-6, $u_1 = 2957.9 \frac{\text{kJ}}{\text{kg}}$

To find u_2 , we need to find x_2 using v_{stop} :

$$x_2 = \frac{v_{stop} - v_f}{v_g - v_f} = \frac{0.183966 \frac{\text{m}^3}{\text{kg}} - 0.001093 \frac{\text{m}^3}{\text{kg}}}{0.37483 \frac{\text{m}^3}{\text{kg}} - 0.001093 \frac{\text{m}^3}{\text{kg}}}$$

$$x_2 = 0.4893 \text{ (48.9\%)}$$

Thus,

$$u_2 = u_f + x u_{fg}$$

$$u_2 = 639.54 \frac{\text{kJ}}{\text{kg}} + (0.4893)(1921.2 \frac{\text{kJ}}{\text{kg}})$$

$$u_2 = 1579.60 \frac{\text{kJ}}{\text{kg}}$$

$$\Rightarrow \Delta U_{sys} = (0.3 \text{ kg}) \left(1579.60 \frac{\text{kJ}}{\text{kg}} - 2957.9 \frac{\text{kJ}}{\text{kg}} \right)$$

$$\Delta U_{sys} = -413.49 \text{ kJ}$$

(15)

substituting ΔU_{sys} and $W_{\text{in}}^{\text{pdx}}$ into the Energy Balance Eqn,

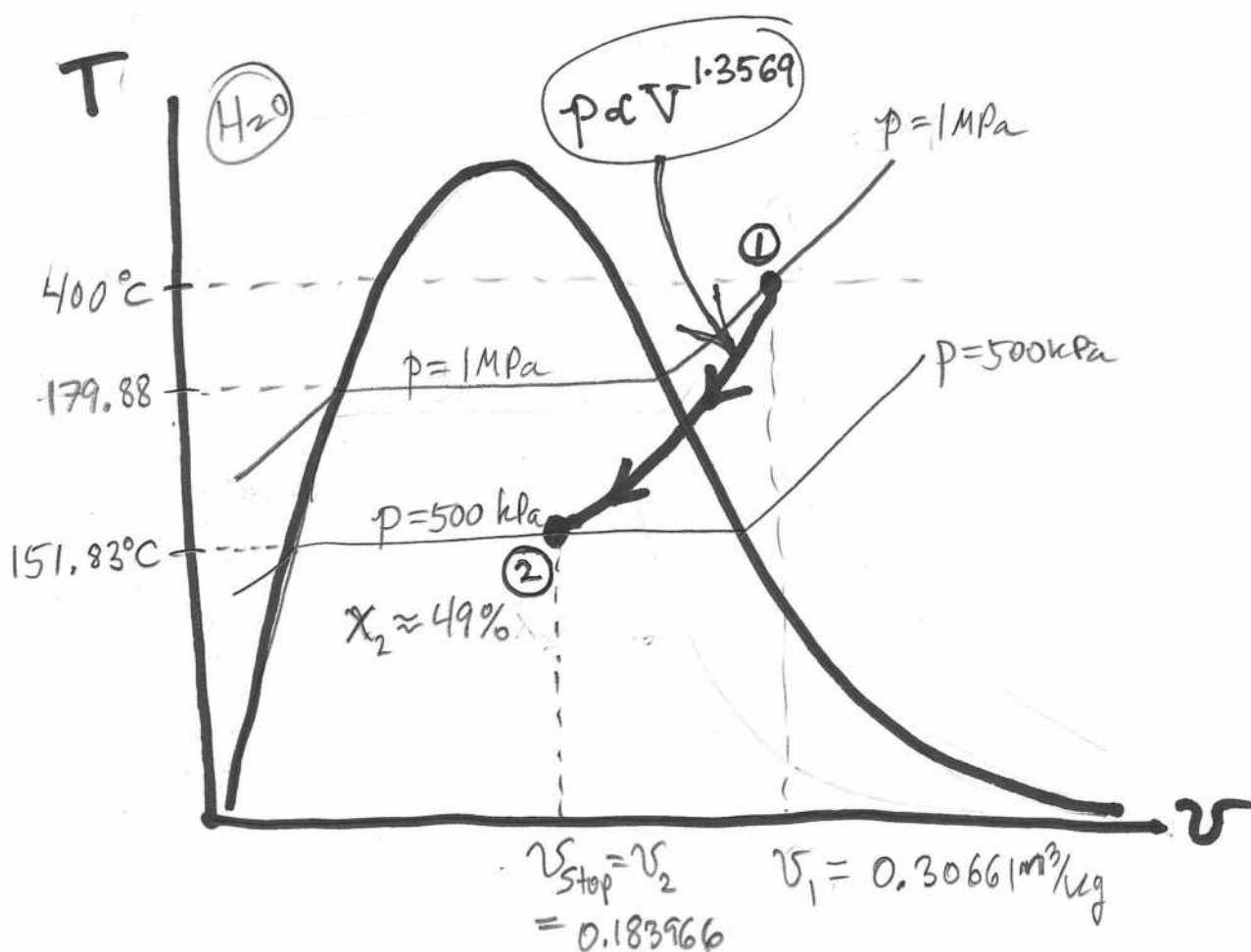
$$Q_{\text{net},\text{in}} = -413.49 \text{ kJ} - 27.32 \text{ kJ}$$

$$Q_{\text{net},\text{in}} \equiv \underset{\substack{\uparrow \\ 0}}{Q_{\text{in}}} - Q_{\text{out}} = -440.81 \text{ kJ}$$

$$\Rightarrow \boxed{Q_{\text{out}} = 440.81 \text{ kJ}}$$

• T_2 . The temperature for T_2 must be the saturation temperature for $p_{\text{sat}} = 500 \text{ kPa}$ since the state is a saturated mixture

$$\Rightarrow \boxed{T_2 = 151.83^\circ\text{C}}$$



Summary of Results

Case	n	W_{in}^{pd+} (kJ)	Q_{out} (kJ)	T_2 (°C)	$\frac{V_2}{V_1}$	Phases
A	$p = \text{const}$ 0	22.16	96.41	250	75%	Steam \rightarrow Steam
B	$p \propto V^{1.3569}$ -1.3569	27.32	440.81	152	60%	Steam \rightarrow Saturated Mixture

Notice
the Work
input is
about the
same

Notice
the Larger (over 4x)
amount of Thermal Energy
that must be rejected
from the System in order
to accomplish Case b process!

Q: Why?

A: You have a phase change during this process so the latent heat must be rejected \rightarrow H_2O has the largest latent heat of vaporization of all known simple substances!

Example 7 An aluminum sphere having a mass of 10 kg and inner diameter of 1.5 m (wall thickness ~ 1 mm) is filled with air at 1 atm and is at 21°C , at a height above the Earth's surface of 4,830 m (~ 3 miles). The sphere is dropped and allowed to free-fall. At impact on the ground, sensors indicate the velocity was constant at 50 m/s (terminal velocity) and the air inside was 0°C (Note that at 4800 m, from Table A-16, the average outside air temperature is -16°C !). Assuming that $g = 9.81 \text{ m/s}^2$ is constant and there is negligible heating of the sphere due to air friction, Determine (a) the Heat transferred (magnitude & direction) into the sphere and (b) the Work done by the sphere (i.e., Energy lost) due to aerodynamic drag during free-fall

Given:

State ①

$$\begin{cases} T_1 = 21^\circ\text{C} = 294 \text{ K} \\ P_1 = 101.325 \text{ kPa} \\ y_1 = 4830 \text{ m} \\ V_1 = 0 \text{ m/s} \end{cases}$$

State ②

$$\begin{cases} T_2 = 0^\circ\text{C} = 273 \text{ K} \\ y_2 = 0 \text{ m} \\ V_2 = 50 \text{ m/s} \end{cases}$$

Find (a) $Q_{\text{net}, \text{in}}$ (kJ)

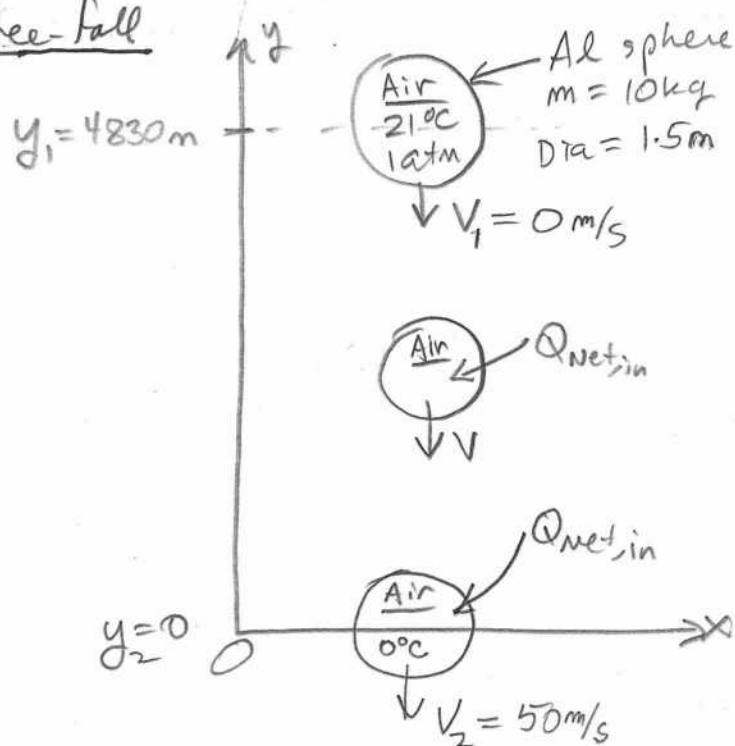
(b) W_{drag} (kJ)

Assume: (1) $g = 9.81 \text{ m/s}^2 = \text{const}$

(2) Air behaves as an Ideal gas

(3) No lifeforms inside to store thermal energy

(4) negligible heating of sphere due to aerodynamic friction



Analysis

(a) The heat transferred into the sphere is due to the local temperature difference between the air inside the sphere (and aluminum shell) and the atmospheric air in the immediate surroundings as it is falling. While the rate of heat transfer is certainly varying with time, the total heat transferred (kJ) is a constant for the process path specified in the problem statement.

⇒ This implies that we can determine $Q_{net,in}$ by choosing a reference frame attached to the sphere ----- as we learned in earlier lectures, the Internal Energy U_{sys} can be assumed independent of reference frame for this class.

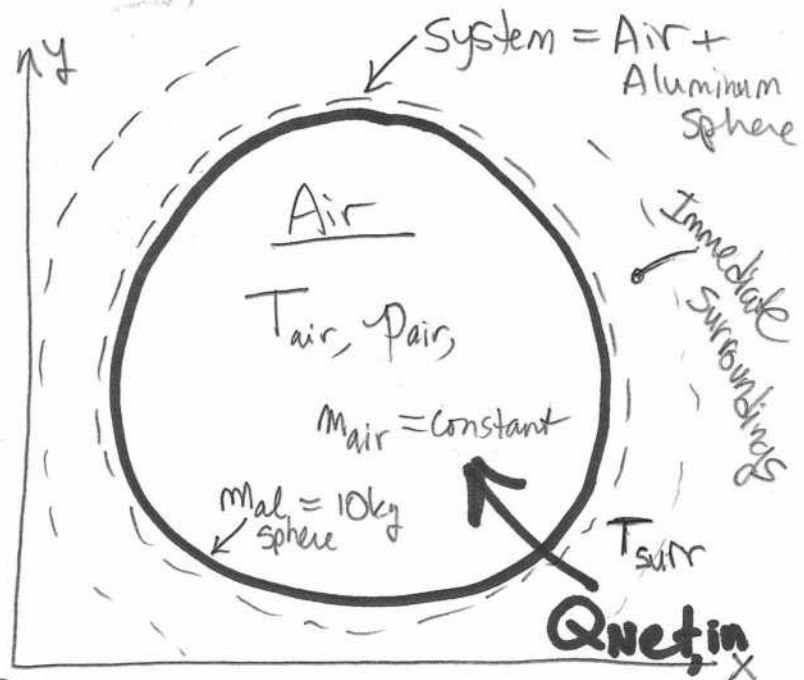
- The Closed System Energy Balance in "Change Form" is

$$\Delta E_{sys} = E_{in} - E_{out}$$

$$\Delta U_{sys} + \cancel{\Delta KE} + \cancel{\Delta PE} = Q_{net,in} - \cancel{W_{net,out}}$$

System is at rest and not falling in this reference frame

No Work performed on or by System in this reference frame



$$\Delta U_{\text{sys}} = Q_{\text{net, in}} \quad \text{Closed System Energy Balance}$$

⇒ Now find ΔU_{sys} for both the inside air and the aluminum sphere:

$$\Delta U_{\text{sys}} = \Delta U_{\text{air}} + \Delta U_{\text{sphere}} \quad \text{Important!}$$

- ⇒ It is important to understand that when there are two or more objects of different phase or material substance comprising the System, you must usually compute ΔU for each one separately unless there is alternative information to use (for example, the H_2O and R-134a Tables for saturated mixtures)
- ⇒ You will see this on a future quiz or exam!

ΔU_{air} Using the known temperatures at States ① & ②, along with Table A-17 for air,

$$@ T_1 = 294 \text{ K} \Rightarrow U_1 = 210.5 \frac{\text{kJ}}{\text{kg}}$$

$$@ T_2 = 273 \text{ K} \Rightarrow U_2 = 187.6 \frac{\text{kJ}}{\text{kg}} \text{ (interpolation)}$$

The mass of air within the sphere is found using state ①:

$$m_{\text{air}} = \frac{p_1 V_1}{R T_1}$$

⇒ What is V_1 ?

For a sphere, $V = \frac{4}{3}\pi r^3 = \frac{\pi}{6}D^3$, so that
with the inner diameter of the sphere $D = 1.5\text{ m}$,

$$V = \frac{\pi}{6}(1.5\text{ m})^3 \Rightarrow \boxed{V = 1.767\text{ m}^3}$$

$$\Rightarrow m_{\text{air}} = \frac{(101.325\text{ kPa})(1.767\text{ m}^3)}{\left(0.287 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(294\text{ K})}$$

$$\boxed{m_{\text{air}} = 2.12\text{ kg}}$$

Thus, $\Delta U_{\text{air}} = m \Delta u_{\text{air}} = m(u_2 - u_1)_{\text{air}}$

$$\Delta U_{\text{air}} = (2.12\text{ kg})\left(187.6 \frac{\text{kJ}}{\text{kg}} - 210.5 \frac{\text{kJ}}{\text{kg}}\right)$$

$$\boxed{\Delta U_{\text{air}} = -48.6\text{ kJ}}$$

ΔU_{sphere} $\Delta U_{\text{sphere}} = m_{\text{sph}} C_{v,\text{Al}} \Delta T$

From Table A-3(b), $C_{v,\text{Al}} \approx 0.887 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$ at $T_{\text{avg}} = \frac{294 + 273\text{ K}}{2} = 283\text{ K}$

So

$$\Delta U_{\text{sph}} = (10\text{ kg})\left(0.887 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(0^\circ\text{C} - 21^\circ\text{C})$$

$$\boxed{\Delta U_{\text{sphere}} = -186.3\text{ kJ}}$$

$$\Rightarrow \Delta U_{\text{sys}} = -48.6\text{ kJ} - 186.3\text{ kJ} = -235\text{ kJ}$$

Using the Energy Balance,

$$Q_{\text{net in}} = Q_{\text{in}} - Q_{\text{out}} = -235\text{ kJ} \Rightarrow$$

$$\boxed{Q_{\text{out}} = 235\text{ kJ}}$$

(b) We want to find the work done by the sphere (21) due to aerodynamic forces (Drag) during the entire free fall.

- In the absence of drag (object falling in a vacuum), the body experiences a constant acceleration due to gravity: $V_2^2 = V_1^2 - 2g(y_2 - y_1)$ [From Newton's 2nd Law, see basic physics text]

Lets consider this simple case first:

re-arranging the equation for constant acceleration,

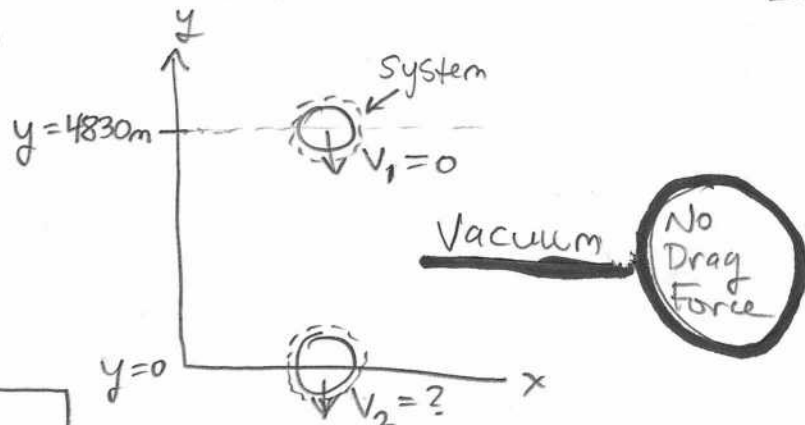
$$\boxed{\frac{1}{2}(V_2^2 - V_1^2) = -g(y_2 - y_1)}$$

[Wow! It says $\frac{\Delta KE}{m} + \frac{\Delta PE}{m} = \Delta e_{mech} = 0$]

The Energy Balance Equation is, taking the System with respect to the Earth frame of reference,

$$\boxed{\Delta E_{sys} = E_{in} - E_{out}}$$

$$\boxed{\Delta U_{sys} + \Delta KE_{sys} + \Delta PE_{sys} = Q_{net,in} - W_{net,out}}$$



For this case in which there is only vacuum outside the sphere, the only relevant form of Heat transfer is by radiation and this we can neglect $\Rightarrow Q_{net,in} = 0$

In part (a), we showed that $Q_{net,in} = \Delta U_{sys}$ so in this case, $\Delta U_{sys} = 0$ also.

The Energy Balance Equation then becomes

22

$$\Delta KE_{sys} + \Delta PE_{sys} = -W_{net,out}$$

Re-arranging,

$$W_{net,out} = -\frac{1}{2}m(V_2^2 - V_1^2) - mg(y_2 - y_1)$$

For the case of constant acceleration due to gravity with free-fall in a vacuum,

$$\frac{1}{2}m(V_2^2 - V_1^2) = -mg(y_2 - y_1) \quad \left(\begin{array}{l} \text{Newton's} \\ 2^{nd} \\ \text{Law} \end{array} \right. \left. \begin{array}{l} \text{Mechanical} \\ \text{Energy} \\ \text{or} \\ \text{Conservation} \end{array} \right)$$

so substituting this into the Closed System Energy Balance:

$$W_{net,out} = -[-mg(y_2 - y_1)] - mg(y_2 - y_1)$$

$$W_{net,out} = mg(y_2 - y_1) - mg(y_2 - y_1)$$

$$W_{net,out} = 0 \text{ kJ}$$

$$W_{in} = W_{out} = 0 \text{ kJ}$$

Q: What does this mean?

A: The System is the Sphere (Air + Aluminum).
This means that the system performed no work on the surrounding Universe as it fell.

⇒ It is important to understand that gravity performed work on the sphere/air system:

[Work done by gravity] $W_{\text{grav on sphere}} = \int_1^2 \delta W_{\text{grav}} = \int_1^2 F_{\text{grav}} dy$

$$= \int_{y_1}^{y_2} -mg dy = -mg \int_{y_1}^{y_2} dy$$

$$W_{\text{grav on sphere}} = -mg(y_2 - y_1) = -mg(0 - 4830 \text{ m})$$

$$W_{\text{grav on sphere}} = -(12.12 \text{ kg})(9.81 \frac{\text{m}}{\text{s}^2})(-4830 \text{ m})$$

$$W_{\text{grav on sphere}} = 574 \text{ kJ}$$

However, this is a different perspective and is not what we want \Rightarrow you will get the same answer in the case of drag forces!

Summary for Vacuum case

$$\Delta KE_{\text{sys}} = -574 \text{ kJ}$$

$$\Delta PE_{\text{sys}} = +574 \text{ kJ}$$

$$\Delta E_{\text{mech}} = 0 \text{ kJ}$$

\Rightarrow Potential Energy was converted perfectly, without losses of energy due to aerodynamic friction and turbulence effects.

Now Let's Consider the Case WITH Drag Forces

The Closed System Energy Balance for the sphere/air is

$$\Delta E_{\text{sys}} = E_{\text{in}} - E_{\text{out}}$$

Expanding into more detail, in the Earth reference frame,

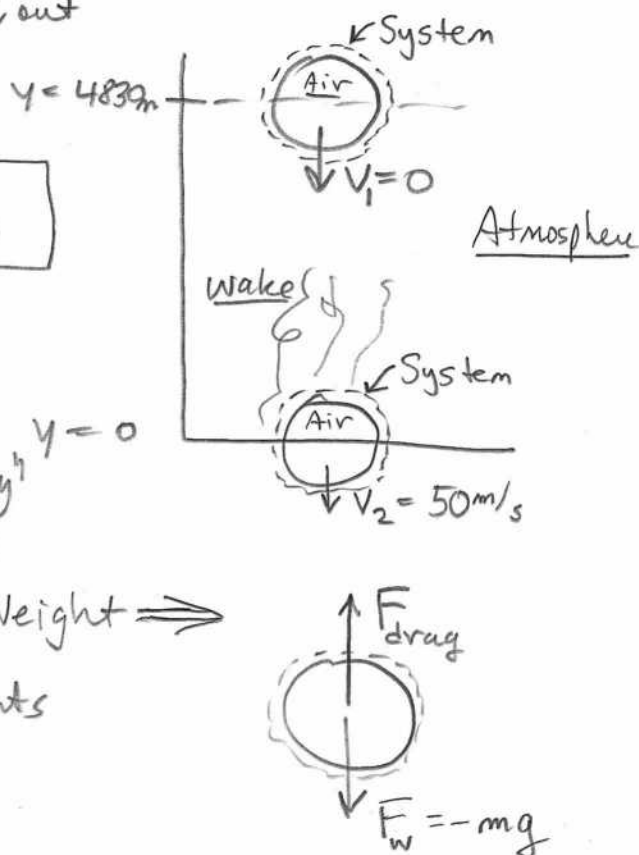
$$\Delta U_{\text{sys}} + \Delta KE_{\text{sys}} + \Delta PE_{\text{sys}} = Q_{\text{net},\text{in}} - W_{\text{net},\text{out}}$$

Now, in part (a) we found that $Q_{\text{net},\text{in}} = \Delta U_{\text{sys}}$ and it cancels out here because Internal Energy, U_{sys} , is practically independent of reference frame:

$$\Delta KE_{\text{sys}} + \Delta PE_{\text{sys}} = -W_{\text{net},\text{out}}$$

re-arranging,

$$W_{\text{net},\text{out}} = -\Delta KE_{\text{sys}} - \Delta PE_{\text{sys}}$$



In this case, the drag forces act upon the sphere so that it reaches a constant "terminal velocity" at some time during free-fall. This just means that the drag force = Weight \Rightarrow

\Rightarrow Here $V_{\text{term}} = 50\text{m/s}$ by measurements

so

$$W_{\text{net},\text{out}} = -\frac{1}{2}(12.12\text{kg})\left[(50\frac{\text{m}}{\text{s}})^2 - 0^2\right] - (12.12\text{kg})(9.81\frac{\text{m}}{\text{s}^2})(0\text{m} - 4830\text{m})$$

$$W_{\text{net},\text{out}} = -(6.06\text{kg})(2500\frac{\text{m}^2}{\text{s}^2}) \times \frac{1\text{kJ}}{1000\frac{\text{m}^2}{\text{s}^2}} + 574273\text{J} \times \frac{1\text{kJ}}{1000\text{J}}$$

$$W_{\text{net},\text{out}} = -15.0\text{kJ} + 574\text{kJ}$$

$$W_{\text{net},\text{out}} = 559\text{kJ}$$

$$(W_{\text{out}} - W_{\text{in}}) = 559\text{kJ} \Rightarrow$$

$$W_{\text{out}} = 559\text{kJ}$$

Next page

Q: What does this mean?!

A: The System is the Sphere (Air inside + Aluminum shell)

This means that the System performed 559 kJ of work on the Surrounding Universe as it fell.

Hold on! We aren't finished yet!

Summary for drag case

$$\Delta KE_{sys} = -15.0 \text{ kJ}$$

$$\Delta PE_{sys} = 574 \text{ kJ}$$

$$\Delta E_{mech} = 559 \text{ kJ} > \Delta E_{mech, vacuum} = 0 \text{ kJ}$$

Potential Energy was also entirely converted — but it went into two distinct parts!

$$\Rightarrow \Delta KE_{sys} + E_{drag loss}$$

⇒ For both cases, $\Delta PE_{sys} = 574 \text{ kJ}$, which is the same as the work done by gravity on the sphere, whether there is vacuum or not.

⇒ The drag on the sphere is related to the different results we got for ΔKE_{sys}

⇒ The maximum ΔKE during free-fall occurs for the vacuum case, $\Delta KE_{sys} = -574 \text{ kJ}$, and the ΔKE_{sys} for the "drag" case is -15 kJ . The deficit in the KE "budget" must be the Energy lost to drag!

(next page)

This deficit in ΔKE between the two cases is

$$\Delta E_{\text{drag}} = \Delta KE_{\text{sys vacuum}} - \Delta KE_{\text{sys drag}}$$

$$\Delta E_{\text{drag}} = -574 \text{ kJ} - (-15 \text{ kJ})$$

$$\underline{\underline{\Delta E_{\text{drag}} = -559 \text{ kJ}}}$$

This is the same answer we got using the Closed System Energy Balance for the aerodynamic drag case!

⇒ In that case, $\boxed{W_{\text{out}} = 559 \text{ kJ}}$ is the work output of the Sphere System on the surrounding Universe — the atmosphere in this case, in the form of aerodynamic energy loss (Drag)!

Pretty Cool, huh?! 😊

Note: I took a "long winded" approach in order to fully explain this — to teach — rather than take a shortcut.

Lecture 16 Open System 1-D Mass Continuity and Mass Balance Equations

Mass Flow Rate

The amount of mass flowing through an Area cross-section per-unit-time (kg/s or lb_m/s) is

$$\dot{m} \equiv \frac{dm}{dt}$$

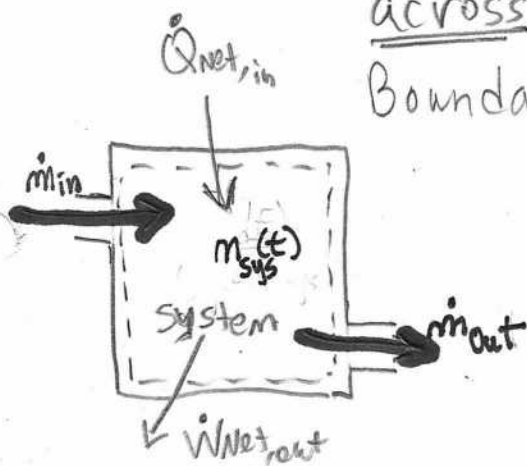


Note: (1) Mass flow rate is both Area and time dependent $\Rightarrow \dot{m}(r,t)$

(2) Within a Control Volume, mass is a property of the System: $m(x,y,z,t)$

But we will deal with mass Flowing across the System Control Volume Boundary, so mass flow rate is an

Interaction with the System, carrying Energy with it, and hence, changing System Properties.



(3) Mass flow in/out of a System through an opening is dependent upon the opening's Area

⇒ mass Velocity distribution is not Spatially Uniform.

⇒ Mass flow rate is an inexact differential because its spatial distribution is not the same for all cases [the "path" in this case is the spatial distribution function $\dot{m}(x, y, z, t)$]

$$\dot{m} = \iint_{\text{Area}} \delta \dot{m} \quad (\text{kg/s})$$

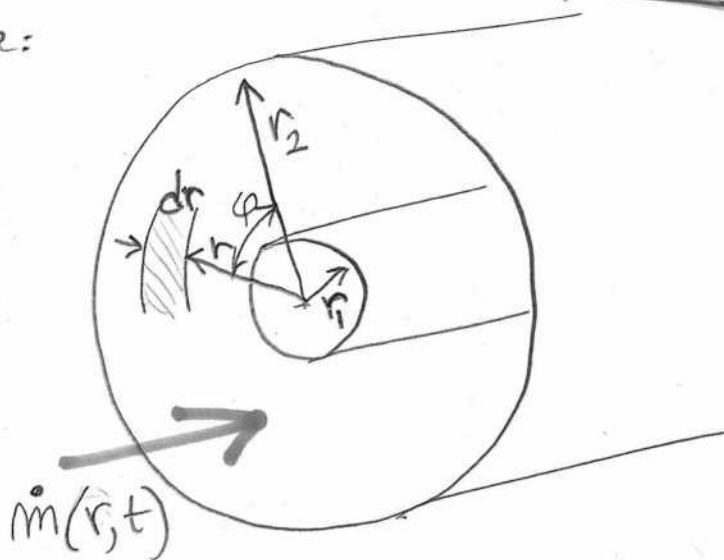
Example 1 Area integration vs. mass flow rate integration

• Consider an annulus pipe:

Area Integration

The exact differential is always the same in this case:

$$dA = r dr d\phi$$

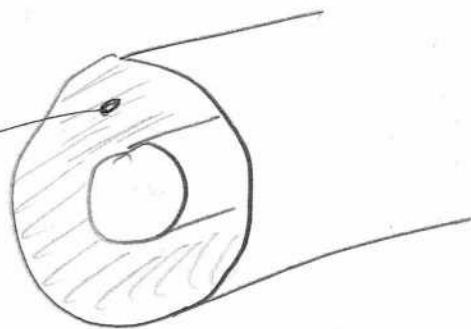


So, we can immediately integrate this:

$$A_{tot} = \int_{r_1}^{r_2} \int_{\phi_1=0}^{\phi_2=2\pi} dA = \int_{r_1}^{r_2} \int_0^{2\pi} r dr d\phi = 2\pi \int_{r_1}^{r_2} r dr$$

$$A_{tot} = 2\pi \left[\frac{1}{2} r^2 \right]_{r_1}^{r_2}$$

$$A_{tot} = \pi [r_2^2 - r_1^2]$$

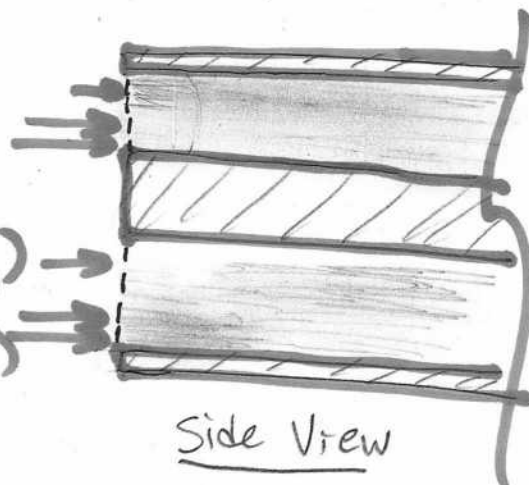


• But mass flow rate depends on the spatial distribution of \dot{m} at the System Entrance:

• Thus, this is an incorrect equation:

~~$$\iint_{Area} \delta \dot{m} = \dot{m}(r_2) - \dot{m}(r_1)$$~~

$$\dot{m}(r_1) \rightarrow$$
$$\dot{m}(r_2) \rightarrow$$



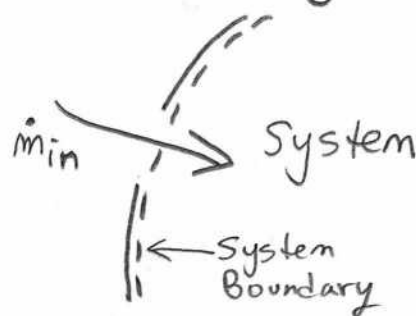
• This is correct:

$$\dot{m}_{in}(t) = \iint_{Area} \delta \dot{m}$$

⇒ You must specify how $\dot{m}(r)$ varies with radius in order to integrate and find $\dot{m}(t)$

The 1-D Mass Continuity Equation Applied to mass crossing System Boundaries

- To solve Open System Flow problems, we need to relate \dot{m} to properties such as ρ , v , $|\vec{V}|$, etc.
- Consider a piece of a System Boundary where mass is flowing in:



- Look at an incremental Volume at this opening \Rightarrow

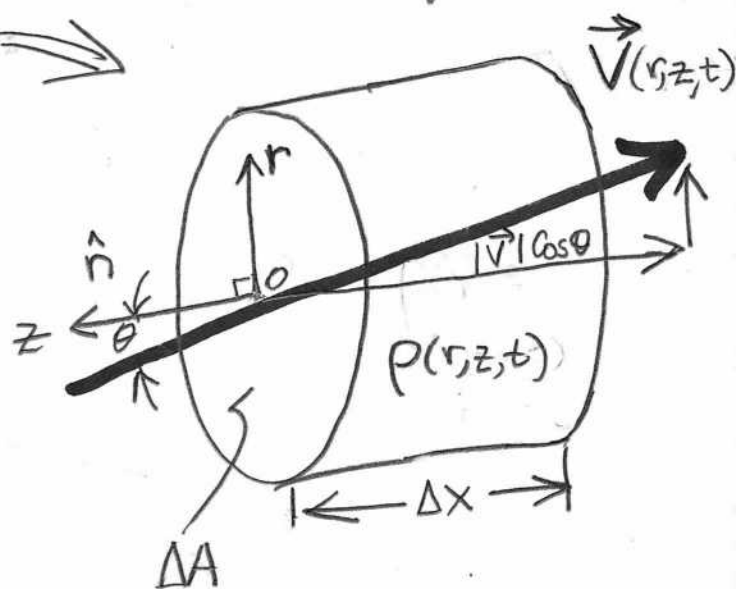
- The mass at time t within this volume is:

$$\Delta m = \underbrace{\rho(r,z,t)}_{\text{mass density}} \times \Delta V$$

- Re-expressing the Volume,

$$\Delta m = \rho(r,z,t) \times \underbrace{\Delta x}_{\text{length}} \underbrace{\Delta A}_{\text{Area}}$$

$$\Delta m = \rho(r,z,t) \underbrace{|\vec{V}| \cos \theta \Delta t}_{\text{length}} \times \Delta A$$



(5)

- Using the unit vector normal (\perp) to the area surface, and opposite to the direction of $|\vec{V}| \cos \theta$, we can express Δm in terms of a dot product:

$$\Delta m = \rho(r, z, t) \vec{V} \cdot \hat{n} \Delta t \Delta A$$

- Dividing by Δt ,

$$\frac{\Delta m}{\Delta t} = \rho(r, z, t) \vec{V} \cdot \hat{n} \Delta A$$

- taking the limit as $\Delta t \rightarrow 0$,

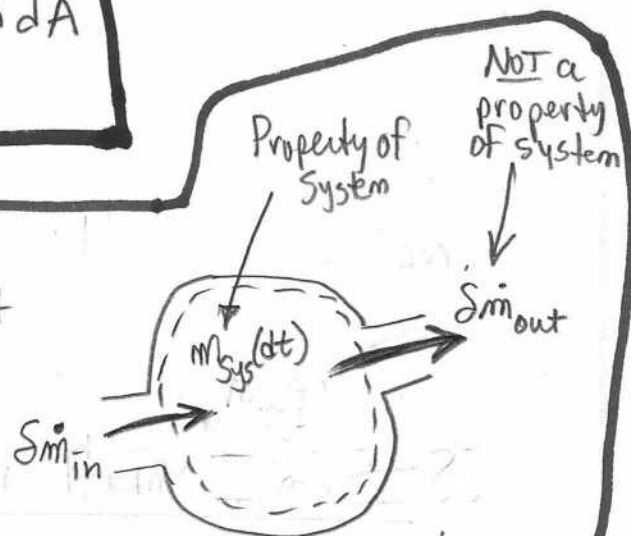
$$\lim_{\Delta t \rightarrow 0} \left(\frac{\Delta m}{\Delta t} \right) = \dot{m} = \rho(r, z, t) \vec{V} \cdot \hat{n} dA$$

- To obtain the total mass flow rate across the opening in the system boundary, we must integrate over the total area of the opening:

$$\dot{m}(t) = \iint_{\text{Area}} \dot{m} = \iint_{\text{Area}} \rho(r, z, t) \vec{V} \cdot \hat{n} dA$$

Important

Notice that \dot{m} is an inexact differential? Mass crossing a system boundary is not a property of the system—it is an interaction! The system mass IS a property, however!



⑥

- In Thermodynamics I and II, we will use a simplified form of this mass flow rate equation:

Assume:

These assumptions make the final Equation valid for 1-D only

- (1) The density is spatially uniform over the Control surface boundary opening:

$$\rho(r, z, t) = \rho_{avg}(t)$$

$$\Rightarrow \dot{m}(t) \approx \rho_{cs, avg}(t) \iint_{cs} \vec{V} \cdot \hat{n} dA$$

- (2) The average flow velocity normal to the control surface Area boundary is

$$V_{avg, cs} \equiv \frac{1}{A_{cs}} \iint_{cs} \vec{V} \cdot \hat{n} dA$$

So that $\iint_{cs} \vec{V} \cdot \hat{n} dA = V_{avg, cs} A_{cs}$

\Rightarrow At the control surface boundary,

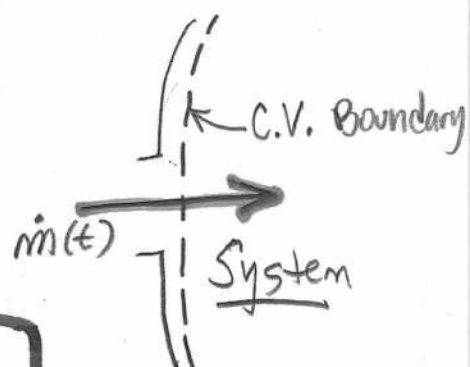
$$\dot{m}(t) \approx \rho_{cs}(t) V_{avg, cs} A_{cs}$$

- We typically drop the subscripts so that

$$\begin{aligned} \dot{m}(t) &= \rho(t) V(t) A \\ &= \frac{1}{\nu(t)} V(t) A \end{aligned}$$

The 1-D Mass Continuity Equation

Commit to Memory!!



VERY Important!!

• Another Very Important Relationship:

7

Volume Flow Rate

The Volume of fluid flowing through a cross-section per unit time

$$\dot{V} = VA \quad \frac{\text{m}^3}{\text{s}} \text{ or } \frac{\text{ft}^3}{\text{s}}$$

Volume Flow rate is derived from the mass continuity equation:

$$\dot{m} = \rho VA$$

$$\frac{\dot{m}}{\rho} = \dot{m} \nu = VA = \dot{V}$$

$\uparrow \quad \uparrow \quad \Rightarrow \quad \frac{\text{m}^3}{\text{s}}$
 $\text{m/s} \quad \text{m}^2$

Note that we can express mass flow rate in terms of Volume Flow rate:

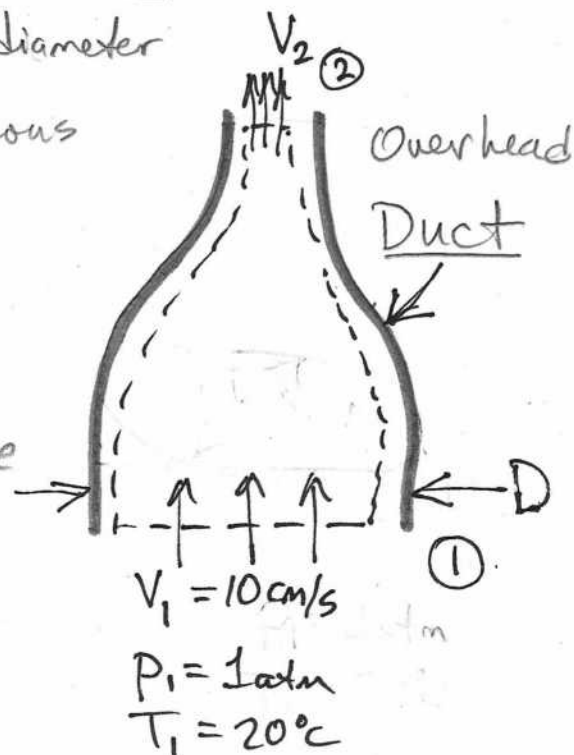
$$\dot{m} = \rho \dot{V} = \frac{\dot{V}}{\nu}$$

An alternate expression of mass continuity

Commit \uparrow this to Memory!
VERY IMPORTANT

Example 1 Air Handling System

An overhead duct having an inlet diameter of 1.5m is used to remove hazardous vapors from a warehouse. The warehouse is at 1 atm and 20°C. The air speed at the duct inlet is 10 cm/s. OSHA regulations state that the Volume flow rate of air must be at least $10 \frac{\text{m}^3}{\text{min}}$.



Find: (a) \dot{V} (m^3/min)
(b) \dot{m} (kg/s)

Assume (1) Air behaves as an Ideal gas
(2) Neglect Hazardous vapors in analysis
(3) Inlet velocity is uniform across the duct and steady

Analysis

(a) $\dot{V} = VA$

$$\dot{V} = \left(10 \frac{\text{cm}}{\text{s}} \times \frac{1 \text{ m}}{100 \text{ cm}}\right) \left(\frac{\pi}{4} [1.5 \text{ m}]^2\right)$$

$$\dot{V} = 0.1767 \frac{\text{m}^3}{\text{s}} \times \frac{60 \text{ sec}}{\text{min}}$$

$$\boxed{\dot{V} = 10.6 \text{ m}^3/\text{min}} \quad \checkmark \text{ meets OSHA Standards}$$

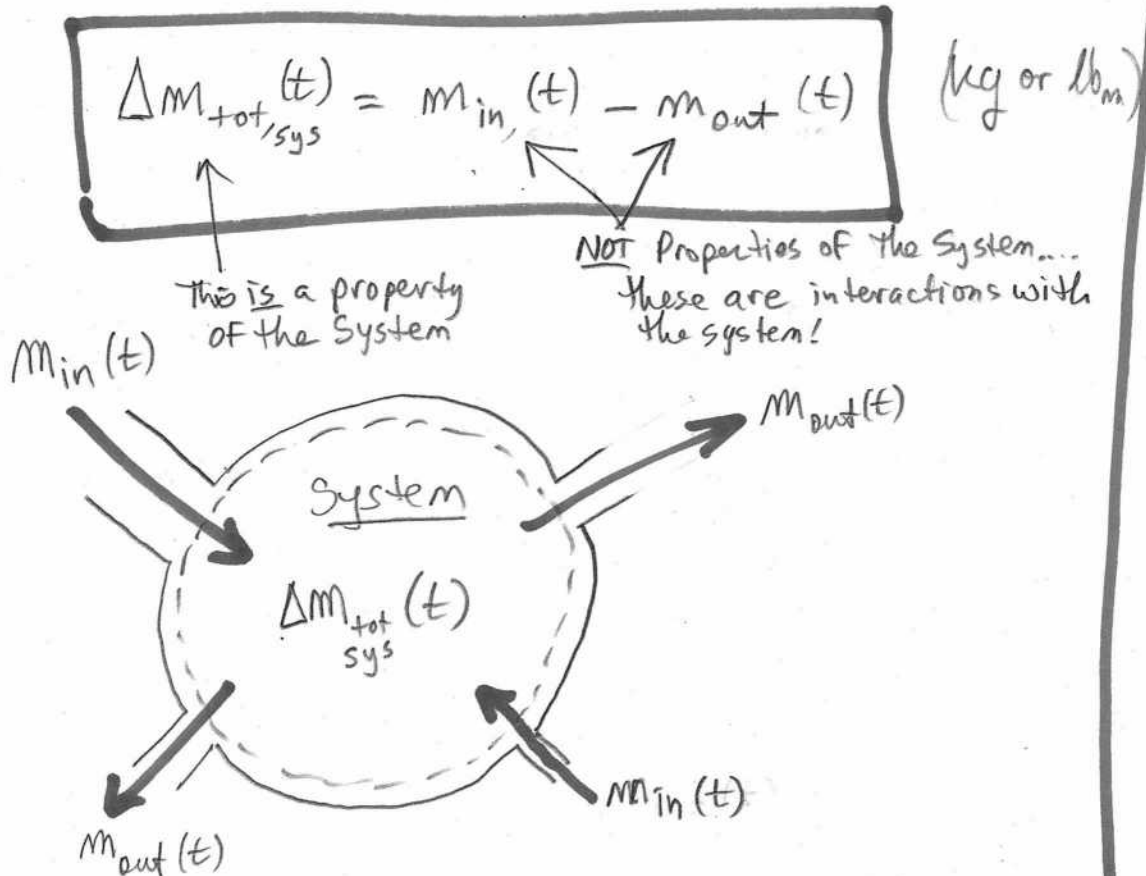
(b) $\dot{m} = \frac{1}{v} \dot{V} = \frac{P}{RT} \dot{V} = \frac{101.325 \text{ kPa}}{\left(0.287 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}\right) (293 \text{ K})} \times 0.1767 \frac{\text{m}^3}{\text{s}}$

$$\boxed{\dot{m} = 0.213 \frac{\text{kg}}{\text{s}}}$$

Open System Mass Balance Equation

9

The Net mass transfer to or from an open System Control Volume during a time Δt is equal to the NET change in the total mass within the Control Volume:



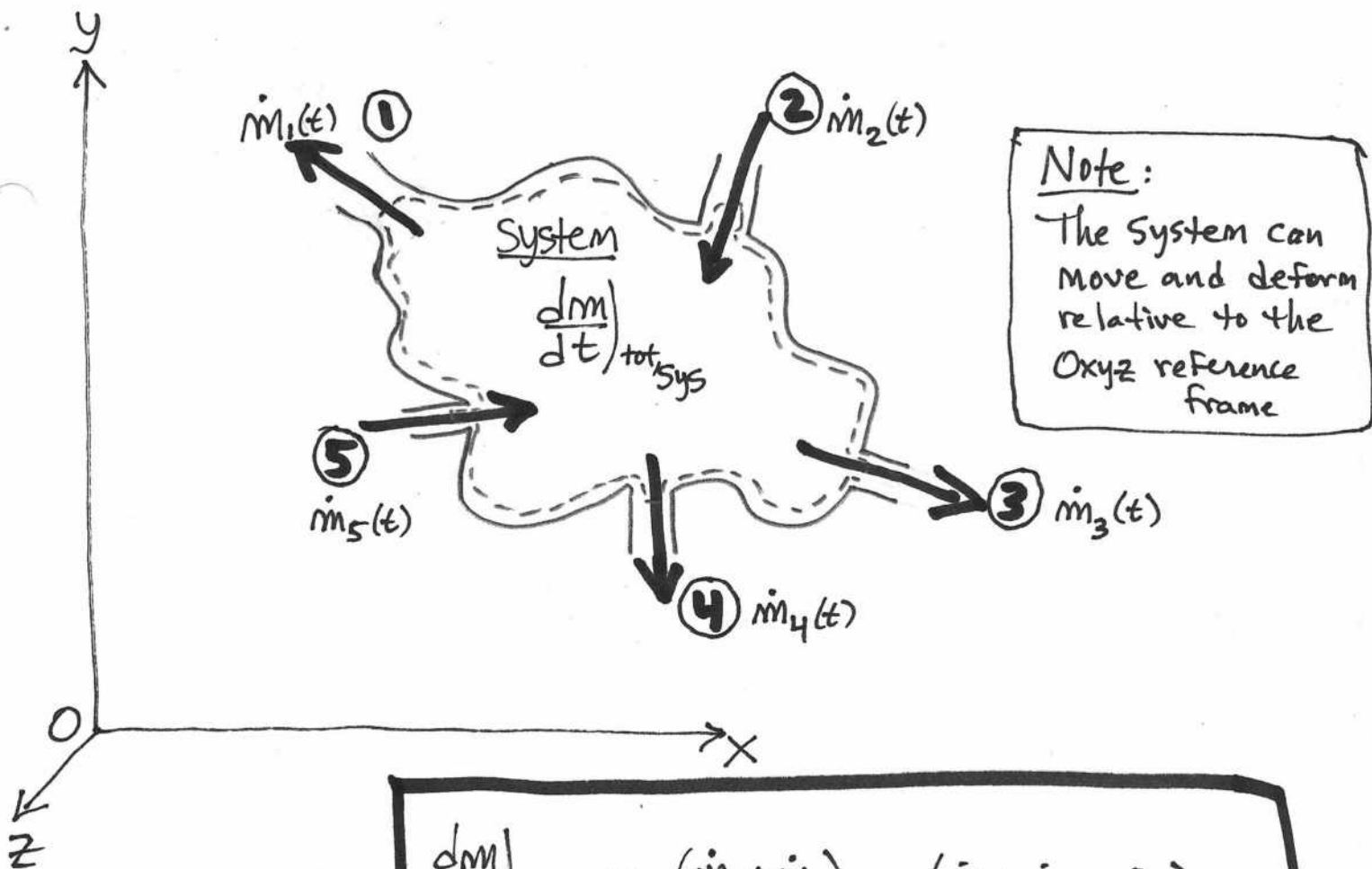
- This states that the system mass (a property) can increase or decrease based on the Net result of mass flow interactions across the System Boundary.
- The mass can be stored/depleted through $\Delta m_{\text{tot, sys}}(t)$ within the System.

Mass Balance for an Open System (Conservation of Mass)

$$\left[\begin{array}{c} \text{Net rate of} \\ \text{change in C.V.} \\ \text{mass} \end{array} \right] = \left[\begin{array}{c} \text{Net rate of} \\ \text{change in mass} \\ \text{LEAVING C.V.} \end{array} \right] - \left[\begin{array}{c} \text{Net rate of} \\ \text{change in mass} \\ \text{ENTERING C.V.} \end{array} \right]$$

$$\frac{dm}{dt}_{\text{tot, sys}} = \dot{m}_{\text{in total}} - \dot{m}_{\text{out total}}$$

(kg/s or lbm/s)



$$\frac{dm}{dt}_{\text{tot, sys}} = (\dot{m}_2 + \dot{m}_5)_{\text{in}} - (\dot{m}_1 + \dot{m}_3 + \dot{m}_4)_{\text{out}}$$

If $\frac{dm}{dt}_{\text{tot, sys}} = 0$ (Steady Flow) $\Rightarrow \boxed{\dot{m}_2 + \dot{m}_5 = \dot{m}_1 + \dot{m}_3 + \dot{m}_4}$

"What goes in must come out"

Mass balance for Multiple Inflows/outflows

11

In general:

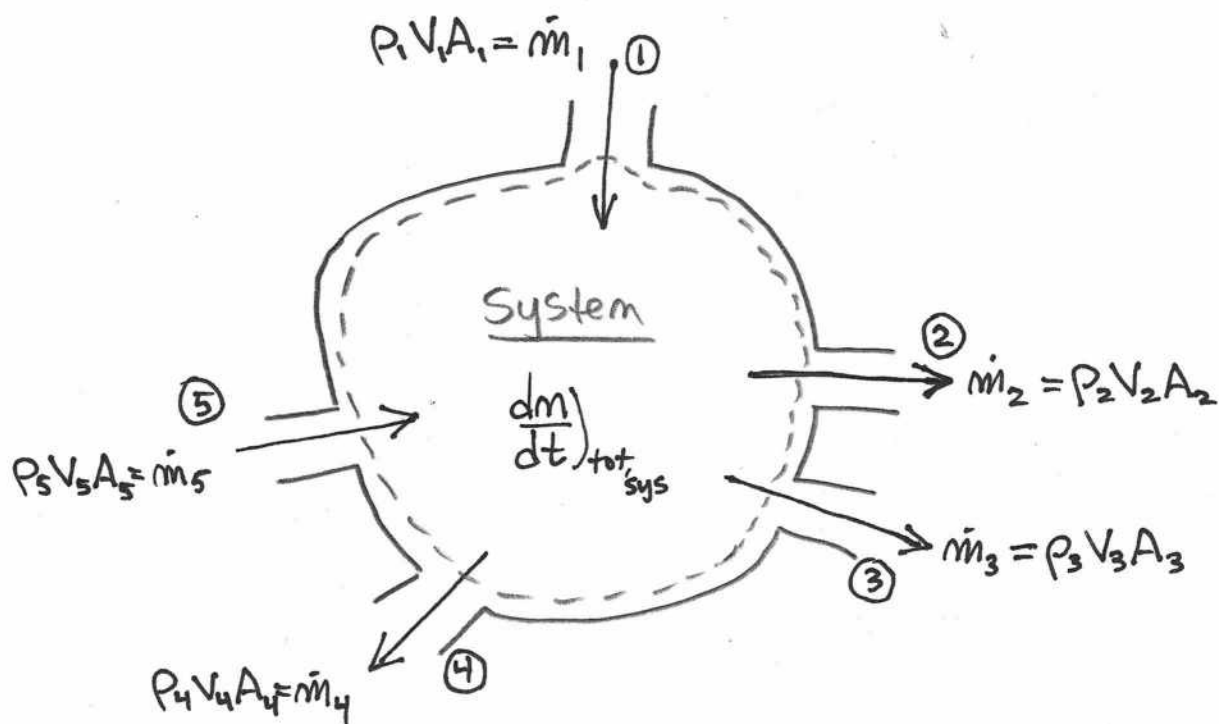
$$\left(\frac{dm}{dt}\right)_{\text{tot, sys}} = \sum_{\text{in}} \dot{m} - \sum_{\text{out}} \dot{m}$$

Mass Balance Equation
(Mass Conservation principle)

where

$$\dot{m} = \rho VA = \frac{VA}{v} = \frac{\dot{V}}{v}$$

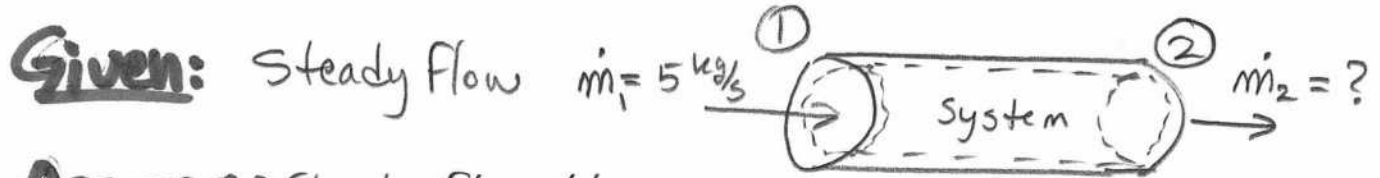
1-D Mass Continuity Equation



$$\left(\frac{dm}{dt}\right)_{\text{tot, sys}} = [\dot{m}_1 + \dot{m}_5] - [\dot{m}_2 + \dot{m}_3 + \dot{m}_4]$$

Example 2 Air flows at steady state into a

10cm diameter duct at mass flow rate of 5 kg/s . What is the mass flow rate of air exiting the duct?



Assume: (1) Steady flow (time independent)

(2) The System Control Surface boundary is the entire inside surface of the duct, including the inlet and outlet openings

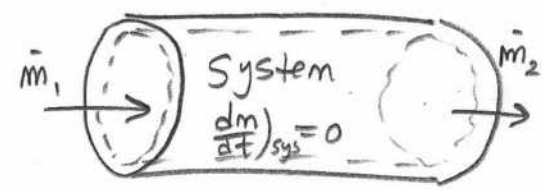
Find: \dot{m}_2

Analysis

The Open System Mass Balance is

~~$\left(\frac{dm}{dt} \right)_{\text{tot, sys}}$~~ = $\sum \dot{m}_{\text{in}} - \sum \dot{m}_{\text{out}}$

Steady flow



$\Rightarrow \sum \dot{m}_{\text{in}} = \sum \dot{m}_{\text{out}}$

Since there is only one inlet and one outlet,

$\dot{m}_1 = \dot{m}_2$

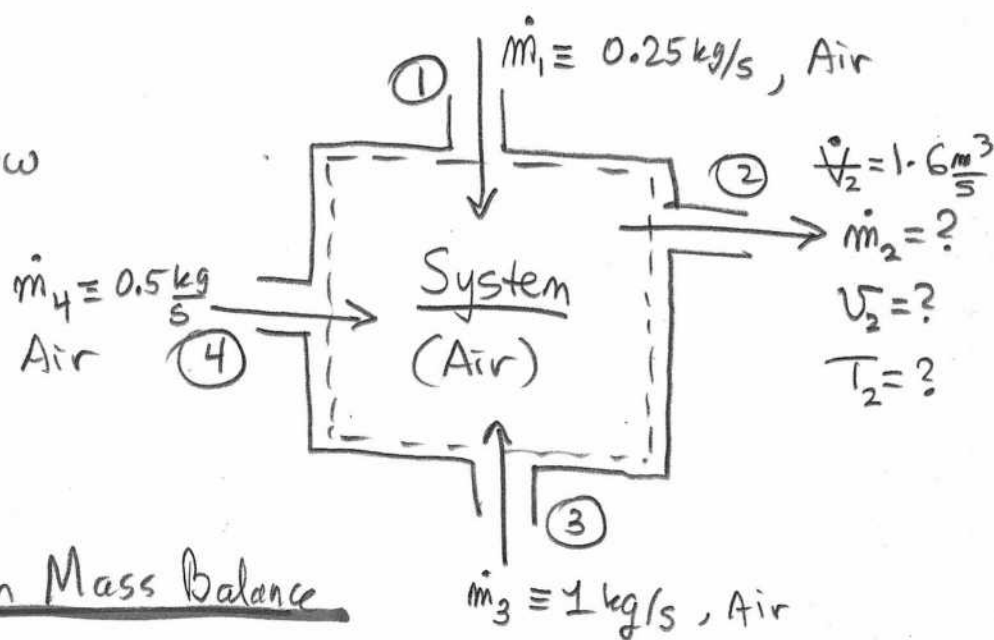
Since $\dot{m}_1 = 5 \text{ kg/s} \Rightarrow \boxed{\dot{m}_2 = \dot{m}_1 = 5 \text{ kg/s}}$

Example 3 Air Flows steadily into the rigid tank according to the picture below.

- (a) What is the outlet mass flow rate?
- (b) If the outlet volume flow rate is $1.6 \frac{\text{m}^3}{\text{s}}$, what is the outlet specific volume?
- (c) If the outlet pressure is 100 kPa, what is the outlet temperature in °C assuming that the air behaves as an Ideal gas at the outlet boundary surface?

Given:

Assume: Steady Flow



Analysis

(a) The Open System Mass Balance

$$\cancel{\frac{dm}{dt}}_{\text{tot/sys}} = \sum_{\text{in}} \dot{m} - \sum_{\text{out}} \dot{m}$$

$$\sum_{\text{in}} \dot{m} = \sum_{\text{out}} \dot{m}$$

$$\dot{m}_1 + \dot{m}_3 + \dot{m}_4 = \dot{m}_2$$

$$\dot{m}_2 = 0.25 \text{ kg/s} + 1 \text{ kg/s} + 0.5 \text{ kg/s}$$

$$\dot{m}_2 = 1.75 \text{ kg/s}$$

(b) The outlet ② $\dot{V} = 1.6 \frac{\text{m}^3}{\text{s}}$

From the relationship for mass flow rate

$$\dot{m}_2 = \rho_2 V A_2 = \frac{V A_2}{v_2} = \frac{\dot{V}_2}{v_2},$$

$$v_2 = \frac{\dot{V}_2}{\dot{m}_2}$$

$$v_2 = \frac{1.6 \text{ m}^3/\text{s}}{1.75 \text{ kg/s}}$$

$$v_2 = 0.94118 \text{ m}^3/\text{kg}$$

(c) Air at outlet ② is an Ideal gas — find T_2 given $P_2 = 100 \text{ kPa}$:

$$P_2 v_2 = R_{\text{air}} T_2$$

$$T_2 = \frac{P_2 v_2}{R_{\text{air}}}$$

$$T_2 = \frac{(100 \text{ kPa})(0.94118 \text{ m}^3/\text{kg})}{0.287 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}}$$

$$T_2 \approx 328 \text{ K}$$

or

$$T_2 \approx 55^\circ \text{C}$$

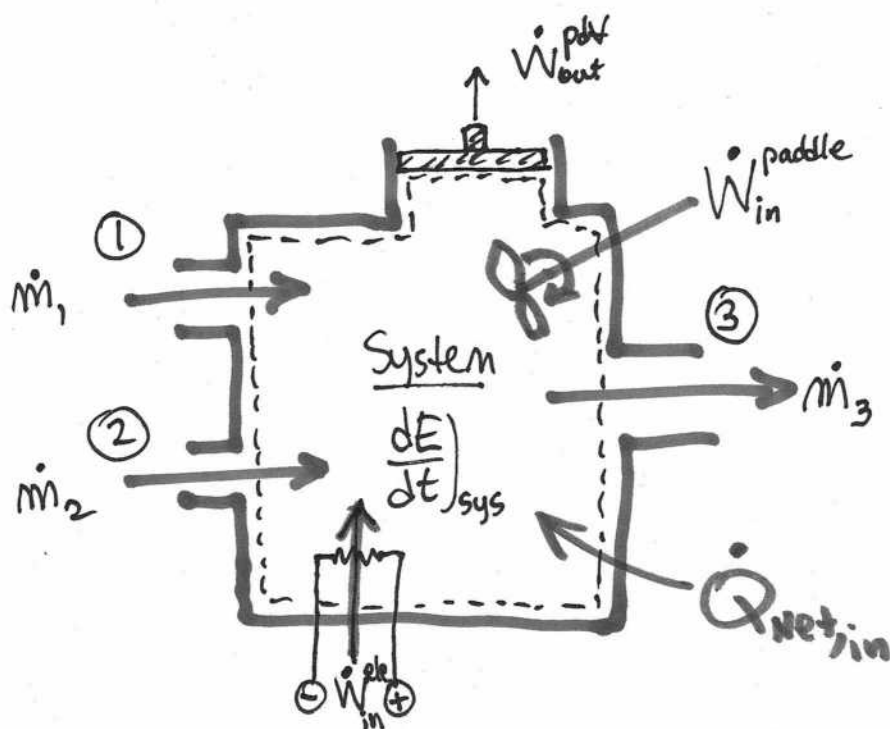
Lecture 17] The Open System Energy Balance Equation

- Your textbook doesn't give a rigorous derivation of the Open System Energy Balance Equation.

This derivation is partly based on the text, Schaum's outline, Moran & Shapiro, and myself.

Derivation of The Open System Energy Balance Equation

- Opposed to a Closed System, we have mass flowing into and out of a System with simultaneous heat and work interactions at and/or across the System Control Volume Boundaries:



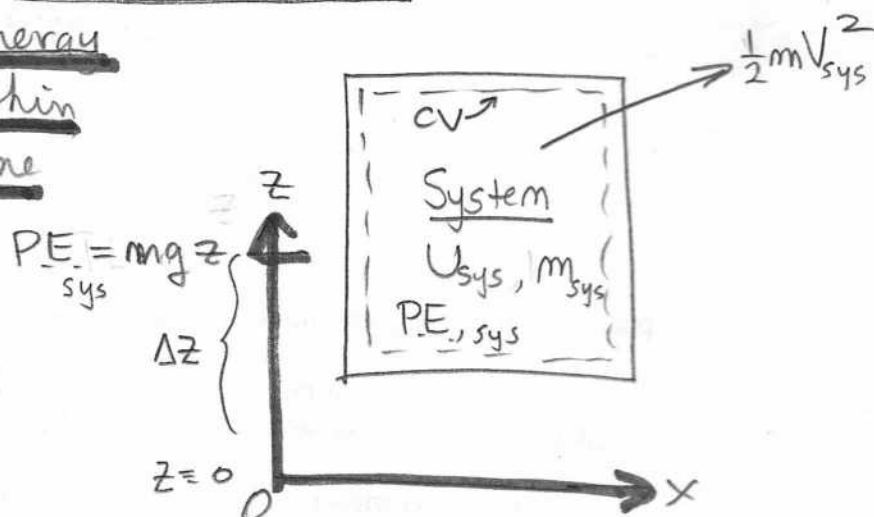
(2)

- In order to make any sense of this thing, we need to go back to the basics!

- From Lecture 6, we know that total System Energy is

$$E_{\text{sys}} = U_{\text{sys}} + K.E._{\text{sys}} + P.E._{\text{sys}} \quad (\text{kJ})$$

which is the Total Energy stored by the mass within any System Control Volume boundary at any time t



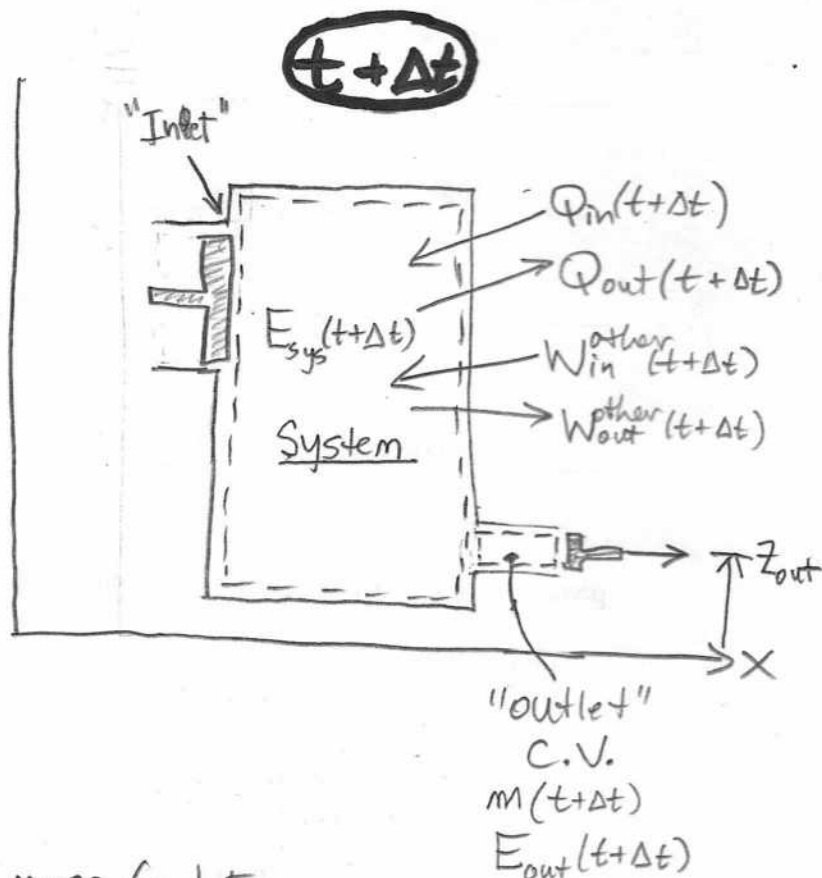
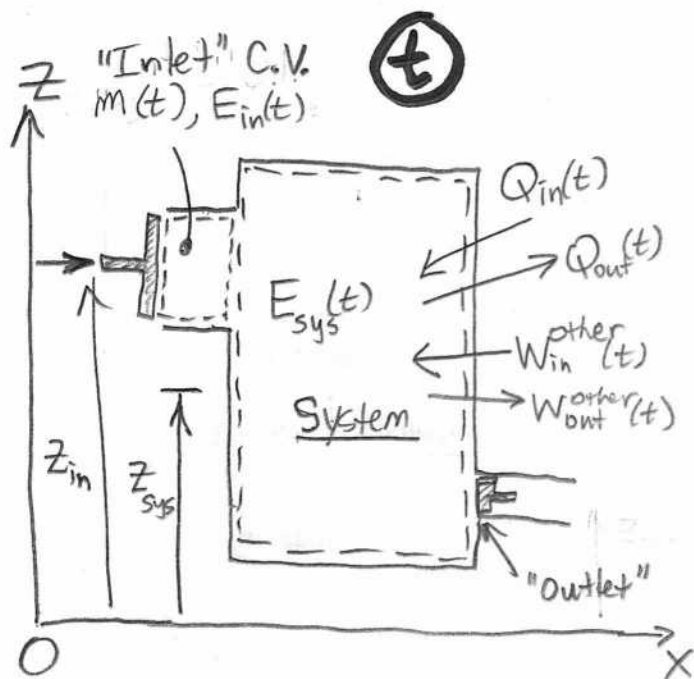
- For open system problems, with mass flow rates in/out, we need to derive the 1st Law on a rate basis.

⇒ the complication here is keeping track of where all the mass is going in time and the energy it carries with it.

- All we are going to do here is evaluate the flowing Open system at two different times, t and $t + \Delta t$, then \div by Δt and take the limit as $\Delta t \rightarrow 0$.

- **Note:** I will first NEGLECT MOVING BOUNDARY WORK ($W_{\text{net, out}}^{\text{pdt}}$) for simplicity, then add it in afterwards.

- Consider a rigid open device at two "Snapshots" in time: t and $t + \Delta t$



Note:

- (1) The pistons push mass (and Energy carried with it) across the System Control Volume boundary and so flowing mass is also considered an Interaction with the system, not a property!
- (2) The pistons are not deforming the System boundary in any way, unlike our previous Closed System problems, but the inlet/outlet Systems are experiencing moving boundary work.
- (3) We actually have 3 Control Volumes: "Inlet", "System", and "Outlet". We must evaluate what happens to the mass and energy within these three C.V.'s as time advances from t to $t + \Delta t$.

Analysis (Look at my diagrams!)

@ time = t

$$E_{sys}(t) = U_{sys}(t) + \frac{1}{2} m_{sys}(t) V_{sys}^2(t) + m_{sys}(t) g z_{sys}(t)$$

$$E_{inlet}(t) = U_{in}(t) + \frac{1}{2} m_{in}(t) V_{in}^2(t) + m_{in}(t) g z_{in}(t)$$

Note that $m_{out}(t) = 0$ and $E_{out}(t) = 0$

and that piston's face is flush against the system boundary surface.

Energy of Both System + Inlet at time = t

$$E_{sys+inlet}(t) = E_{sys}(t) + E_{inlet}(t)$$

@ time = t + Δt

$$E_{sys}(t+\Delta t) = U_{sys}(t+\Delta t) + \frac{1}{2} m_{sys}(t+\Delta t) V_{sys}^2(t+\Delta t) + m_{sys}(t+\Delta t) g z_{sys}(t+\Delta t)$$

$$E_{outlet}(t+\Delta t) = U_{outlet}(t+\Delta t) + \frac{1}{2} m_{out}(t+\Delta t) V_{out}^2(t+\Delta t) + m_{out}(t+\Delta t) g z_{out}(t+\Delta t)$$

Note that $m_{in}(t+\Delta t) = 0$ and $E_{in}(t+\Delta t) = 0$

and that piston's face is flush against the system boundary surface

$$E_{sys+outlet}(t+\Delta t) = E_{sys}(t+\Delta t) + E_{outlet}(t+\Delta t)$$

- It is important to note that in general,

$$(a) \quad m_{inlet}(t) \neq m_{outlet}(t + \Delta t)$$

$$(b) \quad m_{sys}(t) \neq m_{sys}(t + \Delta t)$$

⇒ This means that mass is being compressed and stored within the system during the time interval Δt (or expanded and lost within the system)

⇒ However what IS TRUE is the Conservation of ALL MASS under study:

$$(c) \quad m_{sys}(t) + m_{inlet}(t) = m_{sys}(t + \Delta t) + m_{outlet}(t + \Delta t)$$

⇒ This is a KEY CONCEPT because it allows us to use our friend, the Closed System Energy Balance to evaluate the entire, constant lump of mass at two different times (i.e., two different states) which is what we have been doing all along!

- The Closed System Energy Balance is

$$\underbrace{\Delta E_{c.v.}}_{\text{Change in Properties within Control Volume}} = \underbrace{E_{in} - E_{out}}_{\text{Work, Heat, and Mass flow interactions across C.V. Boundary}}$$

- For the same mass under study but at different times and places, the Closed System Energy Balance gives us:

$$\Delta E_{\text{same mass system + Inlet + outlet}} = Q_{\text{Net, in System}} - W_{\text{Net, out system}}$$

- Plugging in the expressions for $E(t)$, $E(t+\Delta t)$, and Evaluating the RHS Work and Heat transfers at t and $t+\Delta t$, the closed System Energy Balance is:

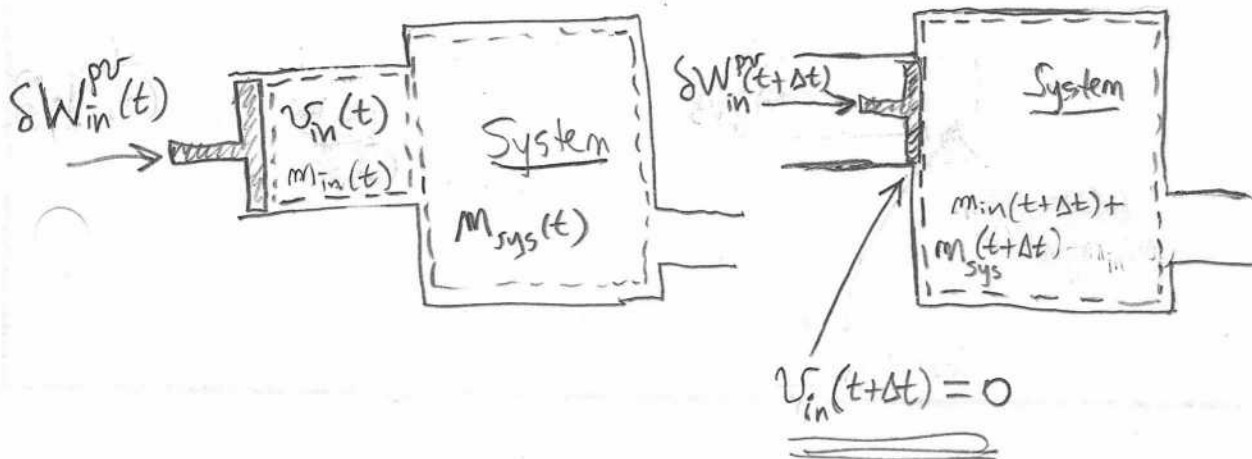
$$E_{\text{system + outlet}}(t+\Delta t) - E_{\text{system + Inlet}}(t) = [Q_{\text{Net, in}}(t+\Delta t) - Q_{\text{Net, in}}(t)] - [W_{\text{Net, out}}(t+\Delta t) - W_{\text{Net, out}}(t)]$$

- Now, at this point I want to focus on the work terms closely and do something that will eventually make our life easier down the road:

- Inlet "pv" Work (Closely associated with $e_{\text{flow}} = pv$ "flow energy" — see Supplement Notes)

time = t

time = $t + \Delta t$



Introduction to a new Property: Flow Energy ($p\upsilon$)

(7)

- For open systems through which a fluid flows, we must take into account a new form of energy called "flow energy", or more precisely, specific flow energy

Specific Flow Energy

Specific flow energy is a form of mechanical energy within fluid flows.

$$e_{\text{flow}} \equiv p\upsilon \quad (\text{kJ/kg})$$

It is closely associated with what some call flow " $p\upsilon$ " work. Flow " $p\upsilon$ " work is the negative change in $p\upsilon$ within the open system fluid flow:

$$\delta W_{\text{flow}}^{p\upsilon} = -de_{\text{flow}} = -d(p\upsilon) \quad (\text{kJ/kg}).$$

Note that " $p\upsilon$ " work is NOT the same as " $p d\upsilon$ " work!

- Flow energy is part of what comprises specific enthalpy:

$$h = u + p\upsilon$$

\uparrow
internal
energy

\uparrow
flow
energy

- For open systems having fluid flow, the total mechanical energy is

$$e_{\text{mech, sys}} = (KE)_{\text{sys}} + (PE)_{\text{sys}} + e_{\text{flow, sys}}$$

$$e_{\text{mech, sys}} = \frac{1}{2}V_{\text{sys}}^2 + gZ_{\text{sys}} + p\upsilon_{\text{sys}}$$

(8)

- From the pictures on page 6 and the definition of "pv" work pg. 7, the work performed by the piston to push the Inlet mass, $m_{in}(t)$, across the System Boundary and completely into the System Control Volume is

$$W_{in}^{pv} = \int_t^{t+\Delta t} \delta W_{in}^{pv} \quad (kJ/kg)$$

- The differential work per unit mass at any given time is

$$\delta W_{in}^{pv} = -d(pv) \quad (kJ/kg) \quad \text{(called "Flow Work")}$$

where $p \neq \text{constant}$, $v \neq \text{constant}$ so we must keep pv together inside the differential.

- Performing the integral for the inlet from t to $t+\Delta t$,

$$W_{in}^{pv} = - \int_{p(t)v(t)}^{p(t+\Delta t)v(t+\Delta t)} d(pv) = - \left[p_{in}(t+\Delta t)v_{in}(t+\Delta t) - p_{in}(t)v_{in}(t) \right]$$

- But wait! $v(t+\Delta t) = 0$ because the volume is zero!

so

$$W_{in}^{pv} = - \left[- p_{in}(t)v_{in}(t) \right]$$

$$W_{in}^{pv} = p_{in}(t)v_{in}(t)$$

- For the Outlet,

$$W_{out}^{pv} = p_{out}(t+\Delta t)v_{out}(t+\Delta t)$$

- On an Energy Basis (kJ) the flow pv work is

9

$$W_{in}^{pv}(t) = m_{in}(t) p_{in}(t) v_{in}(t) \quad (kJ)$$

$$W_{out}^{pv}(t+\Delta t) = m(t+\Delta t) p_{out}(t+\Delta t) v_{out}(t+\Delta t) \quad (kJ)$$

- Now we write the Closed System Energy balance as

$$\underbrace{E(t+\Delta t)}_{\text{System Outlet}} - \underbrace{E(t)}_{\text{System Inlet}} = [Q_{Net,in}(t+\Delta t) - Q_{Net,in}(t)] - \left\{ (W_{Net,out}^{pv}(t+\Delta t) + W_{Net,out}^{other}(t+\Delta t)) - (W_{Net,out}^{pv}(t) + W_{Net,out}^{other}(t)) \right\}$$

- From here, it's all nasty algebraic book-keeping:

$$\left[\underbrace{E(t+\Delta t)}_{\text{sys}} + \underbrace{E(t+\Delta t)}_{\text{outlet}} \right] - \left[\underbrace{E(t)}_{\text{sys}} + \underbrace{E(t)}_{\text{outlet}} \right] = [Q_{Net,in}(t+\Delta t) - Q_{Net,in}(t)] - \left\{ (W_{out}^{pv}(t+\Delta t) + W_{Net,out}^{other}(t+\Delta t)) - (-W_{in}^{pv}(t) + W_{Net,in}^{other}(t)) \right\}$$

- For Simplicity, I do not want to explicitly write down all the terms for E_{sys} , but I do want to plug in the terms for both $E_{outlet}(t+\Delta t)$ and $E_{inlet}(t)$:

$$\left[E_{\text{sys}}(t+\Delta t) + U_{\text{out}}(t+\Delta t) + \frac{1}{2} m_{\text{out}}(t+\Delta t) V_{\text{out}}^2(t+\Delta t) + m_{\text{out}}(t+\Delta t) g z_{\text{out}}(t+\Delta t) \right]$$

$$- \left[E_{\text{sys}}(t) + U_{\text{in}}(t) + \frac{1}{2} m_{\text{in}}(t) V_{\text{in}}^2(t) + m_{\text{in}}(t) g z_{\text{in}}(t) \right]$$

$$= \left[Q_{\text{Net}, \text{in}}(t+\Delta t) - Q_{\text{Net}, \text{in}}(t) \right] - \left\{ (m_{\text{out}}(t+\Delta t) p_{\text{out}}(t+\Delta t) \psi_{\text{out}}(t+\Delta t) + W_{\text{Net}, \text{out}}^{\text{other}}(t+\Delta t)) \right. \\ \left. - (-m_{\text{in}}(t) p_{\text{in}}(t) \psi_{\text{in}}(t) + W_{\text{Net}, \text{in}}^{\text{other}}(t)) \right\}$$

$$\left[E_{\text{sys}}(t+\Delta t) + U_{\text{outlet}}(t+\Delta t) + \frac{1}{2} m_{\text{outlet}}(t+\Delta t) V_{\text{outlet}}^2(t+\Delta t) + m_{\text{outlet}}(t+\Delta t) g z_{\text{outlet}}(t+\Delta t) \right]$$

$$- \left[E_{\text{sys}}(t) + U_{\text{inlet}}(t) + \frac{1}{2} m_{\text{inlet}}(t) V_{\text{inlet}}^2(t) + m_{\text{inlet}}(t) g z_{\text{inlet}}(t) \right]$$

$$= \left[Q_{\text{Net}, \text{in}}(t+\Delta t) - Q_{\text{Net}, \text{in}}(t) \right] - m_{\text{outlet}}(t+\Delta t) p_{\text{outlet}}(t+\Delta t) \psi_{\text{outlet}}(t+\Delta t) - W_{\text{Net}, \text{out}}^{\text{other}}(t+\Delta t)$$

$$+ m_{\text{inlet}}(t) p_{\text{inlet}}(t) \psi_{\text{inlet}}(t) - W_{\text{Net}, \text{out}}^{\text{other}}(t)$$

Re-arranging terms to the RHS,

$$E_{\text{sys}}(t+\Delta t) - E_{\text{sys}}(t) = \left[Q_{\text{Net}, \text{in}}(t+\Delta t) - Q_{\text{Net}, \text{in}}(t) \right] - \left[W_{\text{Net}, \text{out}}^{\text{other}}(t+\Delta t) - W_{\text{Net}, \text{out}}^{\text{other}}(t) \right]$$

$$+ U_{\text{inlet}}(t) + m_{\text{inlet}}(t) p_{\text{inlet}}(t) \psi_{\text{inlet}}(t) + \frac{1}{2} m_{\text{inlet}}(t) V_{\text{inlet}}^2(t) + m_{\text{inlet}}(t) g z_{\text{inlet}}(t)$$

$$- U_{\text{outlet}}(t+\Delta t) - m_{\text{outlet}}(t+\Delta t) p_{\text{outlet}}(t+\Delta t) \psi_{\text{outlet}}(t+\Delta t) - \frac{1}{2} m_{\text{outlet}}(t+\Delta t) V_{\text{outlet}}^2(t+\Delta t) - m_{\text{outlet}}(t+\Delta t) g z_{\text{outlet}}(t+\Delta t)$$

expressing $U = m u$ and factoring out the masses,

$$\begin{aligned}
 E_{\text{sys}}(t+\Delta t) - E_{\text{sys}}(t) &= [Q_{\text{net},\text{in}}(t+\Delta t) - Q_{\text{net},\text{in}}(t)] - [W_{\text{net},\text{out}}^{\text{other}}(t+\Delta t) - W_{\text{net},\text{out}}^{\text{other}}(t)] \\
 &+ m_{\text{inlet}}(t) \left[u(t) + p(t)v(t) + \frac{1}{2}V^2(t) + gz(t) \right]_{\text{inlet}} \\
 &- m_{\text{outlet}}(t+\Delta t) \left[u(t+\Delta t) + p(t+\Delta t)v(t+\Delta t) + \frac{1}{2}V^2(t+\Delta t) + gz(t+\Delta t) \right]_{\text{outlet}}
 \end{aligned}$$

Recognize that $h \equiv u + pv$ so that

$$\begin{aligned}
 E_{\text{sys}}(t+\Delta t) - E_{\text{sys}}(t) &= [Q_{\text{net},\text{in}}(t+\Delta t) - Q_{\text{net},\text{in}}(t)] - [W_{\text{net},\text{out}}^{\text{other}}(t+\Delta t) - W_{\text{net},\text{out}}^{\text{other}}(t)] \\
 &+ m_{\text{inlet}}(t) \left[h(t) + \frac{1}{2}V^2(t) + gz(t) \right]_{\text{inlet}} \\
 &- m_{\text{outlet}}(t+\Delta t) \left[h(t+\Delta t) + \frac{1}{2}V^2(t+\Delta t) + gz(t+\Delta t) \right]_{\text{outlet}}
 \end{aligned}$$

• Now Divide Both sides by Δt :

$$\begin{aligned}
 \frac{E_{\text{sys}}(t+\Delta t) - E_{\text{sys}}(t)}{\Delta t} &= \frac{[Q_{\text{net},\text{in}}(t+\Delta t) - Q_{\text{net},\text{in}}(t)]}{\Delta t} - \frac{[W_{\text{net},\text{out}}^{\text{other}}(t+\Delta t) - W_{\text{net},\text{out}}^{\text{other}}(t)]}{\Delta t} \\
 &+ \frac{m_{\text{inlet}}(t) \left[h(t) + \frac{1}{2}V^2(t) + gz(t) \right]_{\text{inlet}}}{\Delta t} \\
 &- \frac{m_{\text{outlet}}(t+\Delta t) \left[h(t+\Delta t) + \frac{1}{2}V^2(t+\Delta t) + gz(t+\Delta t) \right]_{\text{outlet}}}{\Delta t}
 \end{aligned}$$

- Now here is a "trick" in order to prepare us for taking the $\lim_{\Delta t \rightarrow 0}$ and getting \dot{m} :

$$(1) \quad m_{\text{inlet}}(t) \neq 0 \quad \text{but} \quad m_{\text{inlet}}(t+\Delta t) \equiv 0$$

$$(2) \quad m_{\text{outlet}}(t+\Delta t) \neq 0 \quad \text{but} \quad m_{\text{outlet}}(t) \equiv 0$$

- We now subtract zero = $m_{\text{inlet}}(t+\Delta t) = m_{\text{outlet}}(t)$ from the respective masses:

$$\begin{aligned} \frac{E_{\text{sys}}(t+\Delta t) - E_{\text{sys}}(t)}{\Delta t} &= \frac{[Q_{\text{Net}, \text{in}}(t+\Delta t) - Q_{\text{Net}, \text{out}}(t)]}{\Delta t} - \frac{[W_{\text{Net}, \text{out}}^{\text{other}}(t+\Delta t) - W_{\text{Net}, \text{out}}^{\text{other}}(t)]}{\Delta t} \\ &\quad + \frac{[m_{\text{inlet}}(t) - m_{\text{inlet}}(t+\Delta t)]}{\Delta t} \left[h(t) + \frac{1}{2}V^2(t) + gZ(t) \right]_{\text{inlet}} \\ &\quad - \frac{[m_{\text{outlet}}(t+\Delta t) - m_{\text{outlet}}(t)]}{\Delta t} \left[h(t+\Delta t) + \frac{1}{2}V^2(t+\Delta t) + gZ(t+\Delta t) \right]_{\text{outlet}} \end{aligned}$$

- Taking the limit as $\Delta t \rightarrow 0$ on both sides:

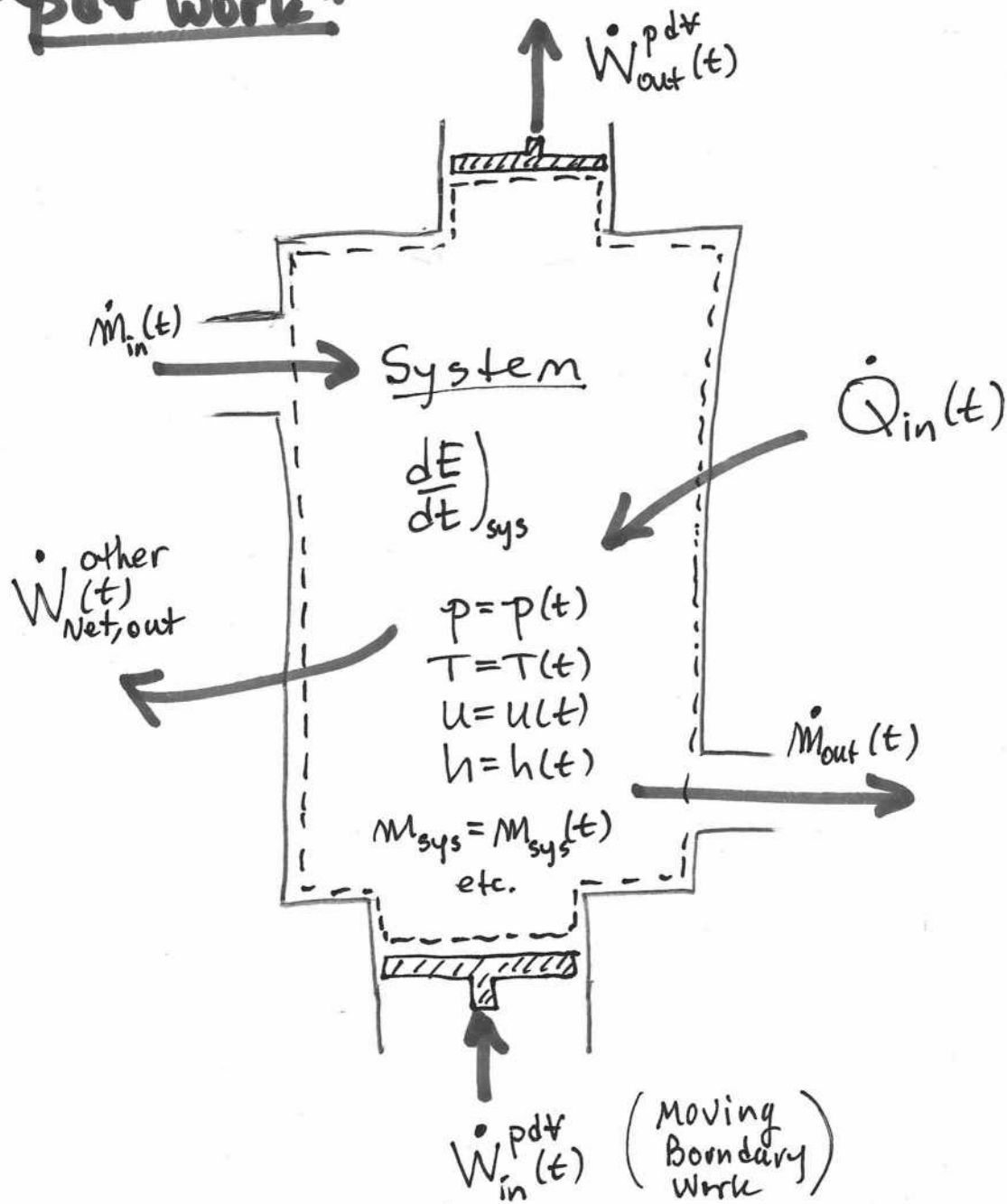
$$\lim_{\Delta t \rightarrow 0} [\text{LHS}] = \lim_{\Delta t \rightarrow 0} [\text{RHS}]$$

$$\boxed{\left(\frac{dE}{dt} \right)_{\text{sys}} = \dot{Q}_{\text{Net}, \text{in}} - \dot{W}_{\text{Net}, \text{out}} + \dot{m}_{\text{in}} \left[h + \frac{1}{2}V^2 + gZ \right]_{\text{in}} - \dot{m}_{\text{out}} \left[h + \frac{1}{2}V^2 + gZ \right]_{\text{out}}}$$

Note that ALL quantities are time dependent

- Now we allow the Control Volume to deform in time: Generalize with "pdv work"

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- With $\dot{W}_{net,other}^{pdv}(t) = \dot{W}_{out}^{pdv}(t) - \dot{W}_{in}^{pdv}(t)$

we will simply add the "pdv" work to $\dot{W}_{net,other}^{other}(t)$

(next page)

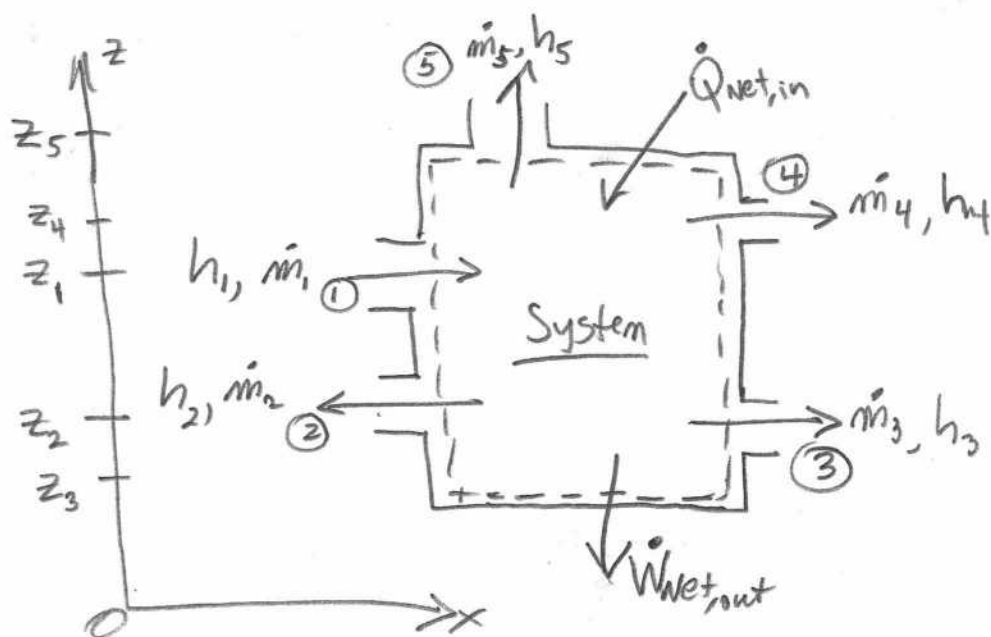
$$\left(\frac{dE}{dt}\right)_{sys} = \dot{Q}_{Net,in} - \left[\dot{W}_{Net,out}^{other} + \underbrace{\dot{W}_{Net,out}^{pd}}_{\substack{\text{"pd"} \\ \text{Boundary} \\ \text{Deformation} \\ \text{Work}}} \right] + \dot{m}_{in} \left[h + \frac{1}{2} V^2 + gz \right]_{in} - \dot{m}_{out} \left[h + \frac{1}{2} V^2 + gz \right]_{out}$$

So

this whole term Now becomes $\dot{W}_{Net,out}$ (kW)

$$\left(\frac{dE}{dt}\right)_{sys} = \dot{Q}_{Net,in} - \dot{W}_{Net,out} + \dot{m}_{in} \left[h + \frac{1}{2} V^2 + gz \right]_{in} - \dot{m}_{out} \left[h + \frac{1}{2} V^2 + gz \right]_{out}$$

• Multiple Inlets/Outlets :



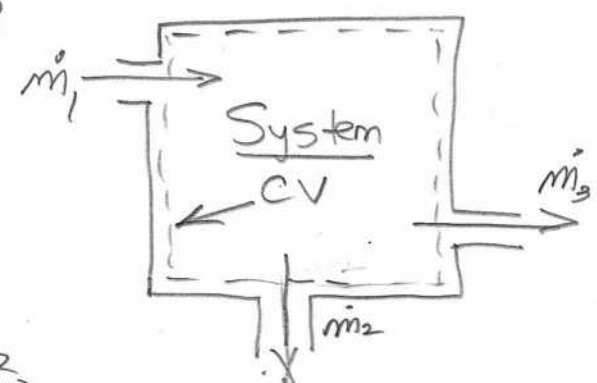
$$\left(\frac{dE}{dt}\right)_{sys} = \dot{Q}_{Net,in} - \dot{W}_{Net,out} + \sum_{in} \dot{m}_{in} \left[h + \frac{1}{2} V^2 + gz \right]_{in} - \sum_{out} \dot{m}_{out} \left[h + \frac{1}{2} V^2 + gz \right]_{out}$$

The Open System Energy Balance Eqn.

Important Info:

(1) What does the $\frac{dE}{dt}_{sys}$ term "look like"?

- This **LHS** of the energy balance represents the time rate of change of properties within the System Control Volume:



- Recall from Lecture 6:

$$E_{sys}(t) = U_{sys}(t) + \frac{1}{2} m_{sys}(t) V_{sys}^2(t) + m_{sys}(t) g z_{sys}(t)$$

taking the time derivative:

$$\frac{dE}{dt}_{sys} = \frac{dU_{sys}}{dt} + \frac{1}{2} \frac{d(m(t)V^2(t))}{dt}_{sys} + g \frac{d(m(t)z(t))}{dt}_{sys}$$

The energy balance equation says that this expression must equal $\dot{E}_{in} - \dot{E}_{out}$ interactions that occur across the System Control Volume Boundary:

$$\underbrace{\frac{dU_{sys}(t)}{dt} + \frac{1}{2} \frac{d(m(t)V^2(t))}{dt}_{sys} + g \frac{d(m(t)z(t))}{dt}_{sys}}_{\frac{dE}{dt}_{sys}} = \underbrace{\dot{Q}_{Net,in} - [\dot{W}_{Net,out}^{other} + \dot{W}_{Net,out}^{pdv}] + \sum_{in} \dot{m}_in [h + \frac{1}{2} V^2 + g z] - \sum_{out} \dot{m}_out [h + \frac{1}{2} V^2 + g z]}_{\text{Interactions with the System}}$$

Time-rate-of-change of total System Energy

(!!)

Solving this Differential Equation is WAY beyond
the scope of our class, and for many systems
of engineering interest, the LHS term $\frac{dE}{dt}_{sys} \equiv 0$
anyway because they operate under Steady Flow
Conditions

⇒ This means that under steady flow,

(1) $\frac{dm}{dt}_{sys} = \sum \dot{m}_{in} - \sum \dot{m}_{out}$

↙ Steady

$\sum \dot{m}_{in} = \sum \dot{m}_{out}$

(2) $\frac{dE}{dt}_{sys} = \dot{E}_{in} - \dot{E}_{out}$

↙

$\dot{E}_{in} - \dot{E}_{out}$

(2) Under Steady Flow Conditions,

$\dot{W}_{Net, out}^{pd\&} = 0$

 ⇒ The C.V. Boundary is Rigid!!

so the only work you need to consider is
that of the "other" — electrical, shaft, a fan,
a pump, etc. Thus, the Energy Balance becomes

$\frac{dE}{dt}_{sys} = \dot{Q}_{Net, in} - \dot{W}_{Net, out}^{other} + \sum \dot{m}_{in} \left[h + \frac{1}{2}V^2 + gz \right]_{in} - \sum \dot{m}_{out} \left[h + \frac{1}{2}V^2 + gz \right]_{out}$

↙ Steady Flow

Lecture 18 Open System Engineering Devices and Analysis Approximations

- Read pages 229 – 242 in your textbook and follow the given Examples.

Open System, Steady-Flow Engineering Devices

- Many important engineering devices operate for long periods of time and the fluid flow through them is constant in time (Steady Flow)
- This means that within the Control Volume, the Open System Energy Balance Equation term on the LHS is

$$\left. \frac{dE}{dt} \right|_{\text{sys C.V.}} = 0$$

- This is a list of devices we will study and the typical approximations used for the Open System Energy Balance Equation

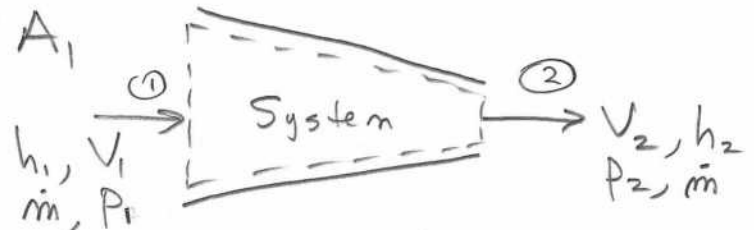
① Nozzles & Diffusers

②

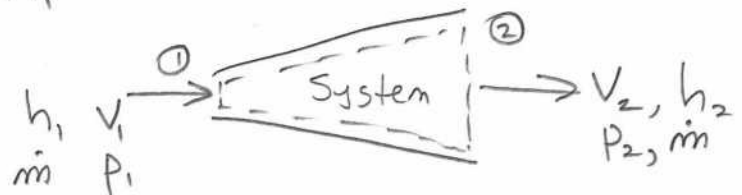
(a) Nozzle A device that increases the velocity of a fluid and lowers the pressure.

- $V_2 \gg V_1$, $P_2 \ll P_1$

- Subsonic case: $A_2 < A_1$



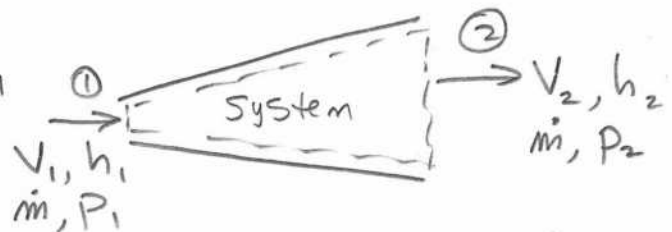
- Supersonic Case: $A_2 > A_1$



(b) Diffuser: A device that decreases the fluid velocity and increases the pressure.

- $V_2 \ll V_1$, $P_2 \gg P_1$

- Subsonic case: $A_2 > A_1$



- Supersonic Case: $A_2 < A_1$



Energy Balance Approximations

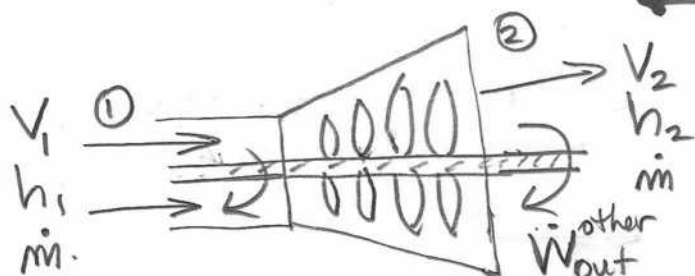
- $\Delta PE = 0$
- $\Delta KE \neq 0$
- $\dot{W}_{\text{net, out}} = 0$

• $\dot{Q}_{\text{net, in}}$ small but not always zero unless specified in the problem

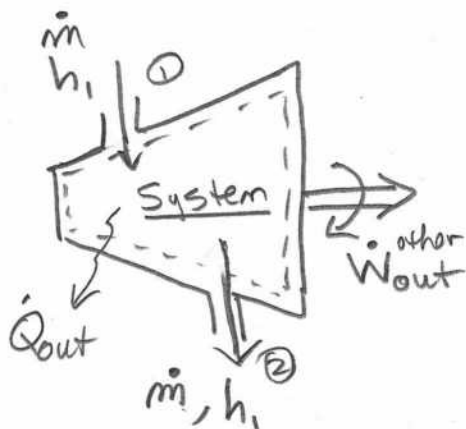
② Turbines, Compressors, Fans, & Pumps

③

(a) Turbines : A device that converts part of the flow enthalpy of a fluid into useful Work Output



See Figure 5-24: The turbine is at the far end of the device near the exit

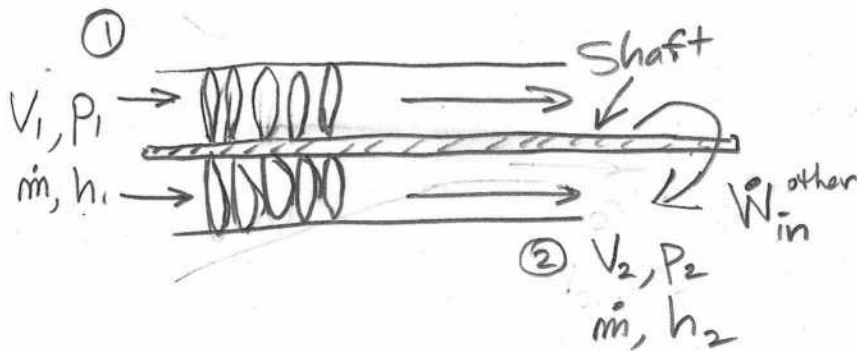


Picture for Analysis

Energy Balance Approximations

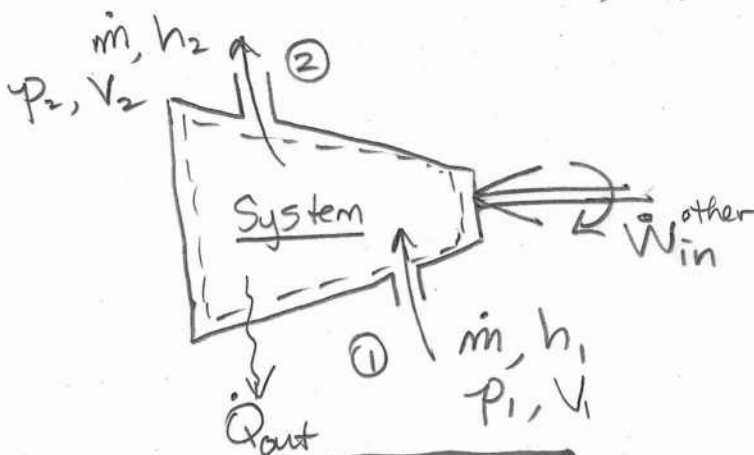
- $\Delta PE = 0$
- $\Delta KE \approx 0$ compared to Δh
- $\dot{W}_{net,out} > 0$
- $\dot{Q}_{out} \approx 0$ • Usually small but not always zero.
 - Many times the problem will say it is Adiabatic, so then $\dot{Q}_{net,in} = 0$
 - Sometimes a turbine is intentionally cooled, then $\dot{Q}_{net,in} \neq 0$. This is stated in the problem.

(b) Compressors : A device that requires Work Input in order to compress a gas to very high pressures.



$$P_2 \gg P_1$$

See Figure 5-24: The Compressor is in the middle section.



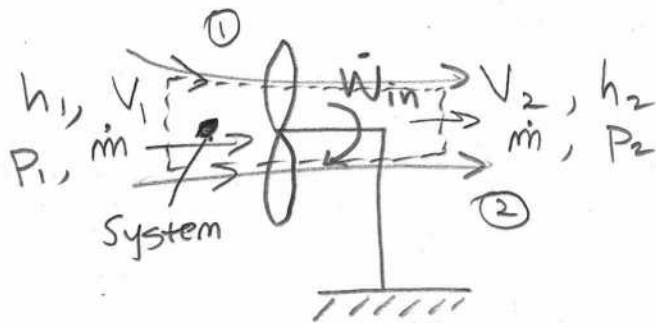
picture for Analysis

Energy Balance Approximations

- $\Delta PE = 0$
- $\Delta KE \approx 0$ compared to Δh
- $\dot{W}_{\text{net}, \text{out}} < 0$
- $\dot{Q}_{\text{out}} \approx 0$
 - Usually small, but not always zero
 - Sometimes a compressor is intentionally cooled, then $\dot{Q}_{\text{net}, \text{in}} \neq 0$. This is stated in the problem.

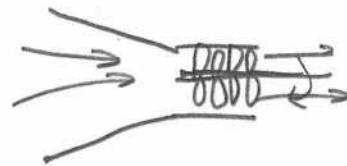
(5)

(c) Fans : A device used to move a gas from one place to another but only increases the pressure slightly.



$$P_2 \approx P_1$$

In Figure 5-24, the fan for that device is located at the front, right after the subsonic diffuser section

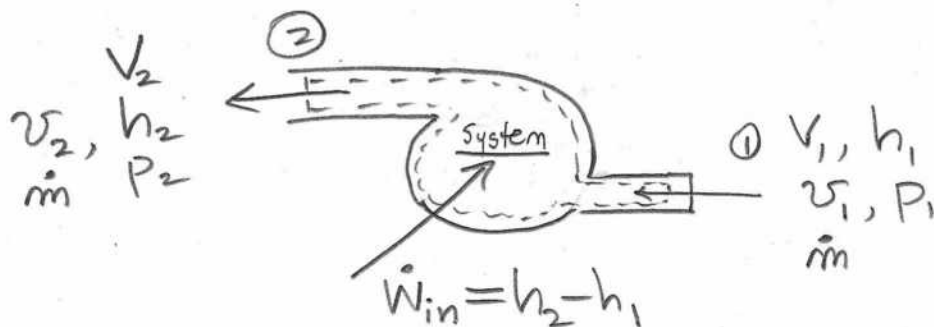


Energy Balance Approximations

- $\dot{Q}_{\text{Net},in} = 0$
- $\Delta PE = 0$
- $\Delta KE = 0$ in comparison to Δh , unless the problem asks for velocities
- $\dot{W}_{\text{Net},out} < 0$

(6)

(d) Pumps : A device that compresses and moves a liquid from one place to another



Energy Balance Approximations

- $\Delta PE = 0$
- $\Delta KE = 0$
- $\dot{W}_{net, out} < 0$
- Usually, $\Delta T = 0$
 - ⇒ we cannot use $\Delta h = c \Delta T$ for this device!
 - ⇒ You must compute the enthalpy change using the compressed liquid approximation

(see Lecture 13)
(and pages 189-190
in textbook)

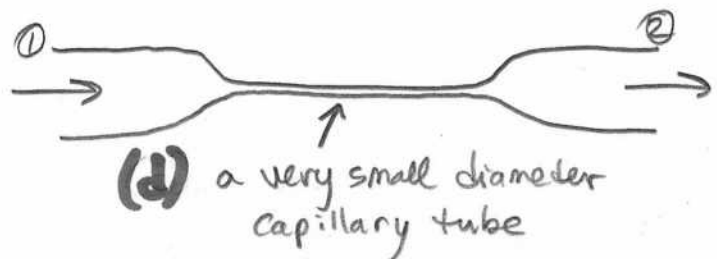
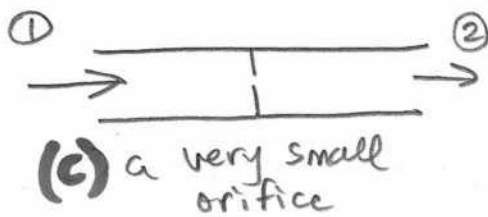
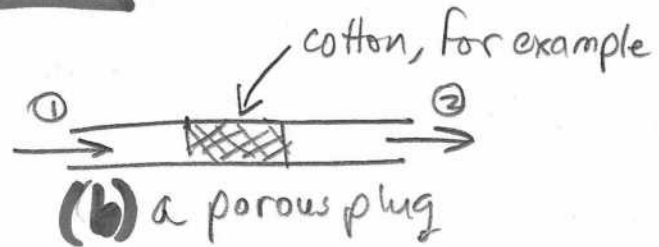
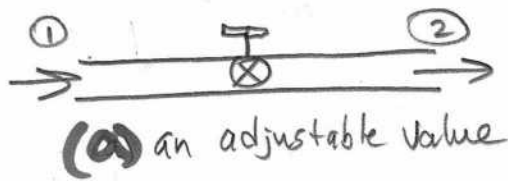
$$h(T, p) \approx h_{f@T} + v_{f@T}(p - p_{sat@T})$$

- the work input is then $\dot{W}_{in} = h_2 - h_1 \approx v_{f@T}(p - p_{sat@T})$
- This is the "pump work approximation"

③ Throttling Valves : A device that restricts the fluid flow in order to cause a significant drop in fluid pressure

⇒ Used in refrigeration and air-conditioning applications

Types of "Throttle Valves"



General pictorial Representation:

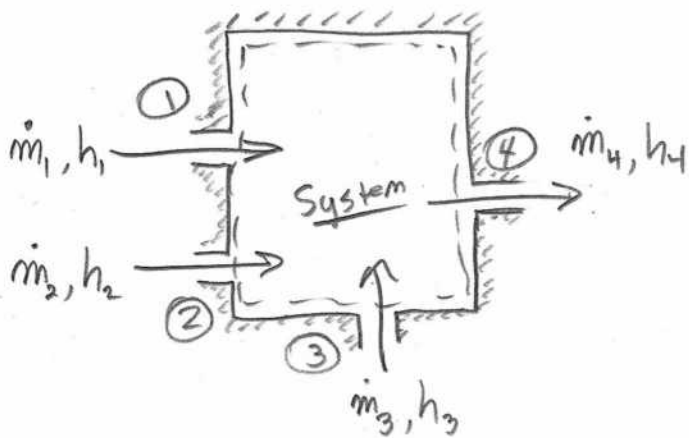


Energy Balance Approximations

- $\Delta P \ll 0$
- $\dot{W}_{Net,out} = 0$
- $\dot{Q}_{Net,in} = 0$
- $\Delta PE = 0$
- $\Delta KE = 0$
- $\Delta T \ll 0$ or maybe > 0
In general
- $\Delta T = 0$ for an Ideal gas
(variable or constant specific heats)

4a) Mixing Chambers ("Direct Contact Heat X-changer")

A device that mixes two or more streams of fluids for a variety of purposes, like the cooling of one fluid by another.

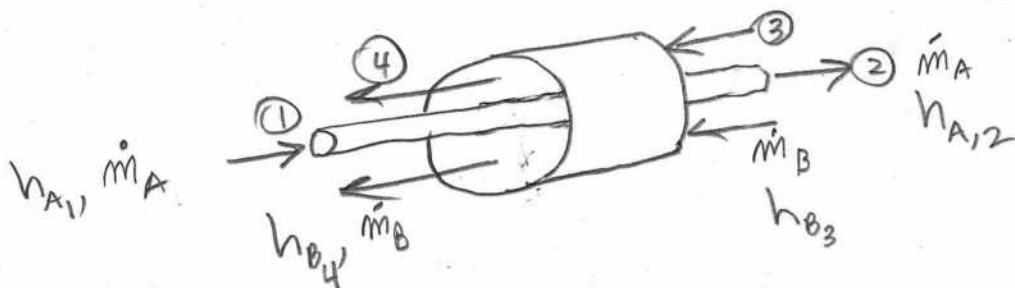


Energy Balance Approximations

- $\dot{Q}_{\text{Net},in} = 0$ (Adiabatic) unless otherwise specified
- $\dot{W}_{\text{Net},out} = 0$
- $\Delta KE = 0$
- $\Delta PE = 0$

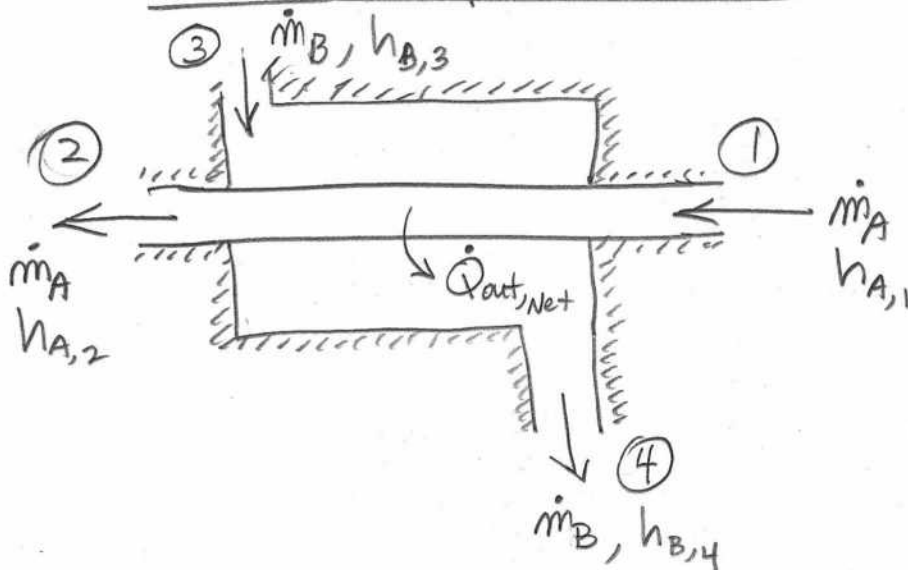
4b) Heat X-changers (Indirect Contact Type)

- A device that allows one moving fluid to lose enthalpy by transferring heat to another moving fluid at a lower temperature.
- Intended for heat transfer between two fluids within the device, with the outer shell usually well insulated
- "Double-tube" or "tube-in-shell" designs:



(9)

Pictorial Representation



Energy Balance Approximations

- $\Delta PE = 0$
 - $\Delta KE = 0$
 - $\dot{W}_{Net,out} = 0$
- } For all Fluid Streams

- $\dot{Q}_{net,in} \rightarrow$ Your answer depends on how you select the System Control Volume for Analysis!

Typical Analysis Strategy

- (1) Select the Entire Device as the System (C.V.) in order to obtain the \dot{m} 's and h 's at each inlet/outlet using both mass & Energy Balances.
- (2) Select one of the fluids as the System (C.V.). Apply the Energy Balance to this C.V. in order to determine the heat transfer rate $\dot{Q}_{net,in}$ for the system.

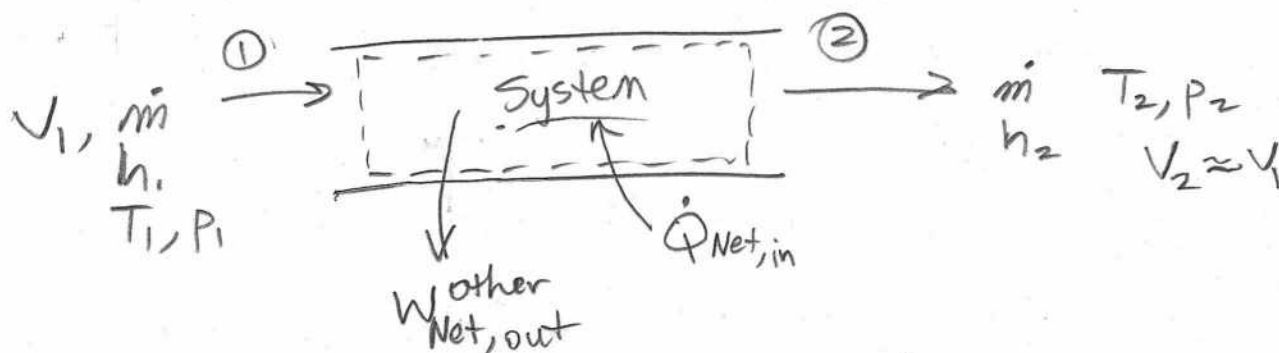
Important Assumption for liquid water inflow/outflows:

- Without any additional information other than the water temperature, we assume that the State is a saturated liquid at $T = T_{\text{sat}}$.
- For open Mixing Chambers (Direct Contact), the pressure is assumed to be constant throughout the device and at the inlets and outlets unless otherwise specified.

⑤ Pipe and Duct Flow

11

A device used to transport liquids, gases, or mixtures of both from one place to another.



Energy Balance Approximations

- $\Delta KE = 0$ if pipe diameter is constant
 $\Delta KE \neq 0$ and comparable to Δh if (1) Diameter changes
(2) \dot{Q} is large
(3) Compressible flow
- $\Delta PE \neq 0$ when there are large changes in elevation between the inlet and outlet, making $\Delta PE \approx \Delta h$
- $\dot{W}_{Net, out}^{other} \neq 0$ if there is a fan, electrical work performed, or a pump inside

Lecture 19: Open System, Steady-State Flow Example Problems

①

Tools :

(1) Conservation of Mass & Continuity Eqn.

$$(a) \left(\frac{dm}{dt} \right)_{sys} = \sum_{in} \dot{m} - \sum_{out} \dot{m} \quad (kg/s)$$

$$(b) \dot{m} = \rho VA = \frac{1}{v} VA = \frac{\dot{V}}{v}$$

where \dot{V} = Volume flow rate (m^3/s)

(2) Open System Energy Balance Eqn.

$$\left(\frac{dE}{dt} \right)_{sys} = \dot{E}_{in} - \dot{E}_{out} \quad (kW)$$

$$\left(\frac{dE}{dt} \right)_{sys} = \dot{Q}_{net,in} - \dot{W}_{net,out} + \sum_{in} \dot{m}_in \left[h + \frac{1}{2} V^2 + gz \right]_{in} - \sum_{out} \dot{m}_{out} \left[h + \frac{1}{2} V^2 + gz \right]_{out}$$

(3) Equation of State

(a) Ideal Gas: $pV = RT$ or Tables

(b) Tables for water or R-134a

(4) Specific Heats:

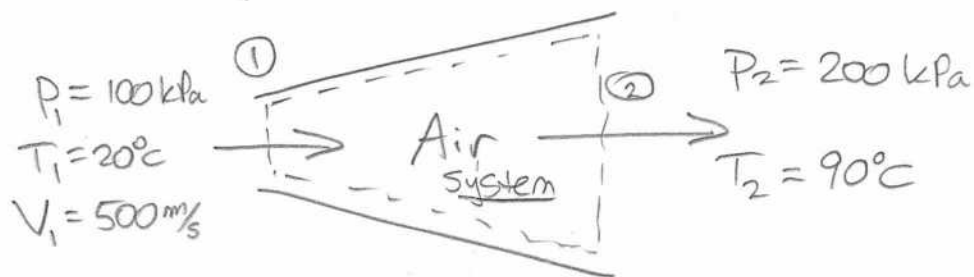
(a) Constant or averaged specific heats: $h = C_p T$
 $\Delta h = C_p \Delta T$

(b) Variable specific heats: Use Tables

Example 1 Diffuser Problem

2

Given: Jet Engine Diffuser, Air.



Find: Velocity of air at exit, V_2

Assume

(1) $\Delta PE_{\text{sys}} = 0$

(2) $\Delta KE_{\text{sys}} \neq 0$

(3) $\dot{W}_{\text{Net, out}} = 0$

(4) $\dot{Q}_{\text{Net, in}} = 0$

(5) Since the air temperatures & pressures are below critical point values \Rightarrow **Ideal gas**

(6) Since ΔT is small, specific heats are constant

(7) Steady Flow

Analysis

Mass Conservation

$$\frac{dm}{dt}_{\text{sys}} = \sum \dot{m}_{\text{in}} - \sum \dot{m}_{\text{out}}$$

Steady flow

$$\sum \dot{m}_{\text{in}} = \sum \dot{m}_{\text{out}}$$

$$\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$$

Open Energy Balance

$$\frac{dE}{dt}_{\text{sys}} = \dot{E}_{\text{in}} - \dot{E}_{\text{out}}$$

$$\frac{dE}{dt}_{\text{sys}} = \dot{Q}_{\text{Net, in}} - \dot{W}_{\text{Net, out}} + \sum \dot{m}_{\text{in}} \left[h + \frac{1}{2} V^2 + g z \right] - \sum \dot{m}_{\text{out}} \left[h + \frac{1}{2} V^2 + g z \right]$$

Steady

$$0 = \sum_{in} \dot{m}_m (h + \frac{1}{2} V^2)_{in} - \sum_{out} \dot{m} (h + \frac{1}{2} V^2)_{out}$$

Since there is only one inlet and outlet,

$$\dot{m}_1 (h_1 + \frac{1}{2} V_1^2) = \dot{m}_2 (h_2 + \frac{1}{2} V_2^2)$$

But from mass conservation, $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$

so $\cancel{\dot{m}} (h_1 + \frac{1}{2} V_1^2) = \cancel{\dot{m}} (h_2 + \frac{1}{2} V_2^2)$

$$h_1 + \frac{1}{2} V_1^2 = h_2 + \frac{1}{2} V_2^2$$

Solving for V_2 ,

$$\frac{1}{2} V_2^2 = \frac{1}{2} V_1^2 + h_1 - h_2$$

$$V_2 = \sqrt{V_1^2 + 2(h_1 - h_2)}$$

Using assumption of Ideal gas with constant c_p ,

$$V_2 = \sqrt{V_1^2 + 2[c_p T_1 - c_p T_2]}$$

$$\boxed{V_2 = \sqrt{V_1^2 + 2c_p [T_1 - T_2]}}$$

We will use the specific heat value of air at an average temperature:

$$T_{avg} = \frac{1}{2} [20^\circ\text{C} + 90^\circ\text{C}]$$

$$T_{avg} = 55^\circ\text{C} \Rightarrow 328\text{K}$$

From Table A-2b, this temperature corresponds to

$$\boxed{c_{p,avg} = 1.007 \frac{\text{kJ}}{\text{kg} \cdot ^\circ\text{C}}}$$

(4)

Thus,

$$V_2 = \sqrt{(500 \text{ m/s})^2 + (2)(1.007 \frac{\text{kJ}}{\text{kg} \cdot ^\circ\text{C}})(20^\circ\text{C} - 90^\circ\text{C}) \times \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)}$$

$$V_2 = 330.2 \text{ m/s}$$

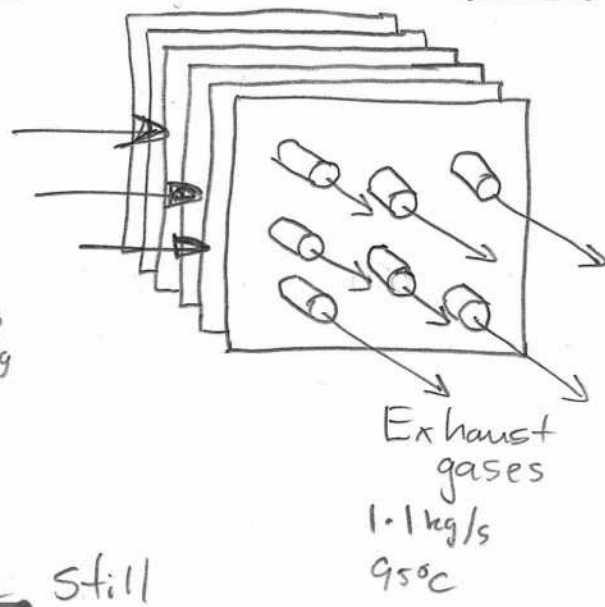
Don't forget
the conversion
factor!!

Example 2 Indirect Contact Heat Exchanger

COMMENTS

(1) DO NOT LET FANCY ARTWORK or FANCY WORDS INTIMIDATE OR MISLEAD YOU!

Air
95 kPa
20°C
0.8 m³/kg

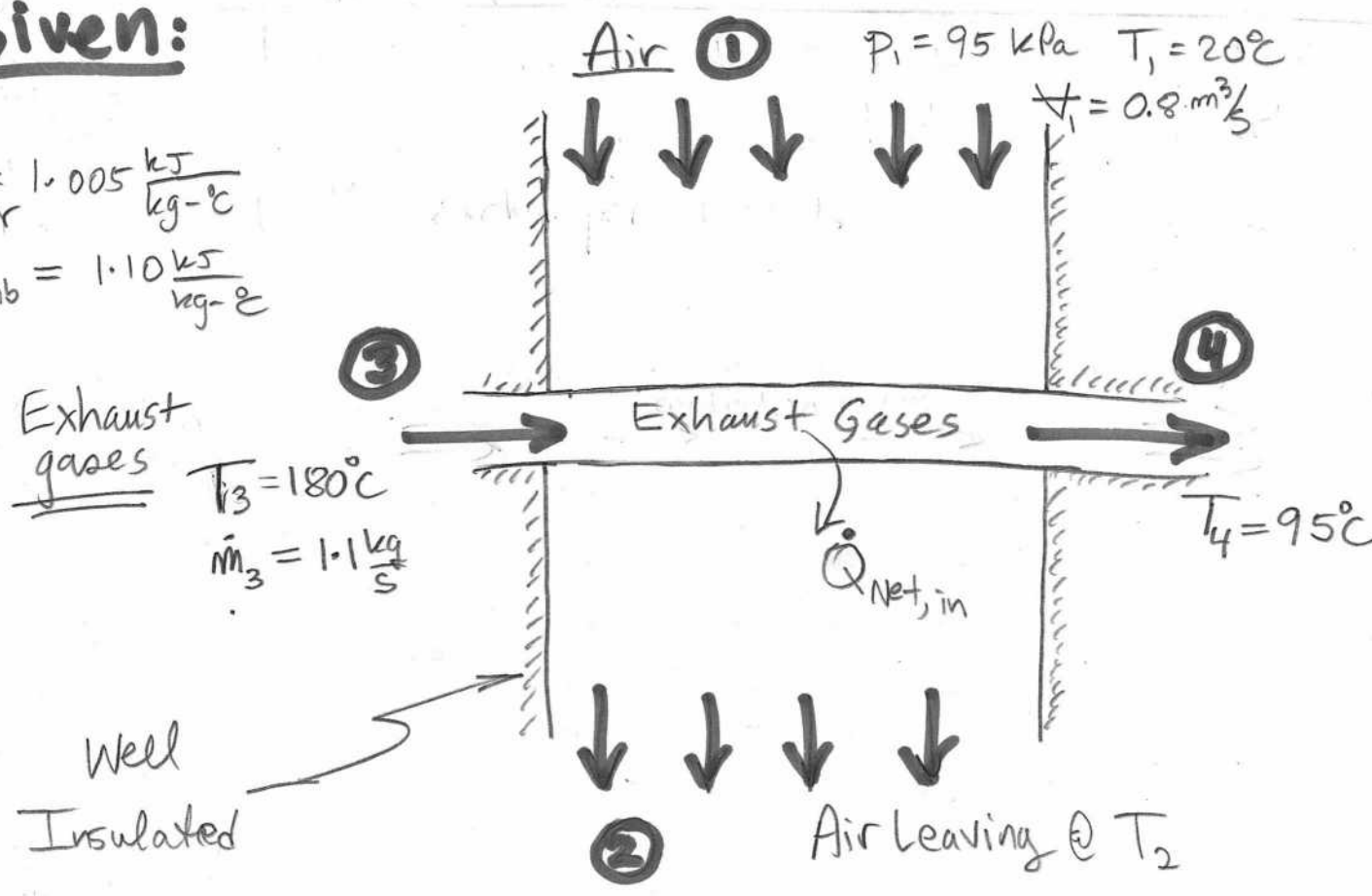


(2) Simplify the picture, while still retaining the important physics, in order to perform the Analysis:

Given:

$$C_{p,air} = 1.005 \frac{kJ}{kg \cdot ^\circ C}$$

$$C_{p,comb} = 1.10 \frac{kJ}{kg \cdot ^\circ C}$$



Assumptions

- (1) Steady Flow
- (2) Constant specific heats \Rightarrow Ideal gas assumption is valid
are given
- (3) $\dot{W}_{\text{net, out}} = 0$
- (4) $\Delta \text{ke} \approx 0$
- (5) $\Delta \text{pe} = 0$
- (6) $\dot{Q}_{\text{net, in}} \Rightarrow$ This depends on the System Control Volume that you choose!

- Find: (a) Rate of heat transfer from the combustion gases to the Air
- (b) Air outlet temperature

Analysis

- (a) As I have stated in Lecture 17B, pg. 9, your choice of the control volume will depend upon the quantity you are seeking to find. Some C.V.'s will not capture the quantity in question!

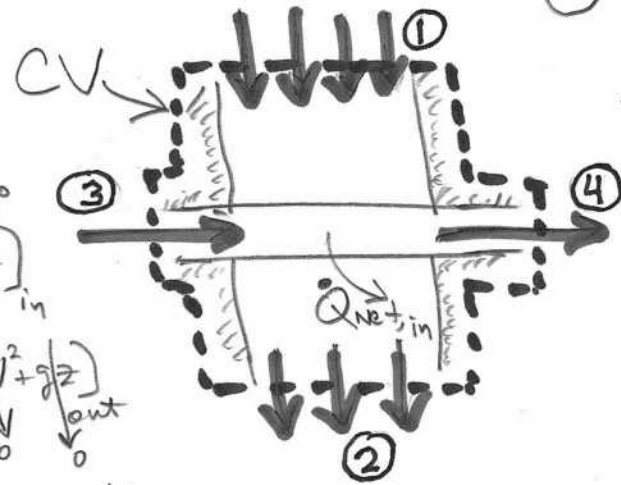
As an illustration, suppose you choose the entire heat exchanger as the System.....

Open System Energy Balance

$$\left(\frac{dE}{dt}\right)_{\text{sys}} = \dot{E}_{\text{in}} - \dot{E}_{\text{out}}$$

$$\frac{dE}{dt}_{\text{sys}} = \dot{Q}_{\text{Net}, \text{in}} - \dot{W}_{\text{Net}, \text{out}} + \sum_{\text{in}} \dot{m}_{\text{in}} \left[h + \frac{1}{2} V^2 + gz \right]_{\text{in}} - \sum_{\text{out}} \dot{m}_{\text{out}} \left[h + \frac{1}{2} V^2 + gz \right]_{\text{out}}$$

$\frac{dE}{dt}_{\text{sys}}$: Steady flow
 $\dot{Q}_{\text{Net}, \text{in}}$: Adiabatic chamber
 $\dot{W}_{\text{Net}, \text{out}}$: No work output by device
 $\Delta KE = 0$ $\Delta PE = 0$



$$0 = \sum_{\text{in}} \dot{m} h - \sum_{\text{out}} \dot{m} h$$

$$0 = \underbrace{\dot{m}_1 h_1}_{\text{air}} + \underbrace{\dot{m}_3 h_3}_{\text{comb. gas}} - \underbrace{\dot{m}_2 h_2}_{\text{air}} - \underbrace{\dot{m}_4 h_4}_{\text{comb. gas}}$$

This is all fine and dandy from a physics viewpoint, but this equation doesn't tell you anything about $\dot{Q}_{\text{Net}, \text{in}}$ Inside between the Exhaust gas pipe and the air!

So we must choose a different control volume that will capture $\dot{Q}_{\text{Net}, \text{in}}$ and we want to choose the CV that has enough given information to solve the problem easily.

⇒ Choose the exhaust gas pipe as the C.V.

The Mass Balance for this C.V. System

is

$$\frac{dm_{sys}}{dt} = \sum \dot{m}_{in} - \sum \dot{m}_{out}$$

Steady flow

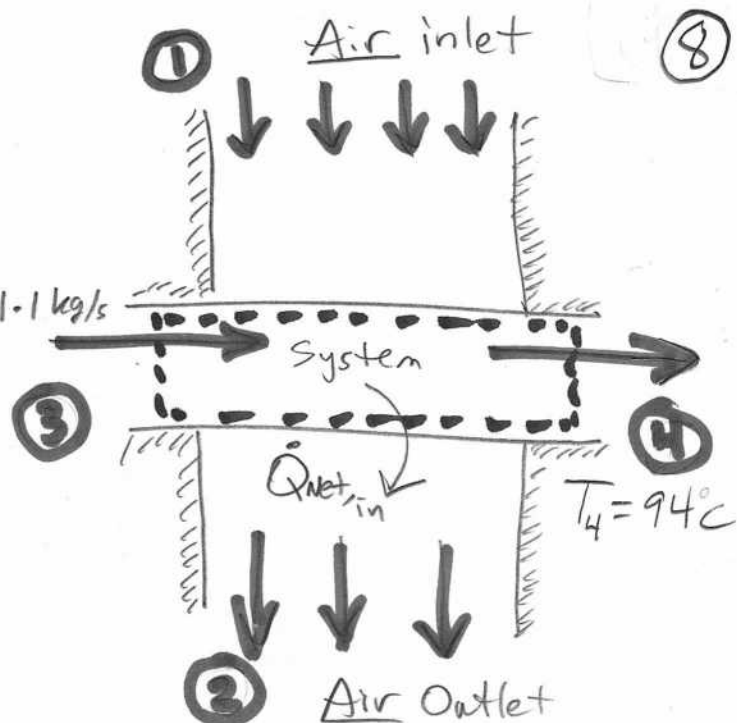
$$\sum \dot{m}_{in} = \sum \dot{m}_{out}$$

$$\dot{m}_3 = \dot{m}_4 \equiv \dot{m}_{comb}$$

$$\dot{m}_3 = 1.1 \text{ kg/s}$$

$$T_3 = 180^\circ\text{C}$$

(comb. gases)



The Energy Balance for this CV is :

$$\frac{dE_{sys}}{dt} = \dot{Q}_{net,in} - \dot{W}_{net,out} + \sum \dot{m}_{in} \left(h + \frac{1}{2}V^2 + gz \right) - \sum \dot{m}_{out} \left(h + \frac{1}{2}V^2 + gz \right)$$

Steady flow No work $\Delta KE = 0$ $\Delta PE = 0$ $\Delta KE = 0$ $\Delta PE = 0$

$$0 = \dot{Q}_{net,in} + \sum \dot{m}_{in} h - \sum \dot{m}_{out} h$$

$$0 = \dot{Q}_{net,in} + \dot{m}_3 h_3 - \dot{m}_4 h_4$$

$$0 = \dot{Q}_{net,in} + \dot{m}_{comb} (h_3 - h_4)$$

Since the gases behave Ideally and $c_{p,comb} = \text{constant}$,

$$0 = \dot{Q}_{net,in} + \dot{m}_{comb} c_{p,comb} (T_3 - T_4)$$

$$(h = c_p T)$$

solving for $\dot{Q}_{net,in}$,

(9)

$$\dot{Q}_{\text{net},in} = \dot{m}_{\text{Comb}} C_{p,\text{Comb}} (T_4 - T_3)$$

Plugging in values,

$$\dot{Q}_{\text{net},in} = \left(1.1 \frac{\text{kg}}{\text{s}}\right) \left(1.0 \frac{\text{kJ}}{\text{kg} \cdot ^\circ\text{C}}\right) (95^\circ\text{C} - 180^\circ\text{C})$$

$$\dot{Q}_{\text{net},in} = -102.85 \text{ kW}$$

⇒ The negative sign means that heat transfer is occurring from the Exhaust gas pipe to the air:

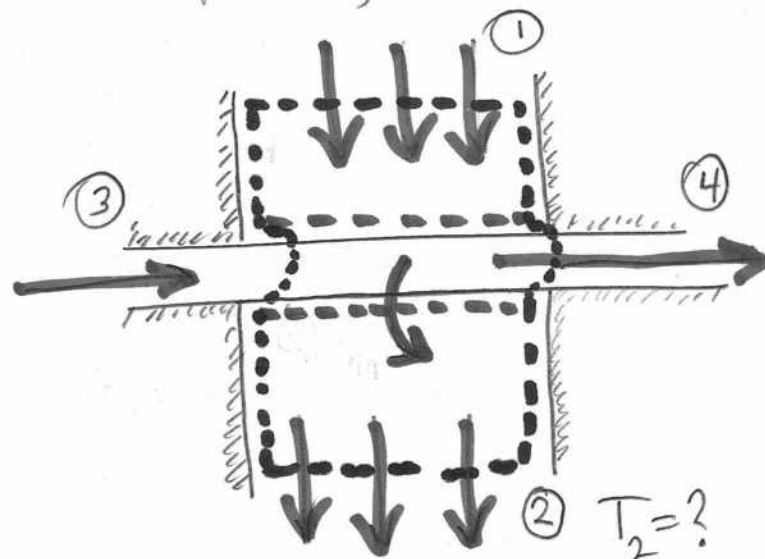
$$\dot{Q}_{\text{net},in} = (\dot{Q}_{\text{in}} - \dot{Q}_{\text{out}})_{\text{pipe}} = -102.85 \text{ kW}$$

$$\dot{Q}_{\text{out},\text{Comb Pipe}} = 102.85 \text{ kW}$$

(b) Find the Air outlet temperature.

In order to find T_2 , we need a CV that includes this physical quantity. So, I must choose:

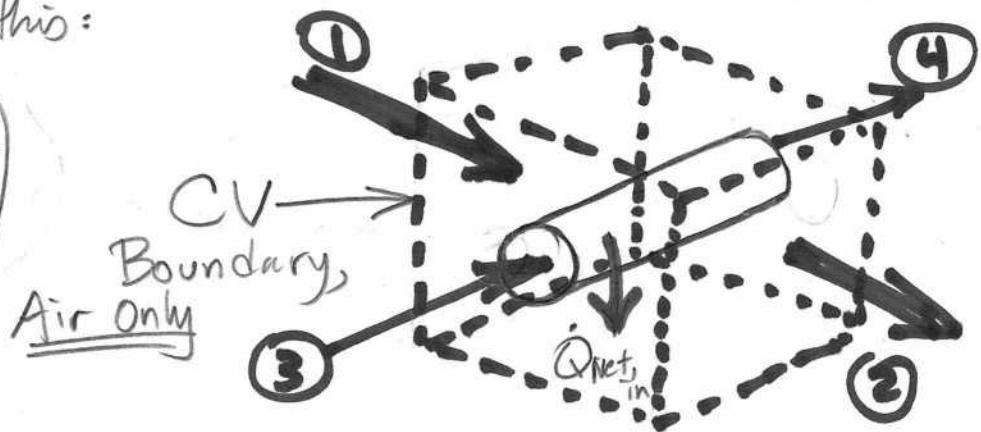
(Note: The Control Volume Boundary surrounds the Exhaust Gas Pipe)



In 3-D, the Control Volume looks something like this:

(10)

The Exhaust pipe cuts through the Control Volume



Mass Balance on CV

$$\dot{m}_1 = \dot{m}_2 \equiv \dot{m}_{\text{air}} \quad (\text{by inspection})$$

Energy Balance on CV

$\frac{dE}{dt}_{\text{sys}}$
 Steady flow

$$= \dot{Q}_{\text{net},in} - \dot{W}_{\text{net},out} + \sum_{in} \dot{m} \left(h + \frac{1}{2} V^2 + g z \right) - \sum_{out} \dot{m} \left(h + \frac{1}{2} V^2 + g z \right)$$

$$0 = \dot{Q}_{\text{net},in} + \dot{m}_1 h_1 - \dot{m}_2 h_2$$

$$0 = \dot{Q}_{\text{net},in} + \dot{m}_{\text{air}} h_1 - \dot{m}_{\text{air}} h_2$$

$$0 = \dot{Q}_{\text{net},in} + \dot{m}_{\text{air}} (h_1 - h_2)$$

Since the air behaves Ideally and $c_{p,\text{air}} = \text{constant}$,
then $h = c_p T$ so

$$0 = \dot{Q}_{\text{net},in} + \dot{m}_{\text{air}} c_{p,\text{air}} (T_1 - T_2)$$

solving for T_2 ,

$$\dot{m}_{\text{air}} c_{p,\text{air}} (T_2 - T_1) = \dot{Q}_{\text{net},\text{in}}$$

$$T_2 - T_1 = \frac{\dot{Q}_{\text{net},\text{in}}}{\dot{m}_{\text{air}} c_{p,\text{air}}}$$

$$T_2 = T_1 + \frac{\dot{Q}_{\text{net},\text{in}}}{\dot{m}_{\text{air}} c_{p,\text{air}}}$$

Now, from the point of view of this C.V.,

$$\dot{Q}_{\text{net},\text{in Air}} = \dot{Q}_{\text{in},\text{air}} - \dot{Q}_{\text{out},\text{air}} = \dot{Q}_{\text{out, Comb gases}} = 102.85 \text{ kW}$$

$$\boxed{\dot{Q}_{\text{in},\text{air}} = +102.85 \text{ kW}}$$

We need \dot{m}_{air} ; so use mass continuity equation:

$$\boxed{\dot{m}_1 = \frac{1}{v_1} \dot{V}_1} \quad \text{where } \dot{V}_1 = 0.8 \text{ m}^3/\text{s}$$

to find v_1 , use the Ideal gas E.O.S.:

$$p_1 v_1 = R_{\text{air}} T_1$$

so

$$\boxed{v_1 = \frac{R_{\text{air}} T_1}{p_1}}$$

then,

$$\boxed{\dot{m}_1 = \frac{p_1 \dot{V}_1}{R_{\text{air}} T_1}}$$

$$\rightarrow 1 \frac{\text{N}}{\text{m}^2} = 1 \frac{\text{J}}{\text{m}^3}$$

$$\Rightarrow \dot{m}_1 = \frac{(95,000 \frac{\text{N}}{\text{m}^2}) (0.8 \text{ m}^3/\text{s})}{(287 \frac{\text{J}}{\text{kg-K}}) (20^\circ\text{C} + 273^\circ\text{C})} = \underline{\underline{0.90378 \frac{\text{kg}}{\text{s}}}}$$

Then

$$T_2 = 20^\circ\text{C} + \frac{102.85 \text{ kW}}{\left(0.90378 \frac{\text{kg}}{\text{s}}\right) \left(1.005 \frac{\text{kJ}}{\text{kg}\cdot^\circ\text{C}}\right)}$$

$$T_2 = 20^\circ\text{C} + 113.2^\circ\text{C}$$

$$\boxed{T_2 = 133.2^\circ\text{C}}$$

Notice that the air temperature increased from 20°C to 133.2°C while the combustion gas temperature decreased from 180°C to 94°C due to 102.85 kW of thermal energy transfer from the comb. gases to the air.

Lecture 20 / Open System Energy Balance Examples

Example 1 / Adiabatic Turbine

Given Steam flows steadily through an Adiabatic Turbine. The given properties at the inlet & outlet are shown in the diagram:

Find: (a) Δke (kJ/kg)
(b) \dot{W}_{out} (MW)
(c) Turbine Inlet Area (m^2)

$p_1 = 10 \text{ MPa}$ $V_1 = 80 \text{ m/s}$ $\dot{m}_1 = 12 \text{ kg/s}$
 $T_1 = 450^\circ\text{C}$

Assumptions

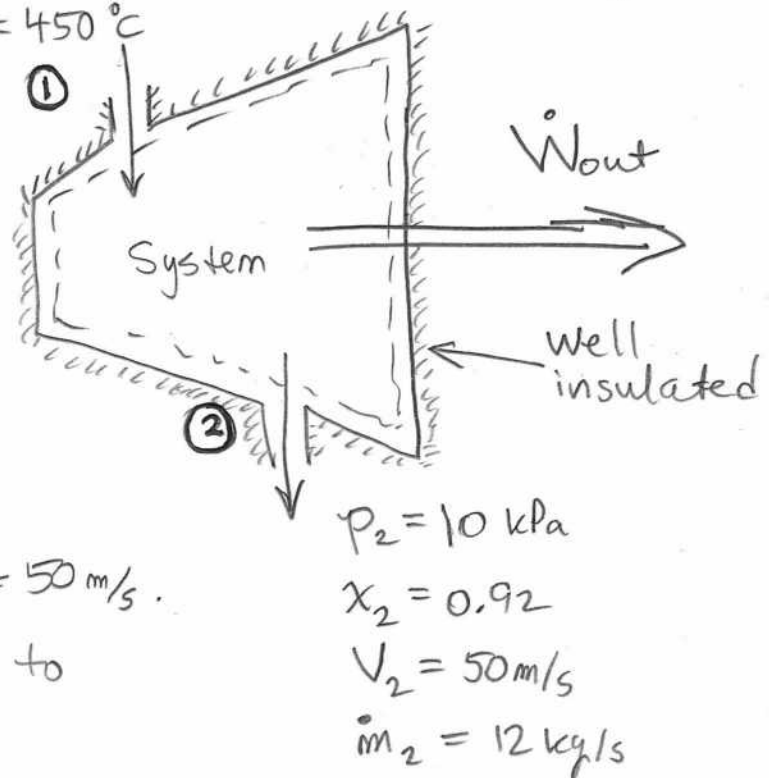
- (1) Steady flow
- (2) Adiabatic $\Rightarrow \dot{Q}_{net, in} = 0$
- (3) $\Delta PE = 0$
- (4) $\dot{W}_{net, out} \neq 0$

Note that $V_1 = 80 \text{ m/s}$ and $V_2 = 50 \text{ m/s}$.

The intent of this problem is to demonstrate to you that

$$\Delta ke \ll \Delta h$$

which leads to the usual assumption that $\Delta ke \approx 0$ for Turbines. In this problem we are to keep the fluid (System) kinetic energy terms within the energy balance equation.



Analysis

- (a) The change in the System's kinetic Energy per unit mass is

$$\Delta ke = \frac{1}{2} (V_2^2 - V_1^2)$$

$$\Delta ke = \frac{1}{2} \left(\left(50 \frac{\text{m}}{\text{s}} \right)^2 - \left(80 \frac{\text{m}}{\text{s}} \right)^2 \right)$$

$$\Delta ke = \frac{1}{2} \left(2500 \frac{\text{m}^2}{\text{s}^2} - 6400 \frac{\text{m}^2}{\text{s}^2} \right)$$

$$\Delta ke = -1950 \frac{\text{m}^2}{\text{s}^2} \times \frac{1 \text{ kg/m}^2}{1000 \frac{\text{m}^2}{\text{s}^2}}$$

Unit Conversion
from textbooks
back cover

$$\Delta ke = -1.95 \frac{\text{kJ}}{\text{kg}}$$

- (b) The turbine's power output can be found using both the Mass and Energy Balance equations:

Mass Balance

$$\frac{dM}{dt}_{\text{sys}} = \sum \dot{m}_{\text{in}} - \sum \dot{m}_{\text{out}}$$

Steady Flow

$$\sum \dot{m}_{\text{in}} = \sum \dot{m}_{\text{out}}$$

$$\dot{m}_1 = \dot{m}_2 = \dot{m} = 12 \text{ kg/s}$$

Open System Energy Balance

$$\frac{dE}{dt}_{\text{sys}} = \dot{E}_{\text{in}} - \dot{E}_{\text{out}}$$

$$\frac{dE}{dt}_{\text{sys}} = \dot{Q}_{\text{net, in}} - \dot{W}_{\text{net, out}} + \sum \dot{m}_{\text{in}} \left(h + \frac{1}{2} V^2 + g z \right) - \sum \dot{m}_{\text{out}} \left(h + \frac{1}{2} V^2 + g z \right)$$

Steady Flow Adiabatic $\Delta PE = 0$ $\Delta PE = 0$

$$0 = -(\dot{W}_{out} - \dot{W}_{in}) + \dot{m}_1 \left(h_1 + \frac{1}{2} V_1^2 \right) - \dot{m}_2 \left(h_2 + \frac{1}{2} V_2^2 \right)$$

But, $\dot{m}_1 = \dot{m}_2 = \dot{m}$

So

$$\dot{W}_{out} = \dot{m} \left[(h_1 - h_2) + \frac{1}{2} (V_1^2 - V_2^2) \right]$$

or,

$$\dot{W}_{out} = -\dot{m} \left[\underbrace{(h_2 - h_1)}_{\Delta h} + \frac{1}{2} \underbrace{(V_2^2 - V_1^2)}_{\Delta ke} \right]$$

$\Delta ke = -1.92 \frac{kJ}{kg}$

We now need to find the Enthalpies at states ① and ②.

State ①

$P_1 = 10 \text{ MPa}$
 $T_1 = 450^\circ\text{C}$
 superheated
 vapor (steam)

From Table A-6

$$h_1 = 3242.4 \frac{kJ}{kg}$$

We know that State ② is a saturated mixture
because we are given $x_2 = 0.92$.

State ②

$P_2 = 10 \text{ kPa}$
 $x_2 = 0.92$

Table A-5

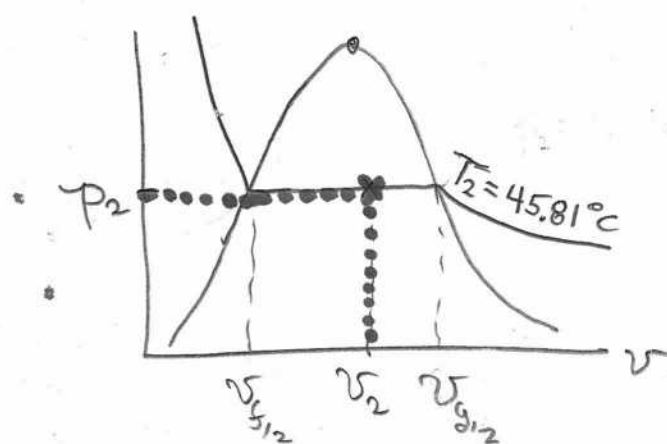
$$h_f = 191.81 \frac{kJ}{kg}$$

$$h_{fg} = 2392.1 \frac{kJ}{kg}$$

So
$$h_2 = h_{f@10kPa} + x_2 h_{fg@10kPa}$$

$$h_2 = 191.81 \frac{kJ}{kg} + (0.92)(2392.1 \frac{kJ}{kg})$$

$$h_2 = 2392.5 \frac{kJ}{kg}$$



(4)

Thus, plugging in all the known quantities,

$$\dot{W}_{out} = -\left(12 \frac{\text{kg}}{\text{s}}\right) \left[\underbrace{(2392.5 - 3242.4) \frac{\text{kJ}}{\text{kg}}}_{\Delta h} + \underbrace{(-1.92 \frac{\text{kJ}}{\text{kg}})}_{\Delta ke} \right]$$

$$\dot{W}_{out} = -\left(12 \frac{\text{kg}}{\text{s}}\right) \left[-849.8 \frac{\text{kJ}}{\text{kg}} - 1.92 \frac{\text{kJ}}{\text{kg}} \right]$$

$$\dot{W}_{out} = \left(12 \frac{\text{kg}}{\text{s}}\right) \left[\underbrace{849.8 \frac{\text{kJ}}{\text{kg}}}_{-\Delta h} + \underbrace{1.92 \frac{\text{kJ}}{\text{kg}}}_{-\Delta ke} \right]$$

Notice that $|h_2 - h_1| \gg |\Delta ke|$ (by a factor ≈ 500)

so yeah, a "quick-n-dirty" analysis of a turbine allows for the assumption that $\Delta ke \approx 0$ compared to Δh .

Finishing the solution,

$$\dot{W}_{out} = 10,220.6 \text{ kW}$$

or

$$\boxed{\dot{W}_{out} = 10.2 \text{ MW}}$$

(c) The turbine inlet area can be found from consideration of the mass flow rate (mass continuity Eqn.)

$$\dot{m} = \frac{1}{v_1} A_1 V_1$$

$$\Rightarrow \boxed{A_1 = \frac{\dot{m} v_1}{V_1}}$$

$$\dot{m} = \text{known} = 12 \text{ kg/s}$$

$$V_1 = \text{known} = 80 \text{ m/s}$$

We need to find v_1

State ①

superheated vapor

$P_1 = 10 \text{ MPa}$

$T_1 = 450^\circ\text{C}$

Table A-6

$v_1 = 0.029782 \text{ m}^3/\text{kg}$

plugging in ,

$$A_1 = \frac{(12 \cancel{\text{kg/s}})(0.029782 \cancel{\text{m}^3/\text{kg}})}{80 \cancel{\text{m/s}}}$$

$$A_1 = 0.00447 \text{ m}^2$$

Example 2 Adiabatic Turbine Using Ideal Gas

⑥

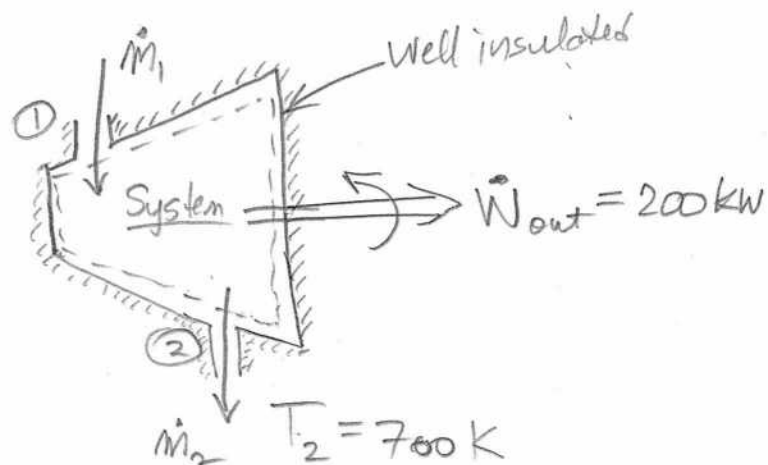
Given: A well insulated turbine operating with some unknown gas. The gas behaves ideally, and the specific heats are given as

$$C_p = 1.13 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}, \quad C_v = 0.83 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

such that $R = 0.30 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$

(recall that $c_p = c_v + R$ for an Ideal gas)

$$P_1 = 600 \text{ kPa}$$
$$T_1 = 1200 \text{ K}$$



Find: The turbine inlet Volume flow rate of the gas in m^3/s required to produce 200 kW of power output.

Assume:

- (1) $\Delta PE_{\text{sys}} = 0$
- (2) $\Delta KE_{\text{sys}} \approx 0$ compared to Δh
- (3) $\dot{Q}_{\text{net, in}} = 0$ (adiabatic)
- (4) $\dot{W}_{\text{net, out}} > 0$
- (5) Ideal Gas EOS is valid
- (6) Constant specific heats given
- (7) Steady Flow

Analysis

We are trying to find the inlet \dot{V} , so using the mass continuity equation,

$$\dot{V}_1 = \dot{m}_1 v_1$$

→ We need to find expressions for both \dot{m}_1 and v_1 since they were not given directly.

Mass Conservation

$$\left(\frac{dm}{dt} \right)_{\text{sys}} = \sum_{\text{in}} \dot{m} - \sum_{\text{out}} \dot{m}$$

steady

$$\sum_{\text{in}} \dot{m} = \sum_{\text{out}} \dot{m}$$

$$\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$$

From our tools listing, we see that there are no further mass conservation relationships we can use to re-express \dot{m} in terms of work and flow parameters — thus, we need to use the Energy balance Eqn.

Open System Energy Balance

$$\left(\frac{dE}{dt} \right)_{\text{sys}} = \dot{E}_{\text{in}} - \dot{E}_{\text{out}}$$

$$\left(\frac{dE}{dt} \right)_{\text{sys}} = \dot{Q}_{\text{Net, in}} - \dot{W}_{\text{Net, out}} + \sum_{\text{in}} \dot{m} \left[h + \frac{1}{2} v^2 + g z \right] - \sum_{\text{out}} \dot{m} \left[h + \frac{1}{2} v^2 + g z \right]$$

Steady Flow Adiabatic $\Delta KE \approx 0$ $\Delta PE = 0$ $\Delta KE \approx 0$ $\Delta PE = 0$

(8)

this leaves us with

$$0 = - \left[\overset{\text{other}}{W_{\text{out}}} - \overset{\text{other}}{W_{\text{in}}} \right] + \dot{m}_1 h_1 - \dot{m}_2 h_2$$

$$0 = -W_{\text{out}} + \dot{m}_1 h_1 - \dot{m}_2 h_2$$

Since $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$,

$$0 = -W_{\text{out}} + \dot{m} h_1 - \dot{m} h_2$$

$$W_{\text{out}} = \dot{m}(h_1 - h_2)$$

Solving for \dot{m} ,

$$\dot{m} = \frac{W_{\text{out}}}{h_1 - h_2}$$

For an Ideal gas having constant specific heats,

$$h_1 - h_2 = c_p(T_1 - T_2)$$

so

$$\dot{m} = \frac{W_{\text{out}}}{c_p(T_1 - T_2)}$$

Now we need to find v_1

Using the Ideal gas E.O.S., $p_1 v_1 = R T_1$,

at state ①:
$$v_1 = \frac{R T_1}{p_1}$$

(9)

then,

$$\dot{V}_1 = \dot{m}_1 v_1$$

$$\dot{V}_1 = \left(\frac{W_{out}}{C_p (T_1 - T_2)} \right) \times \left(\frac{RT_1}{P_1} \right)$$

$$\boxed{\dot{V}_1 = \frac{RT_1 W_{out}}{P_1 C_p (T_1 - T_2)}}$$

Plugging in known quantities

$$\dot{V}_1 = \frac{(0.30 \frac{\text{kJ}}{\text{kg} \cdot \text{K}})(1200 \text{ K})(200 \text{ kW})}{(600 \text{ kPa})(1.13 \frac{\text{kJ}}{\text{kg} \cdot ^\circ\text{C}})(1200 \text{ K} - 700 \text{ K})}$$

$$\dot{V}_1 = \frac{72,000 \frac{\text{kJ}}{\text{kg}} \times \text{kW}}{339,000 \frac{\text{kPa} \cdot \text{kJ}}{\text{kg}}}$$

$$\dot{V}_1 = 0.212 \frac{\text{kW}}{\text{kPa}}$$

$$\dot{V}_1 = 0.212 \times \frac{1000 \text{ J/s}}{1000 \text{ J/m}^3}$$

$$\boxed{\dot{V}_1 = 0.212 \frac{\text{m}^3}{\text{s}}}$$

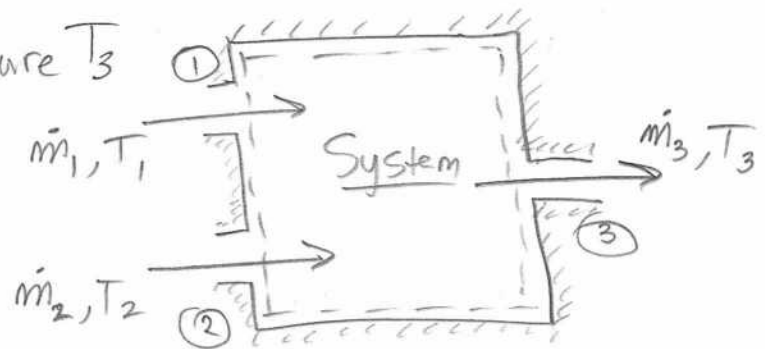
(Remember that)
 $1 \text{ Pa} = 1 \text{ J/m}^3$

Example 3 Direct Contact Heat Exchanger

(10)

Given: Two streams of the same Ideal gas but having two different mass flow rates and temperatures are mixed in a Steady-Flow, Adiabatic Direct-Contact Heat Exchanger. Assume constant specific heats!

Find: Outlet Temperature T_3 in terms of $\dot{m}_1, T_1, \dot{m}_2, T_2$ only.



Assume:

- | | |
|----------------------------------|-----------------------------------|
| (1) $Q_{net,in} = 0$ (Adiabatic) | (4) $\Delta PE_{sys} = 0$ |
| (2) $\dot{W}_{net,out} = 0$ | (5) Steady Flow |
| (3) $\Delta KE_{sys} = 0$ | (6) Ideal gas with constant C_p |

Analysis

Mass Conservation

$\frac{dm_{sys}}{dt} = \sum \dot{m}_{in} - \sum \dot{m}_{out}$

Steady flow $\Rightarrow \sum \dot{m}_{in} = \sum \dot{m}_{out}$

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

Open System Energy Balance:

$$\frac{dE}{dt}_{sys} = \dot{E}_{in} - \dot{E}_{out}$$

(11)

$$\frac{dE}{dt}_{sys} = \dot{Q}_{net,in} - \dot{W}_{net,out} + \sum_{in} \dot{m} \left[h + \frac{1}{2} V^2 + g z \right] - \sum_{out} \dot{m} \left[h + \frac{1}{2} V^2 + g z \right]$$

Steady flow Adiabatic No Work performed on or from device $\Delta KE=0$ $\Delta PE=0$ $\Delta KE=0$ $\Delta PE=0$

$$0 = \sum_{in} \dot{m}_{in} h_{in} - \sum_{out} \dot{m}_{out} h_{out}$$

$$\sum_{in} \dot{m}_{in} h_{in} = \sum_{out} \dot{m}_{out} h_{out}$$

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

substituting for h in terms of temperature, $h = c_p T$,

$$\dot{m}_1 c_p T_1 + \dot{m}_2 c_p T_2 = \dot{m}_3 c_p T_3$$

Solving for T_3 :

$$T_3 = \frac{\dot{m}_1 T_1 + \dot{m}_2 T_2}{\dot{m}_3}$$

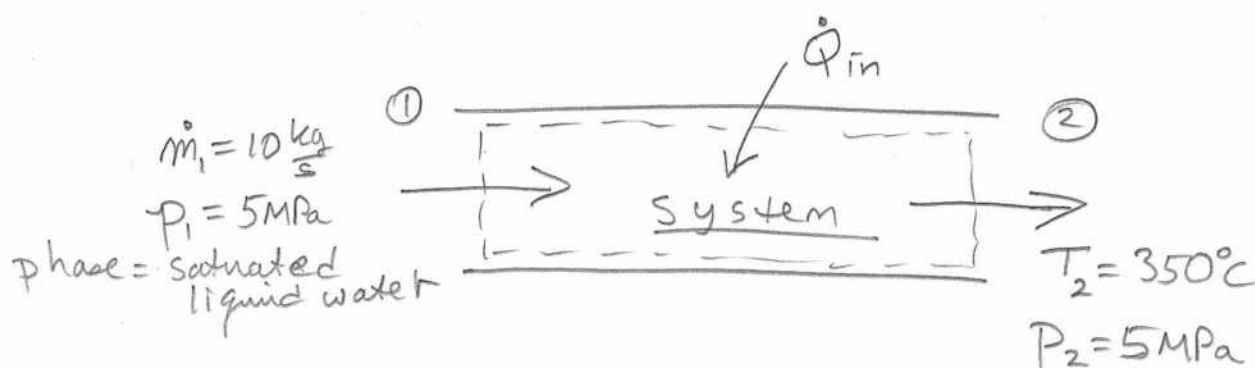
Using the result from Mass Conservation,

$$\dot{m}_3 = \dot{m}_1 + \dot{m}_2,$$

$$T_3 = \frac{\dot{m}_1 T_1 + \dot{m}_2 T_2}{\dot{m}_1 + \dot{m}_2}$$

Example 4 Pipe Flow with Heat Addition

Given: Saturated liquid water is heated in a steady-flow steam boiler at a constant pressure of 5 MPa. The mass flow rate is 10 kg/s. The outlet temperature is 350°C.



Find: \dot{Q}_{in}

Assume:

- (1) Pipe dia. is constant, $\Delta KE = 0$
- (2) $\Delta PE = 0$ since no change in elevation
- (3) $\dot{W}_{net, out} = 0$
- (4) Steady Flow

Analysis

Mass Conservation

$$\frac{dm_{sys}}{dt} = \sum \dot{m}_{in} - \sum \dot{m}_{out}$$

Steady Flow

$$\sum \dot{m}_{in} = \sum \dot{m}_{out}$$

Since there is only one inlet & outlet, $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$

Open System Energy Balance:

$$\left(\frac{dE}{dt} \right)_{\text{sys}} = \dot{E}_{\text{in}} - \dot{E}_{\text{out}}$$

(13)

$$\underbrace{\left(\frac{dE}{dt} \right)_{\text{sys}}}_{\text{Steady Flow}} = \underbrace{\dot{Q}_{\text{net},in}}_{\text{No work interactions}} - \underbrace{\dot{W}_{\text{net},out}}_{\text{No work interactions}} + \underbrace{\sum_{\text{in}} \dot{m} \left[h + \frac{1}{2} V^2 + gz \right]}_{\Delta KE=0} - \underbrace{\sum_{\text{out}} \dot{m} \left[h + \frac{1}{2} V^2 + gz \right]}_{\Delta KE=0}$$

$$0 = \dot{Q}_{\text{net},in} + \dot{m}_1 h_1 - \dot{m}_2 h_2$$

$$0 = [\dot{Q}_{\text{in}} - \dot{Q}_{\text{out}}] + \dot{m}_1 h_1 - \dot{m}_2 h_2$$

$$-\dot{Q}_{\text{in}} = \dot{m}_1 h_1 - \dot{m}_2 h_2$$

$$\dot{Q}_{\text{in}} = \dot{m}_2 h_2 - \dot{m}_1 h_1$$

from mass conservation, $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$, so

$$\boxed{\dot{Q}_{\text{in}} = \dot{m} (h_2 - h_1)}$$

State ①
Saturated water
 $P_1 = 5 \text{ MPa}$

Table A-5 @ 5000 kPa.

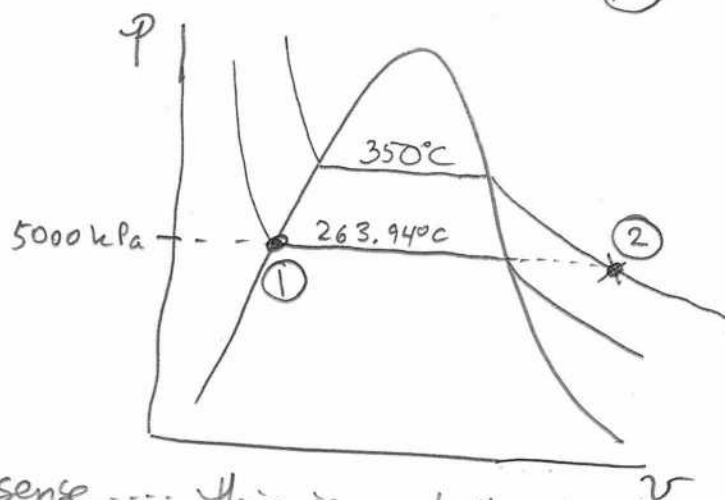
$$T_1 = T_{\text{sat}}_{5000 \text{ kPa}} = 263.94^\circ \text{C}$$

$$h_1 = h_{f@5000 \text{ kPa}} = 1154.5 \frac{\text{kJ}}{\text{kg}}$$

Draw a P-v diagram to help you determine the phase, then the enthalpy at State ②

(14)

Since $T_2 > T_1 = 263.94^\circ\text{C}$,
 you should draw a line
 of constant temperature
above that drawn for
 state ①. \Rightarrow At $p = 5000 \text{ kPa}$,
 state ② must be
superheated vapor \Rightarrow makes sense this is a boiler!



State ②

Superheated vapor

$$T_2 = 350^\circ\text{C}$$

$$P_2 = 5 \text{ MPa}$$

Table A-6

$$h_2 = 3069.3 \frac{\text{kJ}}{\text{kg}}$$

plugging in physical quantities

$$\dot{Q}_{\text{in}} = 10 \frac{\text{kg}}{\text{s}} \left(3069.3 \frac{\text{kJ}}{\text{kg}} - 1154.5 \frac{\text{kJ}}{\text{kg}} \right)$$

$$\dot{Q}_{\text{in}} = 19,150 \text{ kW}$$

or

$$\boxed{\dot{Q}_{\text{in}} = 19.15 \text{ MW}}$$

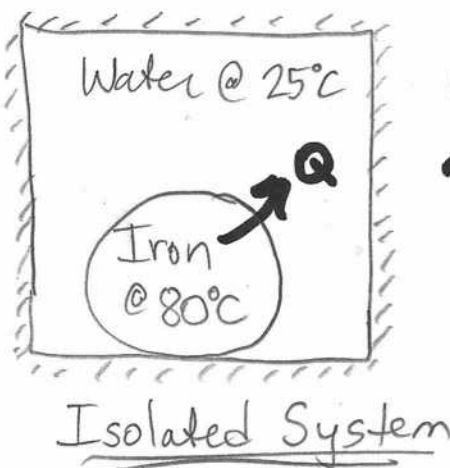
Lecture 21

Introduction to The 2nd Law of Thermodynamics

Q: Why do we need a 2nd Law of Thermo?

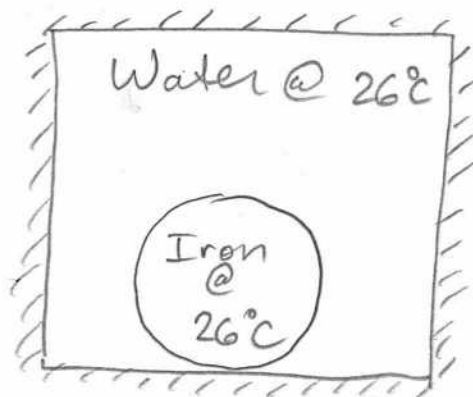
A: Consider a hot iron block dropped into a well insulated, rigid tank of cool water. Using the 1st Law of Thermodynamics (Conservation of Energy), the final temperature of both the water and iron block in Thermodynamic Equilibrium can be computed:

State ①, $t=0$



Non-equilibrium
process

State ②, $t \gg 0$



- The Closed System Energy Balance gives us $T_2 = 26^\circ\text{C}$ for the final equilibrium temperature of both iron and water.
- Now consider this "crazy", non-intuitive process:

State ①, $t=0$



Non-equilibrium
process

State ②, $t \gg 0$



(2)

Notice that the Iron Block's temperature
Actually increased from 80°C to 173°C but the
water's went from 25°C to 24°C !!

This does not violate the 1st Law of Thermo!

Closed System Energy Balance:

$$m_i c_i \Delta T_i = -m_w c_w \Delta T_w$$

$$T_{i,2} = T_{i,1} - \frac{m_w c_w}{m_i c_i} (T_{w,2} - T_{w,1})$$

$$T_{i,2} \approx T_{i,1} - 92.9 (T_{w,2} - T_{w,1})$$

State 1 $T_{i,1} = 80^{\circ}\text{C}$, $T_{w,1} = 25^{\circ}\text{C}$

State 2 $T_{w,2} = 24^{\circ}\text{C}$

then $T_{i,2} \approx 80^{\circ}\text{C} - 92.9 (24^{\circ}\text{C} - 25^{\circ}\text{C})$

$$T_{i,2} \approx 80^{\circ}\text{C} + 92.9^{\circ}\text{C}$$

$$\underline{T_{i,2} \approx 173^{\circ}\text{C}}$$

Crazy!!

This is not what we
observe in the real world, even though the
1st Law, Energy Conservation, says it is "true".

⇒ An additional Law must be added so that
this non-physical process is excluded from
the possible outcomes that the 1st Law Allows.

The 2nd Law of Thermodynamics

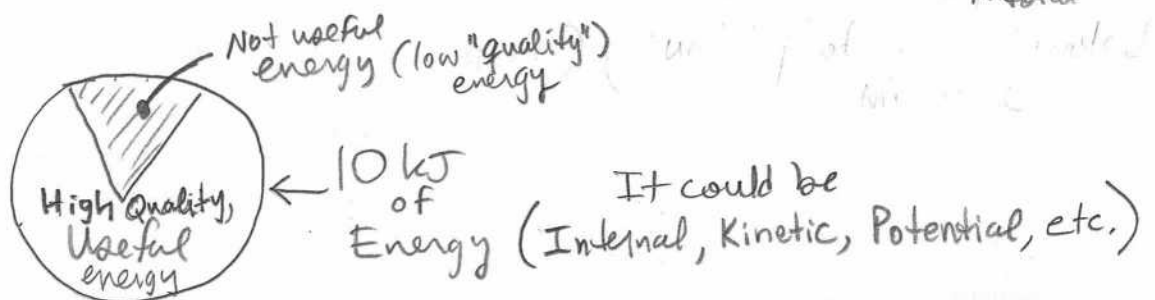
(3)

It is a principle that prohibits or restrains certain processes allowed by The 1st Law of Thermodynamics.

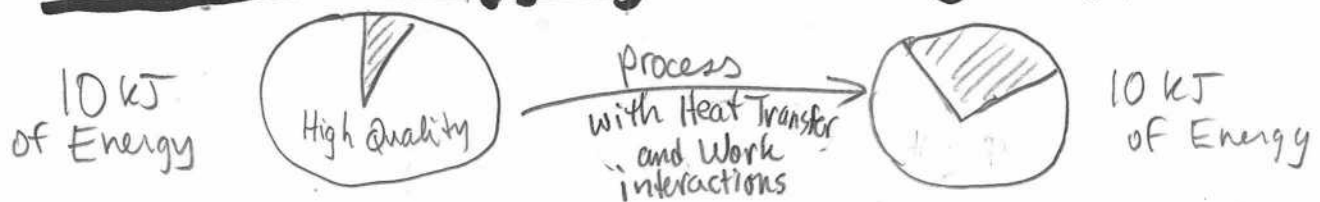
- (1) The 2nd Law does not allow **Spontaneous** heat transfer (i.e., on it's own without external interactions) from a Cold body to a Hotter Body when they comprise an **Isolated System**.

The 2nd Law is based on experimental observation over the past 10,000 years of humanity, plus astronomical observation (13 Billion years), and has NEVER been disproved.

- (2) The 2nd Law asserts that Energy has a "Quality" (or "usefulness") as well as a Quantity (or magnitude). (This "quality" is NOT the same as $x = \frac{m_{\text{vapor}}}{m_{\text{total}}}$)



It also provides tools to determine how much the usefulness of energy degrades during a process:



4

(3) The 2nd Law is used to determine theoretical upper limits on the efficiency of heat engines and refrigerators ($< 100\%$ Always)

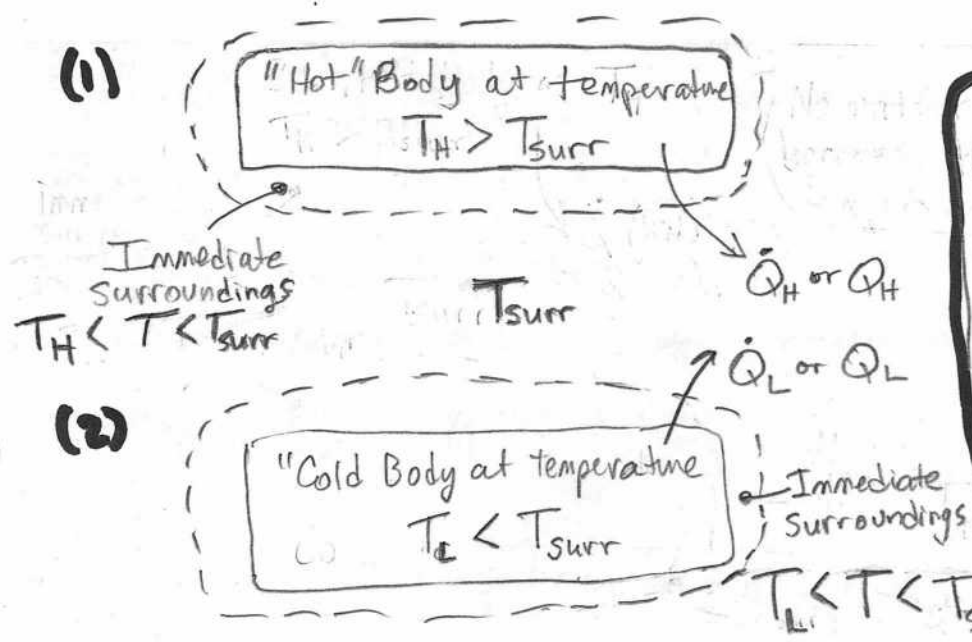
(4) The 2nd Law is used to predict whether a chemical reaction or phase change is possible and if so, the extent of its completion.

Thermal Energy Reservoir

A fictional body having a relatively large thermal energy capacity (mc_v), such that it can absorb finite amounts of heat from another body without undergoing any change in temperature.

Note: A thermal reservoir can be "large" or "small", depending upon the situation you are analyzing.

⇒ It is a physical body whose thermal capacity is large relative to the amount of energy it supplies or absorbs, so that its temperature never changes



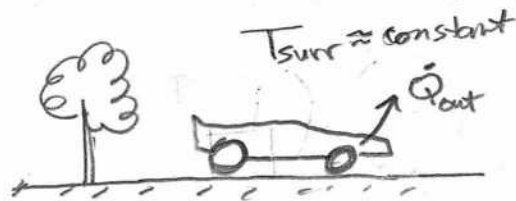
Note that I have drawn spontaneous heat transfer processes here
 ⇒ I have invoked the 2nd Law of Thermo by defining the direction of spontaneous heat transfer!

Real World Examples of a Thermal Reservoir Approximation

(6)

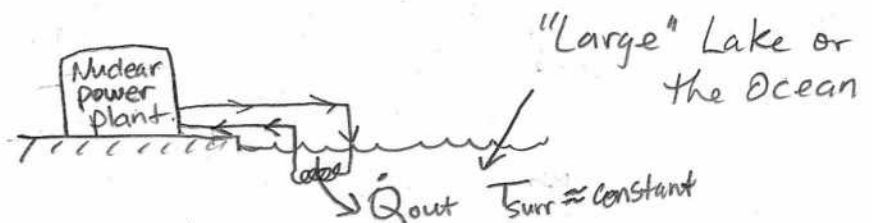
(1) The Atmosphere

- The atmosphere is an approximate thermal reservoir only when its local temperature does not change due to heat transfer to or from an engineering device.

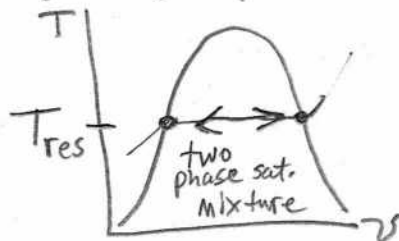


(2) A Body of Water

- Lake or Ocean is an approximate thermal reservoir



- ## (3) A two phase System can be a thermal reservoir by absorbing and releasing large quantities of heat isothermally



(4) An Industrial Furnace

(5) A speck of dust if you are analyzing a micro-machine

⇒ What you consider to be a thermal reservoir is an engineering judgement based on the magnitude of \dot{Q} or Q relative to surroundings

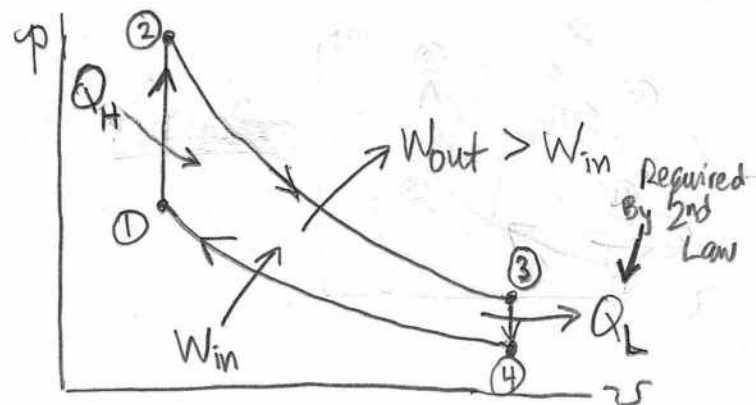
Heat Engine (Work Producing)

A thermodynamic System that operates with a cyclical process in which heat is transferred from a high temperature reservoir to a low one, and net work is delivered

Every Heat Engine (H.E.)

- (1) Receives thermal energy Q_H from a thermal reservoir at a temperature, T_H , higher than the H.E. itself.
- (2) A H.E. converts part of the received thermal energy, Q_H , into net work output $W_{\text{net, out}}$.
- (3) Every H.E. must reject the remaining waste heat, Q_L (of lower usefulness than Q_H), to a thermal reservoir at temperature T_L which is lower than the H.E. itself.

- (4) Every H.E. operates on a Thermodynamic Cycle



Basic Idea of Cyclical Energy Conversion

Burn Fuel(\$\$)

T_H

Q_H

Transfer Thermal Energy (Heat) to some substance (increase enthalpy) a Lot!!

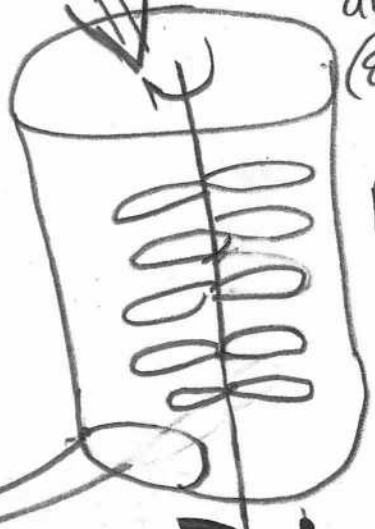
$T_H - \Delta T$
Substance

Use as much of the Heat absorbed (Enthalpy) to perform Work (turbine)

Prepare Substance to be heated again (example: increase pressure a lot, but not a large enthalpy increase)



Win



Wout

Substance $T_L + \Delta T$

Substance now has low Enthalpy ($u + p v$)

⇒ Waste Energy of lowered usefulness

Q_L

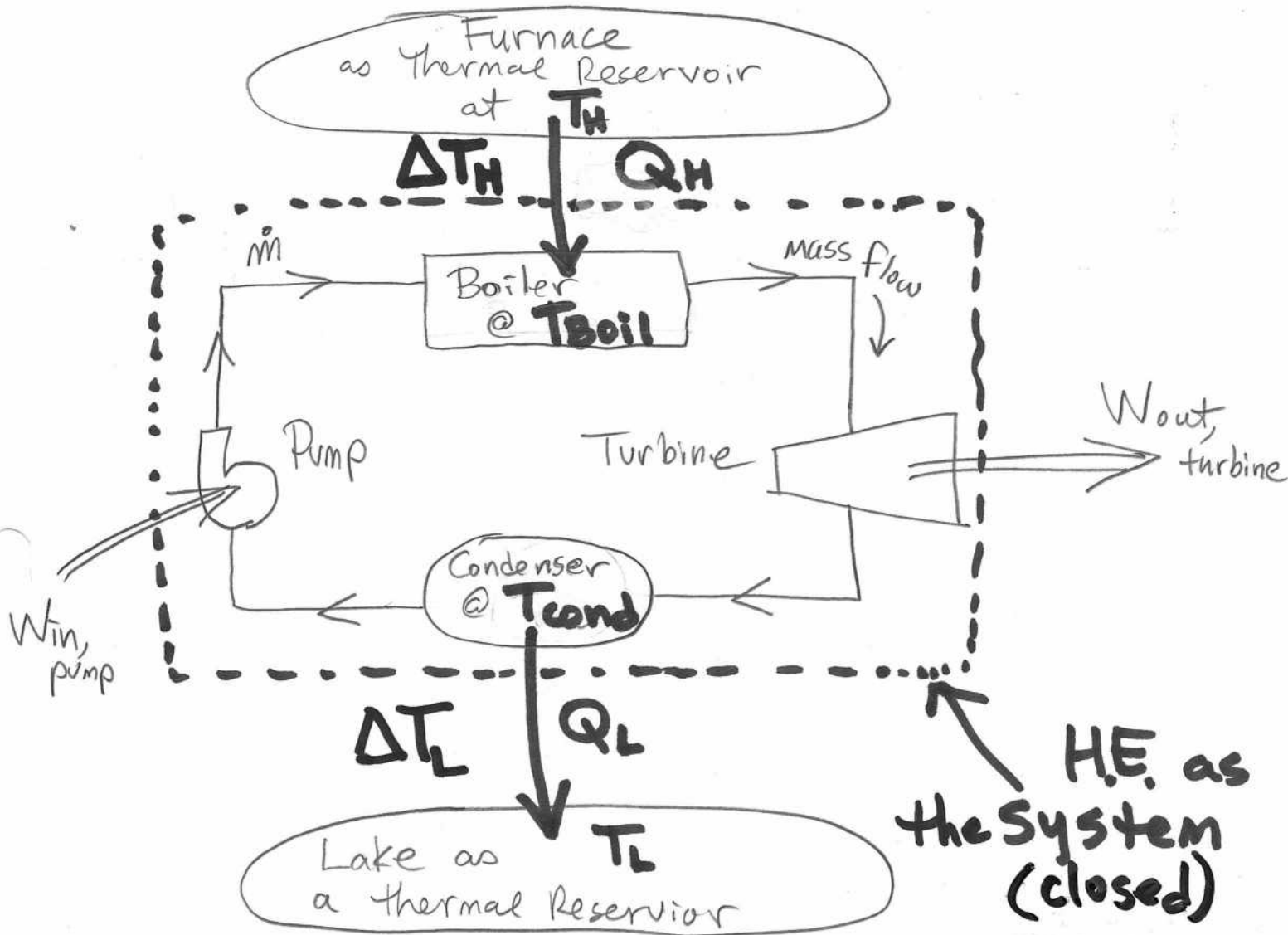
T_L Ohio River

Electricity

For this to be a cycle,
The 2nd Law
States that we MUST reject the Waste Energy to a Cold Reservoir outside of the System

Example Details of a Heat Engine: A Steam Power Plant

Working Fluid = H_2O in form of liquid, vapor, and both (saturated mixture)



Important!! Notice:

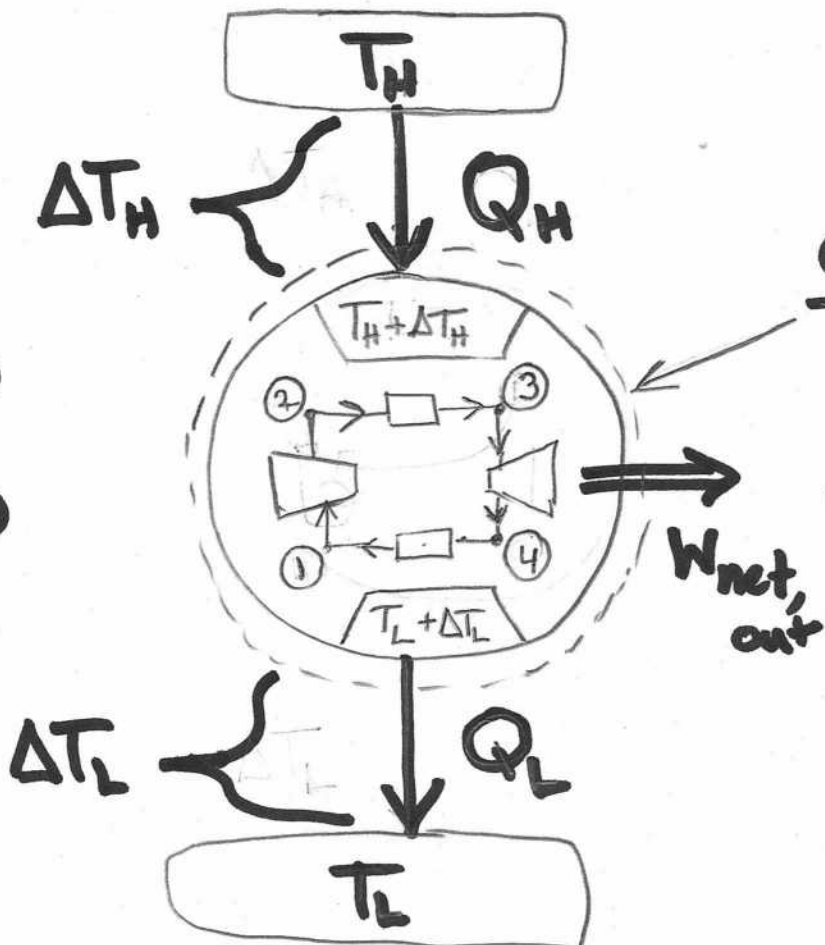
$$(1) \Delta T_H \equiv T_H - T_{Boil} > 0$$

$$(2) \Delta T_L \equiv T_{cond} - T_L > 0$$

The 2nd Law Requires this!!

Model Picture of a Generic Heat Engine

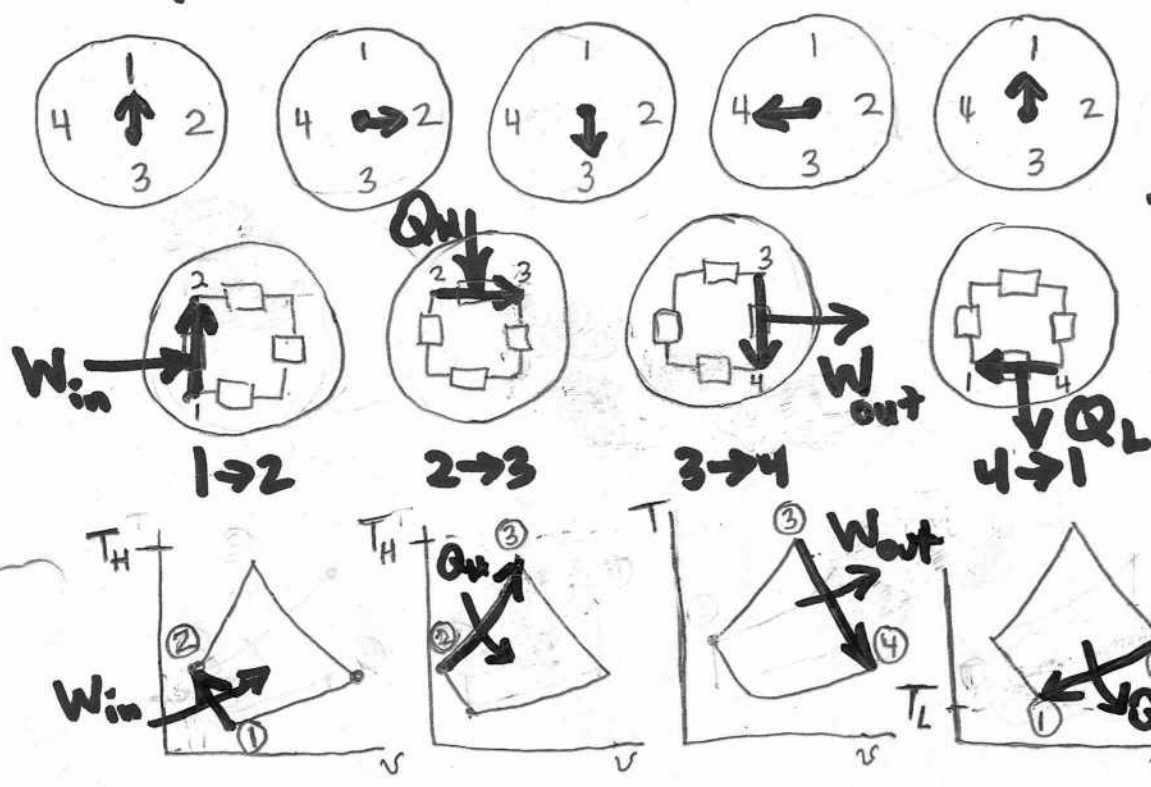
$\Delta T_H > 0$
 $\Delta T_L > 0$



Control Volume

- Closed if no mass transfer with Q_H, Q_L
- Open if there is mass transfer with Q_H, Q_L (if the problem requires mass transfer to be taken into account)

• A cycle envisioned as "clockwork"



time t →

**The 2nd Law
 REQUIRES 4→1
 HEAT REJECT
 to complete the
 cycle!**

*T-v, p-v, and
 T-s diagrams
 represent the
 System Only!*

Efficiency

$$\text{Efficiency} \equiv \frac{\text{Desired Output}}{\text{Required Input}}$$

(What you get for what you are required to pay (\$))

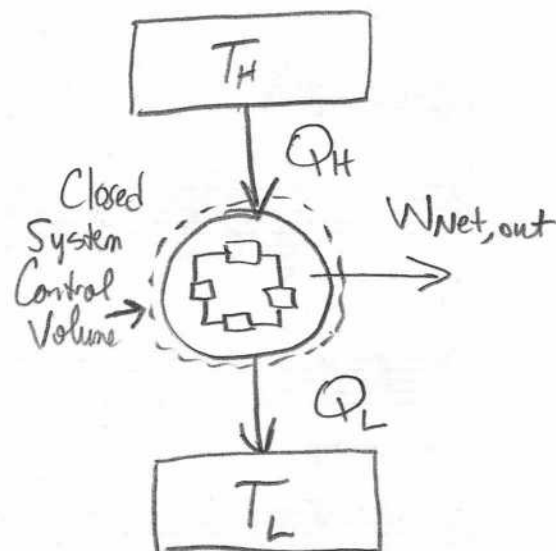
Heat Engine Thermal Efficiency

By definition, thermal efficiency is

$$\eta_{th} \equiv \frac{\text{Desired Output}}{\text{Required Input}} = \frac{W_{net,out}}{Q_H}$$

- Taking the H.E. as a closed system and using the 1st Law Closed System Energy Balance Equation,

$$\Delta E_{sys} = Q_{net,in} - W_{net,out}$$



- Since the H.E. operates on a cycle,

$$\cancel{\Delta E}_{sys} = Q_{net,in} - W_{net,out}$$

for one period: $(1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1)$.



$$W_{net,out} = Q_{net,in}$$

Very important to remember!

Forms of Energy Balance for cycle:

- (1) "Change" Form: $W_{\text{net,out}} = Q_{\text{net,in}}$ (kJ or Btu)
- (2) "Rate" Form: $\dot{W}_{\text{net,out}} = \dot{Q}_{\text{net,in}}$ (kW or hp)
- (3) Per unit mass: $w_{\text{net,out}} = q_{\text{net,in}}$ ($\frac{\text{kJ}}{\text{kg}}$ or $\frac{\text{Btu}}{\text{lbm}}$)
 $\dot{w}_{\text{net,out}} = \dot{q}_{\text{net,in}}$ ($\frac{\text{kW}}{\text{kg}}$ or hp/lbm or $\frac{\text{Btu}}{\text{lbm-s}}$ etc.)

Thermal Efficiency for H.E.

- We denote $Q_{\text{in}} \equiv Q_H$ from the High temperature reservoir
 $Q_{\text{out}} \equiv Q_L$ from the Low temperature reservoir

- From $W_{\text{net,out}} = Q_{\text{net,in}}$ for a cycle, we can express $W_{\text{net,out}}$ as

$$W_{\text{net,out}} = Q_H - Q_L$$

- Substituting this into the definition of efficiency,

$$\eta_{\text{th}} = \frac{W_{\text{net,out}}}{Q_H} = \frac{Q_H - Q_L}{Q_H} = \frac{Q_H}{Q_H} - \frac{Q_L}{Q_H}$$

$$\eta_{\text{th}} = 1 - \frac{Q_L}{Q_H}$$

Thermal Efficiency
for an actual H.E.

$$\eta_{\text{th}} < 100\%$$

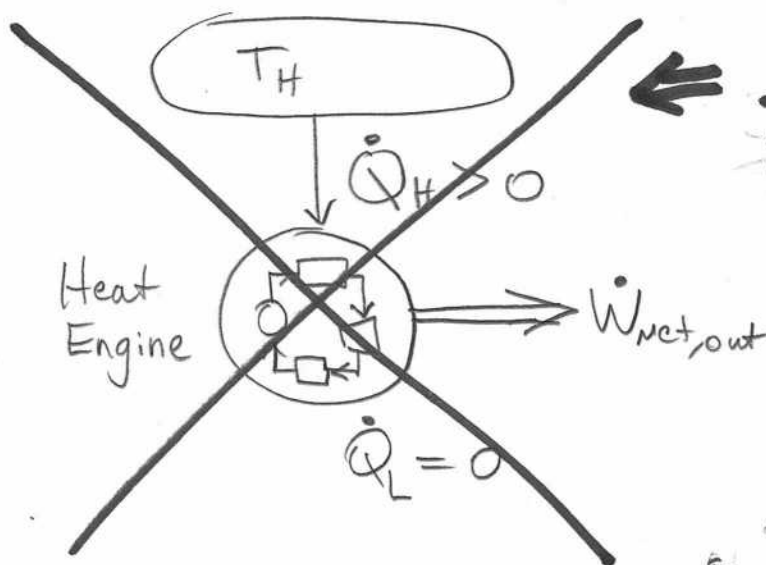
because $Q_L < Q_H$
required by 2nd Law

- Q_L = Rejected waste Heat
of Degraded usefulness relative to Q_H
(thermal poop!)

2nd Law of Thermo for Heat Engines

Kelvin-Planck Statement

It is impossible for any device that operates on a cycle to receive heat from a single reservoir and produce a net amount of work



This is Impossible!

because

$$\dot{Q}_L = 0$$

which means

$$\eta_{th} = 1 - \frac{0}{\dot{Q}_H}$$

so that η_{th} would be 100% efficient

Reality: $\eta_{th} < 100\%$
 $\dot{Q}_L \neq 0$

Plain English

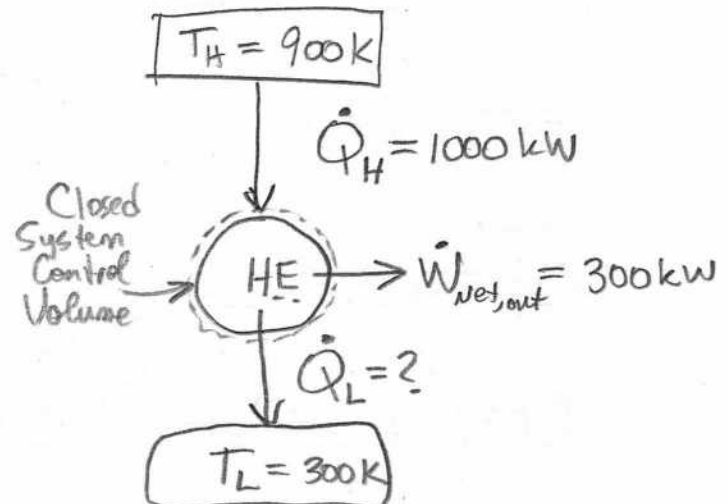
For a cyclical device to produce net power output, it Must reject some heat to a cold reservoir

Example 1 Basic Heat Engine Problem

A Heat Engine operates between 900K and 300K. The measured net power output is 300 kW, while the Heat input is 1000 kW. Determine (a) the necessary Heat rejection rate (kW) and the cycle thermal efficiency (%).

Find: (a) \dot{Q}_L (kW)
(b) η_{th} (%)

Assume: steady cyclical operation



Analysis

(a) \dot{Q}_L thermal pump

- From the Closed system Energy Balance for any cyclical process,

$$\left(\frac{dE}{dt} \right)_{sys} = \dot{Q}_{net,in} - \dot{W}_{net,out}$$

Steady cyclical operation

So that $\boxed{\dot{W}_{net,out} = \dot{Q}_{net,in}} \text{ (kW)}$

- To find the necessary heat rejection to achieve this Power output for the given heat transfer rate in,

$$\dot{W}_{net,out} = \dot{Q}_{in} - \dot{Q}_{out} \equiv \dot{Q}_H - \dot{Q}_L$$

or

$$\boxed{\dot{Q}_L = \dot{Q}_H - \dot{W}_{net,out}}$$

Thermal pump \Rightarrow

$$\dot{Q}_L = 1000 \text{ kW} - 300 \text{ kW} \rightarrow$$

$$\boxed{\dot{Q}_L = 700 \text{ kW}}$$

Waste Heat Rejection rate

(b) $\eta_{th} (\%)$

- By definition, for any Heat Engine,

$$\eta_{th} \equiv \frac{\text{Desired Output}}{\text{Required Input}} = \frac{\dot{W}_{net,out}}{\dot{Q}_{in}}$$

- For our HE,

$$\eta_{th} = \frac{\dot{W}_{net,out}}{\dot{Q}_H} = \frac{300 \text{ kW}}{1000 \text{ kW}} = 0.3 \Rightarrow \boxed{\eta_{th} = 30\%}$$

- Alternatively, you can compute η_{th} this way:

$$\eta_{th} = \frac{\dot{W}_{net,out}}{\dot{Q}_H} = \frac{\dot{Q}_H - \dot{Q}_L}{\dot{Q}_H}$$

$$\boxed{\eta_{th} = 1 - \frac{\dot{Q}_L}{\dot{Q}_H}}$$

$$\eta_{th} = 1 - \frac{700 \text{ kW}}{1000 \text{ kW}}$$

$$= 1 - 0.7 \rightarrow$$

$$\boxed{\eta_{th} = 0.3 = 30\%}$$

\Rightarrow Only $\frac{1}{3}$ of the heat transferred to the HE was converted to net power output, and 70% of \dot{Q}_H (what you paid money \$ for!) was rejected as waste heat as required by the 2nd Law of Thermodynamics.

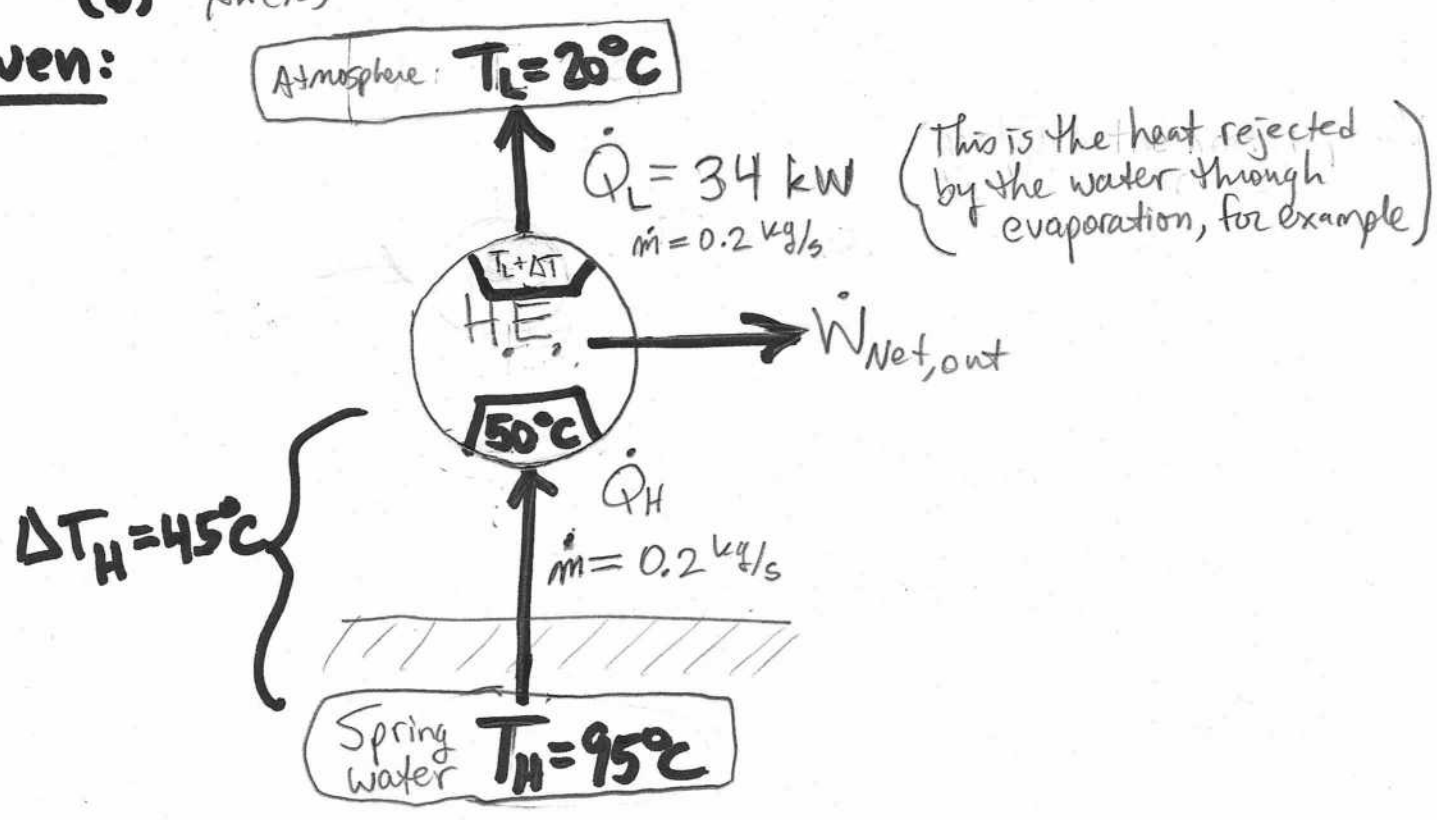
Example 2 Geothermal Energy Conversion

Hot ground water from an underground spring at 95°C is used to produce net power output from a heat engine. The hot water mass flow rate into the heat engine is 0.2 kg/s

The temperature difference between the hot spring water and the heat engine thermal inlet is 45°C , and the total rate of heat rejection of thermal energy to the atmosphere (20°C) is 34 kW . Assume that the specific heat of water is $4.22 \frac{\text{kJ}}{\text{kg}\cdot^{\circ}\text{C}}$

Find: (a) $\dot{W}_{\text{net,out}}$ (kW)
(b) η_{th} (%)

Given:



Assume: (a) Steady cyclical operation

(b) $C_w = 4.22 \frac{\text{kJ}}{\text{kg}\cdot^{\circ}\text{C}}$

(c) For the water, $\Delta KE = 0$, $\Delta PE = 0$

Analysis

(a) $\dot{W}_{net,out}$

- Since there is mass flow into and out of the Heat Engine, Let's try using an open system control volume for the Energy Balance:

$$\frac{dE}{dt}_{sys} = \dot{Q}_{net,in} - \dot{W}_{net,out} + \sum_{in} \dot{m}(h + \cancel{kE} + \cancel{pE}) - \sum_{out} \dot{m}(h + \cancel{kE} + \cancel{pE})$$

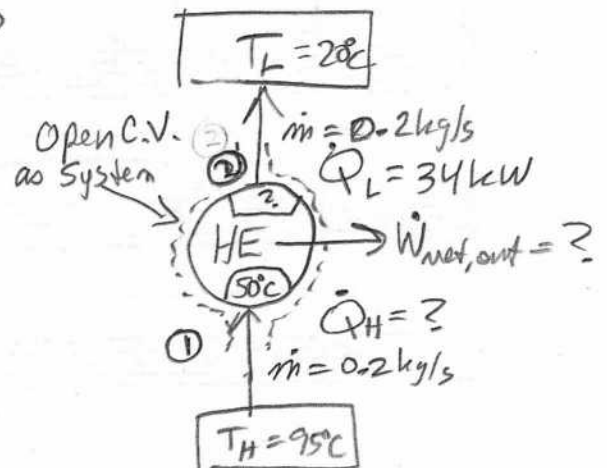
$\frac{dE}{dt}_{sys} = 0$ (steady)

$$\dot{W}_{net,out} = \dot{Q}_{net,in} + \dot{m}(h_{in} - h_{out})$$

using $\Delta h = c \Delta T$,

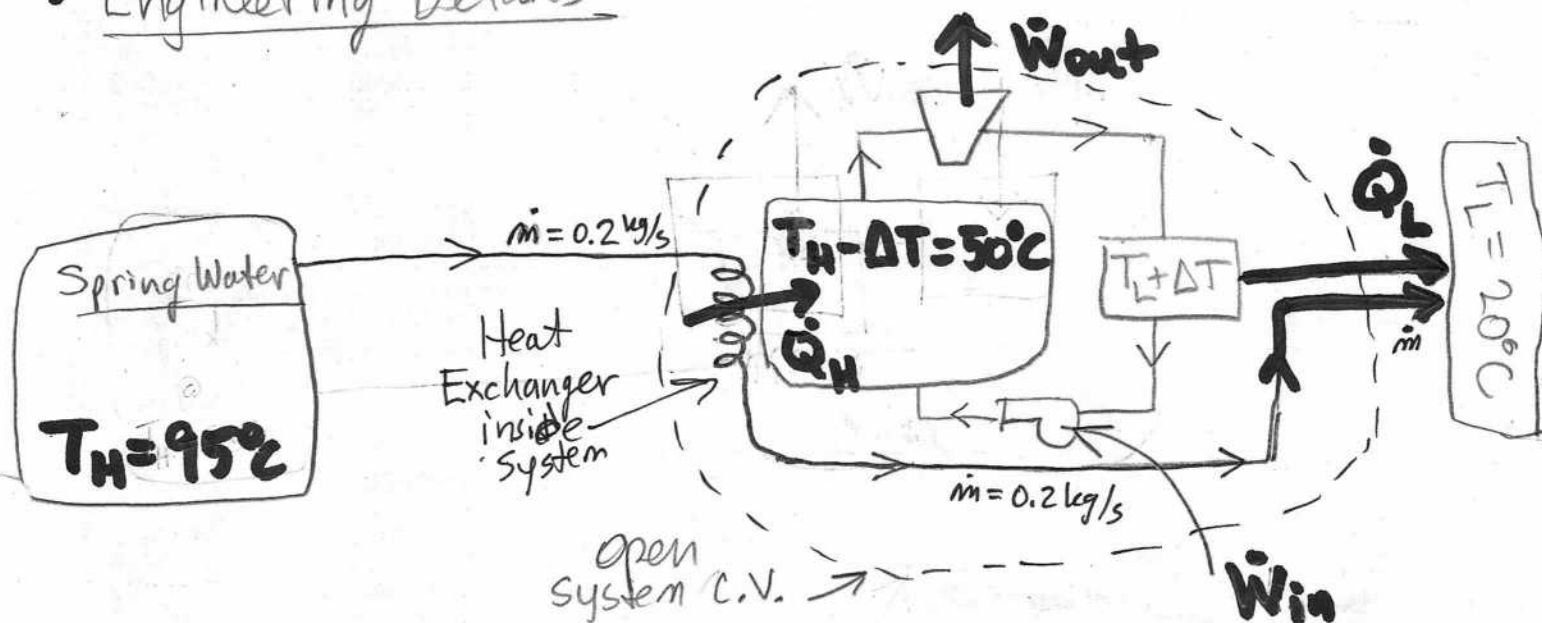
$$\dot{W}_{net,out} = \dot{Q}_H - \dot{Q}_L + \underbrace{\dot{m}c}_{\text{known}} (\underbrace{T_{in}}_{50^\circ\text{C}} - \underbrace{T_{out}}_{\text{Unknown!}})$$

\uparrow Unknown!
 \uparrow known
 \uparrow known 50°C
 \uparrow Unknown!

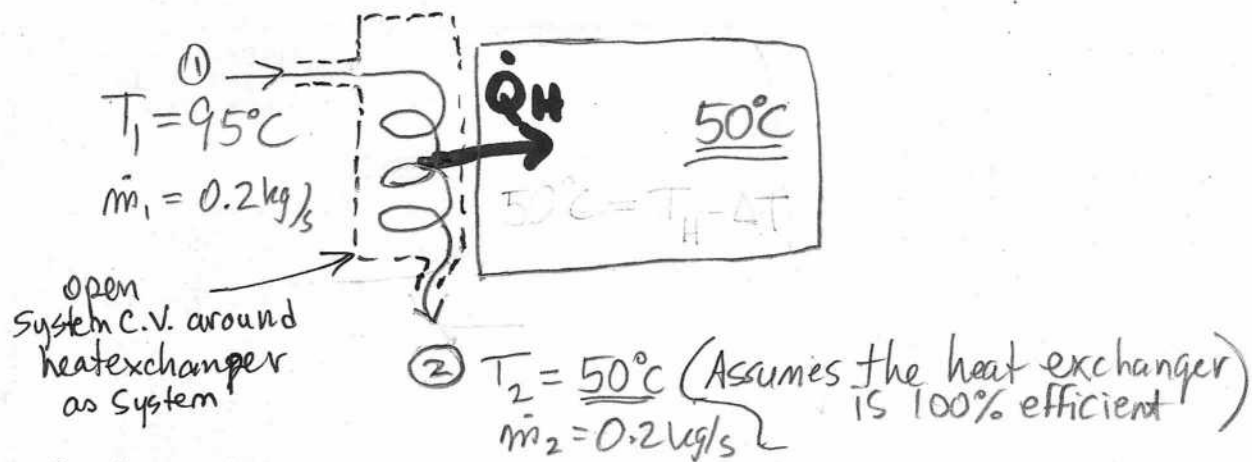


- This Approach will not work! We do know the temperature difference between the Hot spring and H.E. inlet so we can first find \dot{Q}_H , then we can find $\dot{W}_{net,out} = \dot{Q}_H - \dot{Q}_L$
- You are not given enough information about the engineering details of how heat is transferred from the hot reservoir at 95°C to the heat engine inlet at 50°C !!

- A heat exchanger is required to perform this thermal energy transfer from the spring water at 95°C to the H.E. thermal inlet device to the H.E. inlet at 50°C .
- Engineering Details:



- Details of Heat Exchange



Energy Balance Eqn.

Energy Balance Eqn.

$$\frac{dE}{dt}_{sys} = \dot{Q}_{net,in} - \dot{W}_{net,out} + \sum_{in} \dot{m} \left(h + \frac{1}{2} V^2 + gz \right) - \sum_{out} \dot{m} \left(h + \frac{1}{2} V^2 + gz \right)$$

$$0 = (\dot{Q}_{in} - \dot{Q}_{out}) + \dot{m} h_{in} - \dot{m} h_{out}$$

$$0 = -\dot{Q}_{out} + \dot{m}(h_1 - h_2)$$

$$\dot{Q}_{out} = \dot{Q}_H = \dot{m}(h_1 - h_2)$$

$$\boxed{\dot{Q}_H = \dot{m} c \Delta T}$$

- Solving for \dot{Q}_H ,

$$\dot{Q}_H = (0.2 \text{ kg/s}) \left(4.22 \frac{\text{kJ}}{\text{kg} \cdot ^\circ\text{C}} \right) (45^\circ\text{C}) \rightarrow \boxed{\dot{Q}_H = 38 \text{ kW}}$$

Q: What about \dot{Q}_L ?

A: \dot{Q}_L represents both the heat rejected by the Heat Engine at $T_L + \Delta T$ AND the spring water stream at 50°C to the environment at 20°C !

\Rightarrow Since $\dot{Q}_L \equiv 34 \text{ kW}$ we don't need to worry about these details.

- We can now solve for $\dot{W}_{net,out}$ with the information that $\dot{Q}_H = 38 \text{ kW}$ and given $\dot{Q}_L = 34 \text{ kW}$

$$\frac{dE}{dt}_{sys} = \dot{Q}_{net,in} - \dot{W}_{net,out}$$

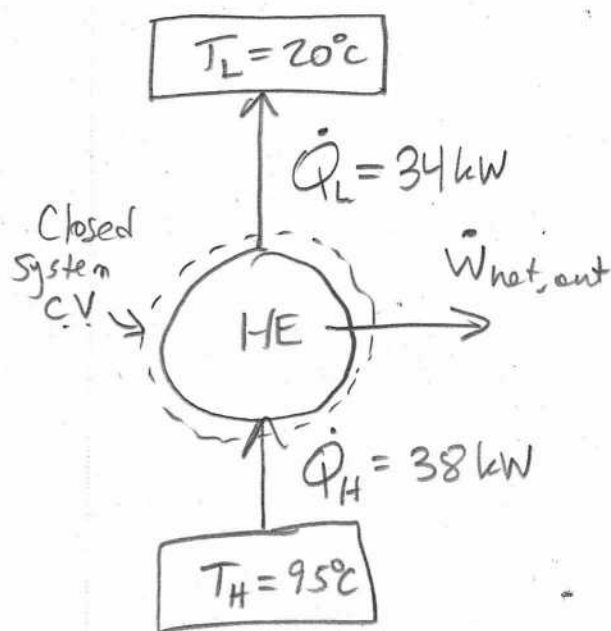
0 steady

$$\dot{W}_{net,out} = \dot{Q}_{net,in}$$

$$\dot{W}_{net,out} = \dot{Q}_H - \dot{Q}_L$$

$$\dot{W}_{net,out} = 38 \text{ kW} - 34 \text{ kW}$$

$$\dot{W}_{net,out} = 4 \text{ kW}$$



(b) η_{th}

$$\eta_{th} = 1 - \frac{\dot{Q}_L}{\dot{Q}_H}$$

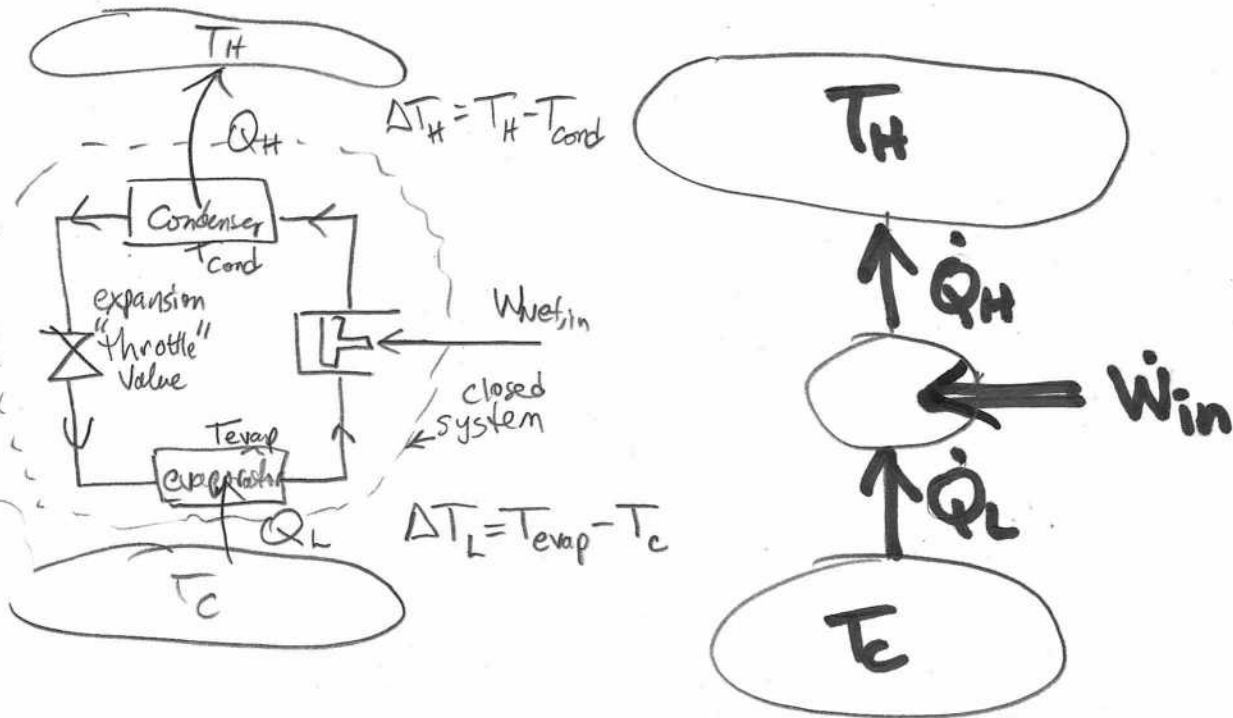
$$\eta_{th} = 1 - \frac{34 \text{ kW}}{38 \text{ kW}} = 1 - 0.895 \rightarrow \eta_{th} \approx 10.5\%$$

• Alternatively,

$$\eta_{th} = \frac{\dot{W}_{net,out}}{\dot{Q}_H} = \frac{4 \text{ kW}}{38 \text{ kW}} = 0.105 \rightarrow \eta_{th} = 10.5\%$$

Refrigerators & Heat Pumps (work consuming)

Both are cyclical devices that transfer Heat from a low temperature medium to a high temperature medium

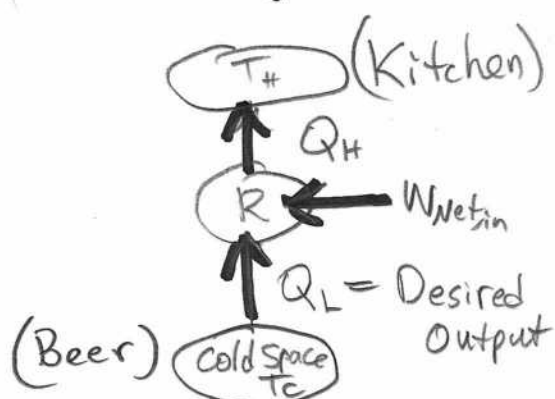


- They differ only in their objectives:

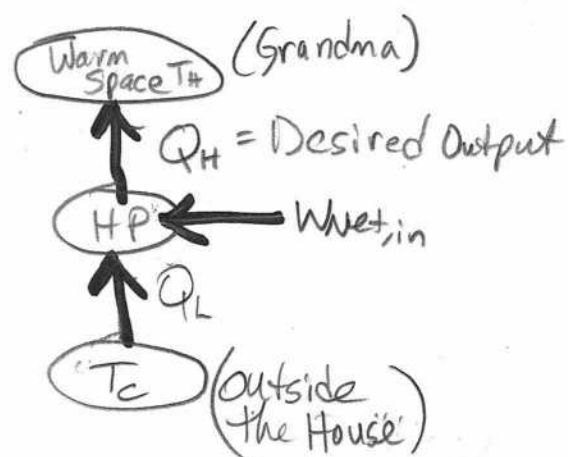
Refrigerator : To maintain a cold space at a desired low temperature T_C

Heat Pump : To maintain a warm space at a desired high temperature T_H

Refrigerator



Heat Pump



Efficiency

$$\text{Coefficient of Performance} \equiv \frac{\text{Desired Output}}{\text{Required Input}}$$

COP

Closed System Energy Balance around the R or HP:
over all steps in the cycle:

$$\Delta E_{\text{sys}} \equiv Q_{\text{net,in}} - W_{\text{net,out}}$$

cyclical

$$W_{\text{net,out}} = Q_{\text{net,in}}$$

$$W_{\text{out}} - W_{\text{in}} = Q_{\text{in}} - Q_{\text{out}}$$

$$W_{\text{in}} = Q_{\text{out}} - Q_{\text{in}}$$

Required Input: $W_{\text{in}} = Q_H - Q_L$

$$\text{COP}_R \equiv \frac{Q_L}{W_{\text{in}}} = \frac{Q_L}{Q_H - Q_L} \Rightarrow$$

$$\text{COP}_{\text{HP}} \equiv \frac{Q_H}{W_{\text{in}}} = \frac{Q_H}{Q_H - Q_L} \Rightarrow$$

Note:

$$\boxed{\text{COP}_{\text{HP}} = 1 + \text{COP}_R}$$

← **very useful!!**

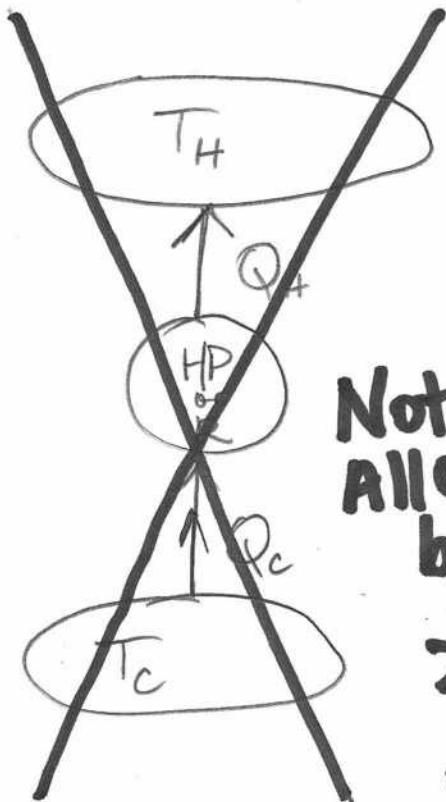
$\text{COP}_R = \frac{1}{\frac{Q_H}{Q_L} - 1}$
$\text{COP}_{\text{HP}} = \frac{1}{1 - Q_L/Q_H}$

2nd Law of Thermo for Refrigerators and Heat Pumps

It is impossible to construct a device that operates in a cycle and produces no effect other than the transfer of heat from a lower temperature body to a higher temperature body

Clausius Statement

Plain English You must have work input to a System in order to transfer heat from Cold to Hot Spaces



Not Allowed by 2nd Law

⇒ You must provide Work input

$$\text{COP}_R \geq 0$$

$$\text{COP}_{HP} > 1$$

$$\text{COP}_{HP} = 1 + \text{COP}_R$$

How Industry Expresses Heat Pump, Air Conditioning, and Refrigeration Performance

- Instead of COP_H and COP_R , Industry has standardized testing procedures which cast performance of heat pumps, air conditioners, and refrigerators in terms of

• **EER \equiv Energy Efficiency Ratio ($\frac{Btu}{Wh}$)**
 (Measure of instantaneous performance of Device)

• **SEER \equiv Seasonal Energy Efficiency Ratio ($\frac{Btu}{Wh}$)**
 (Measure of device performance over an entire Season)

$$EER \approx SEER = 3.412 \cdot COP$$

- You will find that Heat Pumps and Air Conditioners (HVAC) are rated using SEER values, while Refrigerators are rated by EER values.
- "Good" SEER and EER values of today's Devices
 - **HVAC**: SEER values range from 13 to 20 $\frac{Btu}{Wh}$
 - **Refrigeration**:
 - Meat and Dairy EER 8.9 to 10.2 (COP_R 2.6 to 3)
 - Frozen Food EER 4.1 to 5.1 (COP_R 1.2 to 1.5)
 - Ice Cream Unit EER 3.4 to 4.1 (COP_R 1.0 to 1.2)

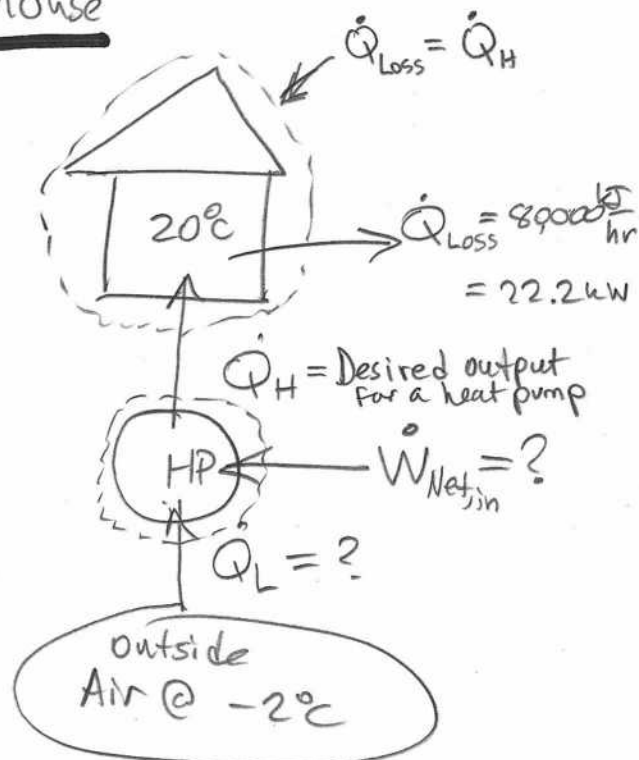
Example 3 Heat Pump for a house

Given: Picture +
 $COP_{HP} = 2.5$

Find:

(a) \dot{W}_{in} (kW) (c) SEER value
 (Btu/Wh)

(b) \dot{Q}_L (kW)



Analysis

(a) By definition

$$COP_{HP} = \frac{\dot{Q}_H}{\dot{W}_{Net,in}} \leftarrow \text{Desired output for HP}$$

so

$$\dot{W}_{Net,in} = \frac{80,000 \frac{\text{kJ}}{\text{h}}}{2.5} = 32,000 \frac{\text{kJ}}{\text{h}} = 8.9 \text{ kW}$$

(b) From Closed System Energy Balance,

$$\dot{W}_{Net,in} = \dot{Q}_H - \dot{Q}_L$$

$$\dot{Q}_L = \dot{Q}_H - \dot{W}_{Net,in}$$

In order to maintain steady flow of Heat into the House, compensating for the loss through the walls,

$$\dot{Q}_{Loss} = \dot{Q}_H = 80,000 \frac{\text{kJ}}{\text{hr}}$$

Perform a closed system energy balance on the house

$$\Rightarrow \dot{Q}_L = 80,000 \frac{\text{kJ}}{\text{hr}} - 32,000 \frac{\text{kJ}}{\text{h}} = 48,000 \frac{\text{kJ}}{\text{h}}$$

(c) $SEER = COP_{HP} \times 3.412 \frac{\text{Btu}}{\text{Wh}} =$

$$SEER = 8.5 \frac{\text{Btu}}{\text{Wh}} \text{ (poor!)}$$

$$\dot{Q}_L = 13.3 \text{ kW}$$

Example 4 A Household Freezer Problem

Given: A household freezer operates between $T_H = 30^\circ\text{C}$ and $T_L = 0^\circ\text{C}$. The power required to operate the refrigerator is 40W . The rate of heat transfer out of the cool-space is 52W .

Find: (a) \dot{Q}_H (W), the rate of heat rejection to the kitchen environment required by the 2nd Law of Thermodynamics

(b) COP_R (c) EER value

Assume: Steady Cyclical operation

Analysis:

(a) \dot{Q}_H (W)

- Performing a closed system Energy Balance Analysis on the Refrigerator:

$\frac{dE}{dt}_{\text{sys}} = \dot{Q}_{\text{net},\text{in}} - \dot{W}_{\text{net},\text{out}}$

Steady operation $\rightarrow \frac{dE}{dt}_{\text{sys}} = 0$

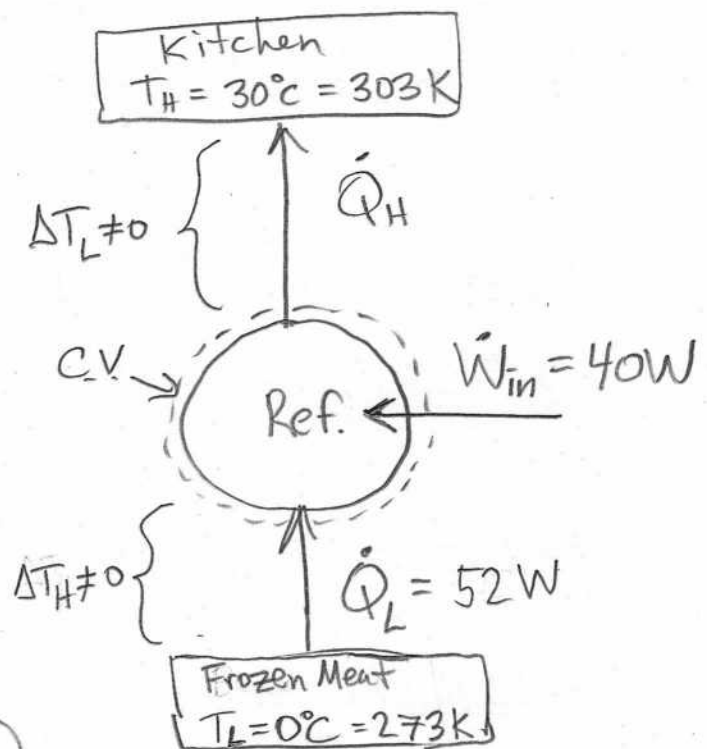
$$0 = (\dot{Q}_L - \dot{Q}_H) - (\dot{W}_{\text{out}} - \dot{W}_{\text{in}})$$

$$0 = \dot{Q}_L - \dot{Q}_H + \dot{W}_{\text{in}}$$

$$\dot{Q}_H = \dot{Q}_L + \dot{W}_{\text{in}}$$

$$\dot{Q}_H = 52\text{W} + 40\text{W} \longrightarrow$$

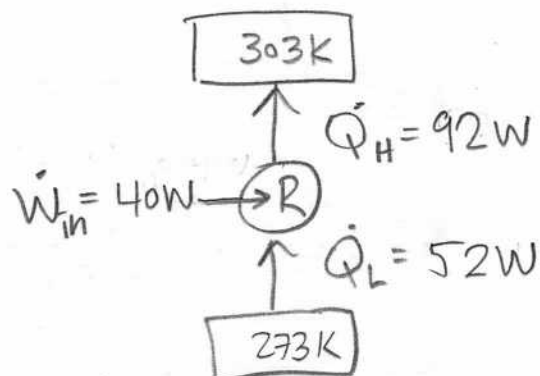
$$\dot{Q}_H = 92\text{W}$$



(b) COP_R

- By definition, $COP_R = \frac{\text{Desired Output}}{\text{Required Input}} = \frac{\dot{Q}_L}{\dot{W}_{in}}$
- From a Closed System Energy Balance Analysis for a Refrigerator (or air Conditioner),

$$COP_R = \frac{1}{\frac{\dot{Q}_H}{\dot{Q}_L} - 1}$$



- Substituting values,

$$COP_R = \frac{1}{\frac{192W}{52W} - 1} = \frac{1}{1.77 - 1} = \frac{1}{0.77} = 1.3$$

$$COP_R = 1.3$$

This is a typical value for a household Freezer

(c) EER value

- $EER \approx COP_R \times 3.412 \frac{Btu}{Wh}$

$$\approx 1.3 \times 3.412 \frac{Btu}{Wh}$$

$$EER \approx 4.4 \frac{Btu}{Wh}$$

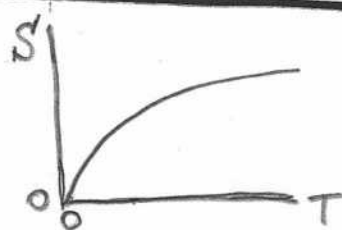
Lecture 22 Entropy (Disorder), Reversible, and Irreversible Processes

Entropy

- Entropy can be viewed as a measure of randomness, or "disorder".
- In thermodynamics, Entropy is a measure of molecular randomness, or "disorder"
- Designated by the Symbol S ($\frac{kJ}{K}$) or s ($\frac{kJ}{kg \cdot K}$)
Extensive Property \nearrow Intensive Property \nearrow

3rd Law of Thermodynamics

- Also known as Nerst's Theorem
- The Entropy (disorder) of a pure, isolated crystalline solid at absolute zero, $T=0K$ is defined as $S = 0 \frac{kJ}{kg \cdot K}$
- The 3rd Law provides an absolute (not relative) reference point for the determination of Entropy
- $S = 0 \frac{kJ}{kg \cdot K}$ at $T=0K$ represents a theoretical state of minimum energy and minimum disorder (Maximum order)



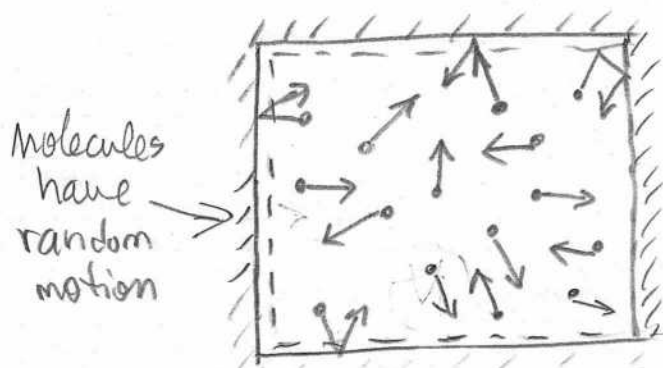
Entropy, Molecular Disorder, and the 2nd Law

I) Entropy as a Property of a System

- Thermal energy, U (kJ), always has a certain degree of random molecular/atomic disorder which is quantified by the amount of Entropy, S ($\frac{\text{kJ}}{\text{kg}}$), of the System.
- Consider a perfectly insulated, rigid tank filled with some gas and is at a State of Thermodynamic Equilibrium.

State Properties of System

$$\begin{aligned} T &= 300 \text{ K} \\ P &= 100 \text{ kPa} \\ S &= 250 \frac{\text{kJ}}{\text{K}} \\ U &= 310 \text{ kJ} \\ V &= 2 \text{ m}^3 \\ m &= 3 \text{ kg} \end{aligned}$$



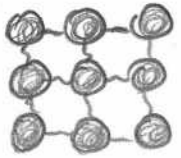
Nature is a Lazy Slob! \Rightarrow

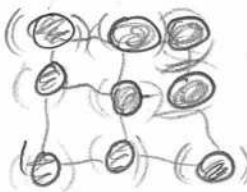
The 2nd Law states that nature seeks the maximum possible disorder for a given state $p, T, V, U \Rightarrow$ Maximum S

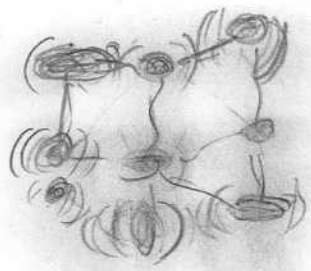
- Entropy is inherently a concept based on probability and Statistics, in which the 2nd Law states that when the molecules in a system achieve thermodynamic equilibrium then they have "searched" and achieved a state of maximum possible disorder (Entropy)

- As the temperature of the pure crystal is increased above $T = 0 \text{ K}$, the Entropy of the crystal increases because the molecules vibrate about their equilibrium positions, and the uncertainty in their positions and Energies increases

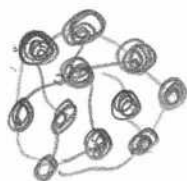
Solid:


 $T = 0 \text{ K}, S = 0 \frac{\text{kJ}}{\text{kg-K}}$
 (only theoretically possible)

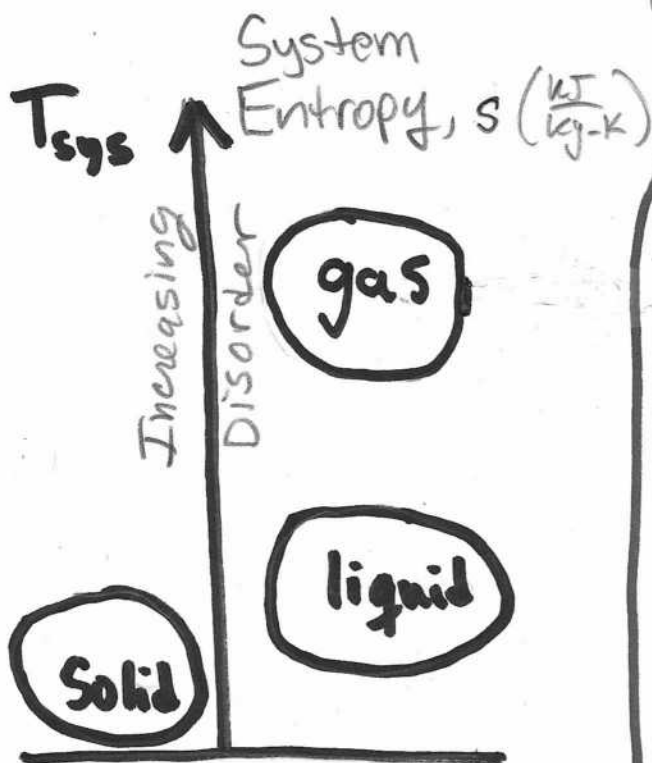
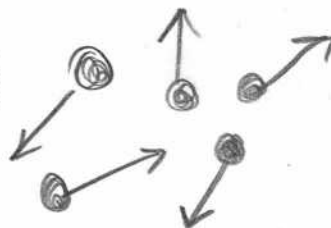

 $T > 0 \text{ K}$


 $T \gg 0 \text{ K}$

Liquid:



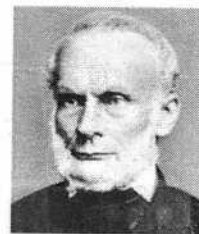
Gas:



S as a Property

- The Entropy of a System increases with increasing T_{sys}
- Note that the System Entropy S can also vary with p and v

- Rudolf Clausius (1822-1888) discovered this new Thermodynamic Property in 1857, calling it "Entropy" (from the Greek $en + tropin$, meaning "transformation content")



- Ludwig Boltzmann (1844-1906) linked this Property of a System to Probability and Statistics of atomic Energy States and discovered his famous equation for absolute Entropy between 1872 to 1875:

$$S_{\text{system}} = k_B \ln(W)$$

Entropy as a Property of System at Thermodynamic Equilibrium

Boltzmann's Constant

Maximum number of possible energy states that a molecule can probably occupy at thermodynamic equilibrium

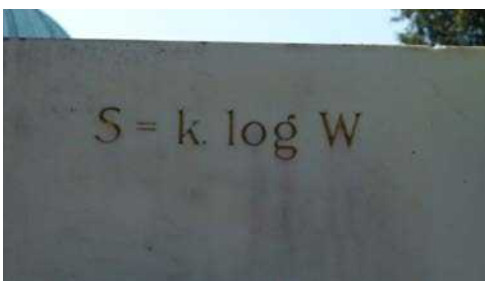


Note: if all molecules occupy the same single energy state then $W=1 \Rightarrow S = k_B \ln(1) = 0$

- Boltzmann's use of both atoms and discrete energy levels in his work were not experimentally proven facts, and he was ridiculed by his colleagues even after quantum mechanics was discovered by Max Planck (1900).
- Boltzmann was famous by 1905, but he suffered from bipolar depression and in 1906 he committed suicide by hanging. Some colleagues still disbelieved him and he felt his work useless.

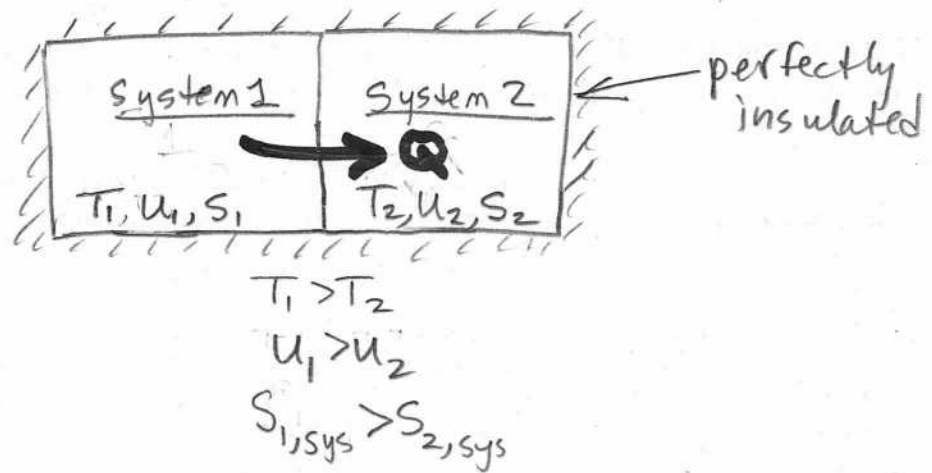


Vienna Central Cemetery (Zentralfriedhof) with Mozart, Beethoven, and many others.

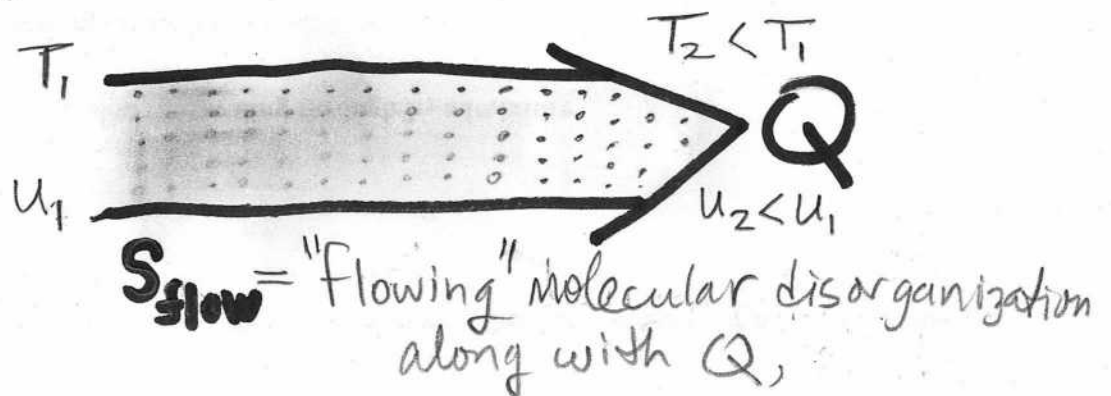


(II) Entropy as an Interaction With the System

- Consider heat transfer with a solid material, or between solid materials (for example).



- Heat transfer is the "flow" of random thermal energy from a region of greater molecular agitation to a region of lower molecular agitation
- Heat transfer is a form of disorganized Energy, and some disorganization (Entropy) "flows" with it
- This "flowing" Entropy, S_{flow} , along with Q is an interaction with the System, NOT a Property of the System



(III) The Generation of NEW Entropy, S_{gen}

- Consider a perfectly insulated, rigid tank that has gas molecules on one side and a vacuum on the other side, separated by a partition:



- First, consider the System to be the side with the gas molecules, at initial state ①
- The system has Properties V, p, T, S, U

Q: If we take away the partition, which situation is most probable to occur?

(a) $t \gg t_1$

All molecules move to one corner and stay there
[Entropy/Disorder Decreases Spontaneously]

(b) $t \gg t_1$

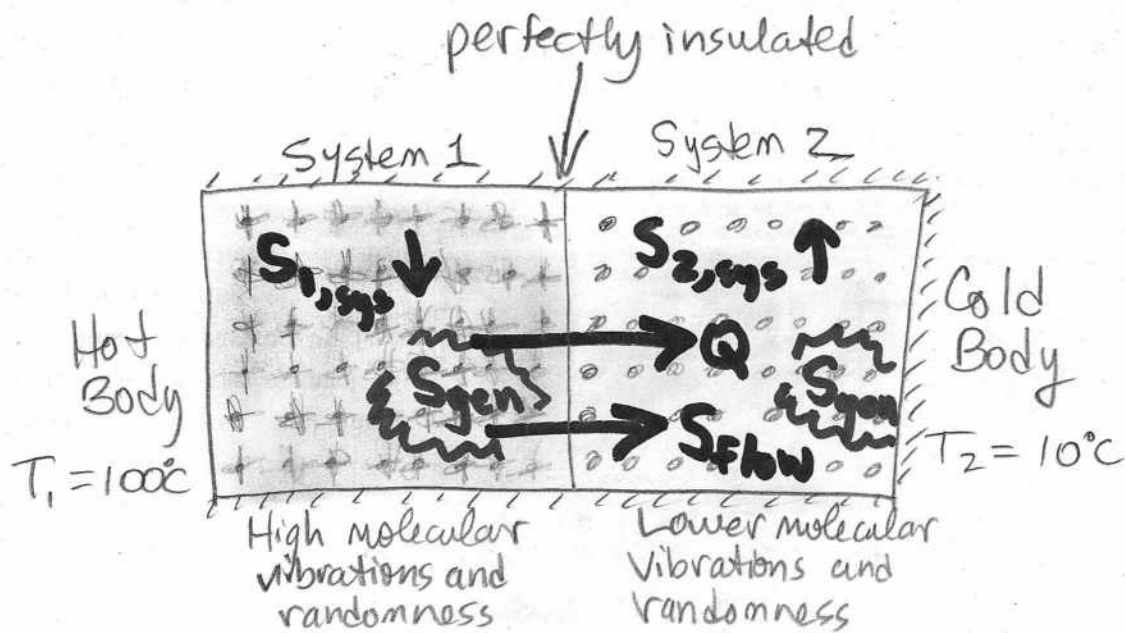
$S_2 > S_1$
 $V_2, P_2, S_{gen} > 0$

All molecules uniformly occupy the new larger volume at state ②
[Entropy/Disorder Spontaneously Increase]

- A:**
- The 2nd Law (and $S = k_B \ln(w)$) requires that (b) must be most probable — in fact, the odds are astronomical!
 - NEW Entropy was also generated (S_{gen}) because the larger volume presented a higher probability of maximizing the disorder of the initial system in state ①

(8)

Consider two solid blocks of material next to each other with one block hotter than the other:

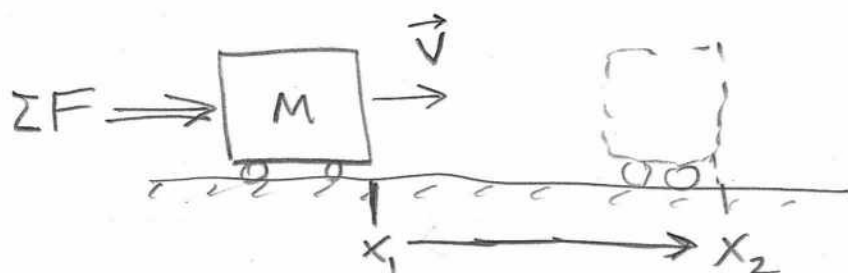


- The 2nd Law demands that NEW Entropy, S_{gen} , must be generated during a heat transfer process, where there is a temperature gradient $\frac{dT}{dx} < 0$
- Considering the Combined System 1 + System 2 above, the NET Entropy must increase according to the 2nd Law $\Rightarrow S_{gen} > 0$
- After reaching thermodynamic equilibrium, the increase in system 2 Entropy, S_2 , is much greater than the decrease in System 1 Entropy, S_1
- Total Entropy is not conserved: $(S_1 + S_2)_{t=0} \ll (S_1 + S_2)_{t \gg 0}$
- The Entropy of the Universe Always Increases !!

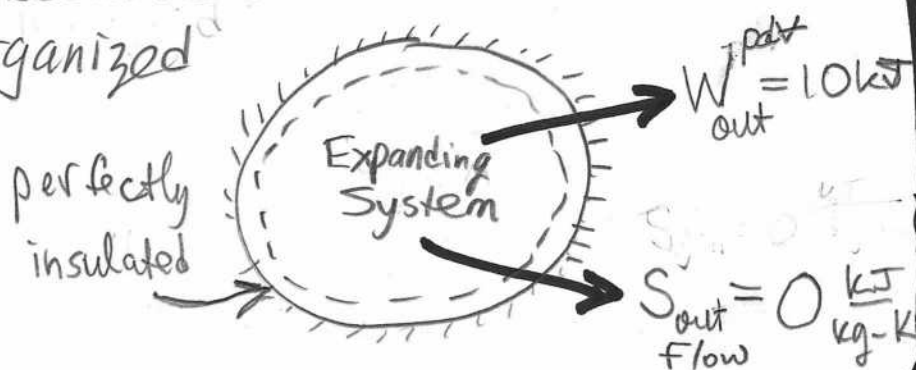
Work

No Entropy Flows Across System Boundary with Work Interactions

- Work is a form of purely organized Energy without any Entropy "flow" transfer
- $W = \text{Force} \times \text{distance}$



- All of the bodies' molecules move in the same direction under the same Net Forces
- During a Work interaction, No Entropy is transferred (or "Flows") across the System Control Volume because Work is a purely organized form of Energy



Types of Processes

Important to Engineering
Thermodynamics and to
the understanding of how
to improve device efficiency

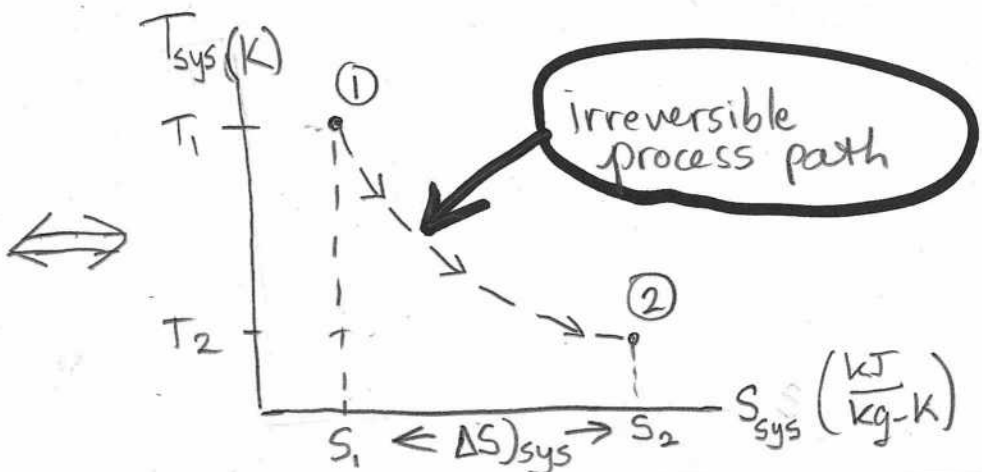
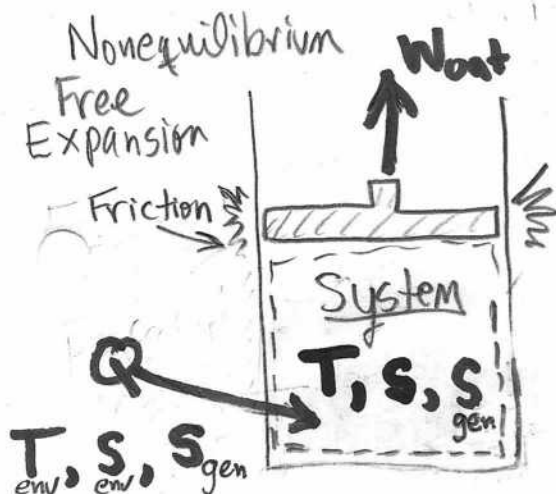
Visualizing Changes in System Entropy on a "T-s" Diagram

- At this point in our studies, it is appropriate to introduce a new type of Property diagram: the "T-s" diagram
- Entropy is a property of a System (and of the surroundings, too). In Chapter 7 we will discover that the new Entropy generated by irreversibilities is not a property.

⇒ (1) Process with Solid line = Quasiequilibrium and no irreversibilities within the System.

(2) Process with Dashed Line = Irreversibilities occur within the System regardless of quasiequilibrium or non-equilibrium process

⇒ T-s diagrams can only show the System Entropy (property) and cannot show any Entropy changes or generation outside of the System!



Irreversible Processes That Generate New Entropy S_{gen}

Irreversible Process

- A process that generates (or "produces") NEW Entropy within:

- (1) The System only
- (2) The Surroundings Only
- (3) Both the System and Surroundings

- This newly generated Entropy adds to the Entropy already existing in the Universe and never goes away

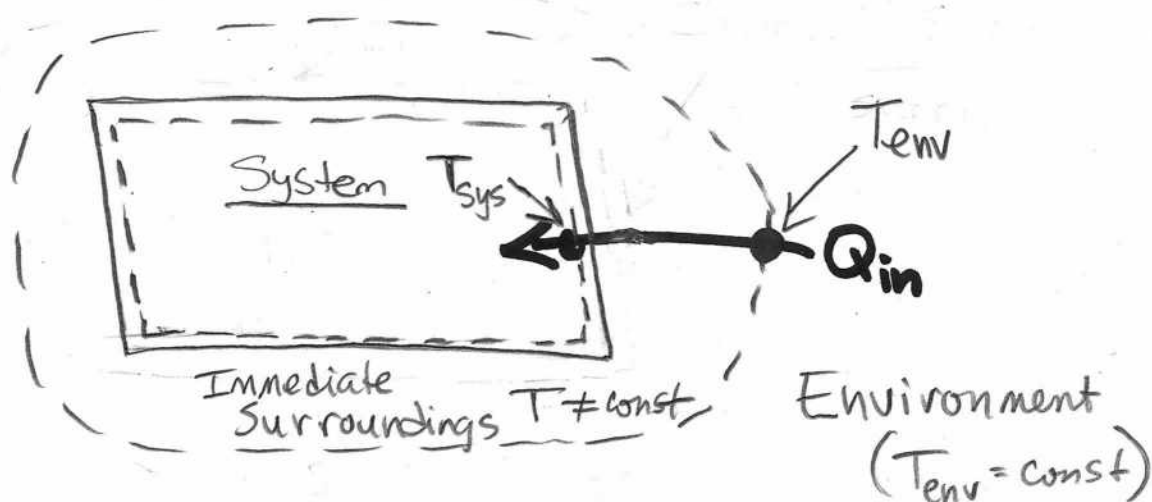
⇒ An irreversible process leaves a net positive "footprint" of Entropy on the System, the surroundings, or both — i.e., the Universe.

- All actual processes are, in reality, irreversible processes. Any System can be taken through a series of irreversible processes and returned to its original state (a cycle). But, in this case the surroundings are left with a "footprint" — a net positive increase in new Entropy generated.

Types of Irreversible Processes

- (1) Any non-equilibrium process (example: very fast "free" expansion)
- (2) Processes with Friction
- (3) Mixing of Substances
- (4) Chemical Reactions
- (5) Inelastic Deformation of Solids
- (6) Heat transfer across a System Boundary through a finite temperature difference:

$$\Delta T = T_{\text{env}} - T_{\text{sys}} > 0 \quad (\text{for heat flow into System})$$



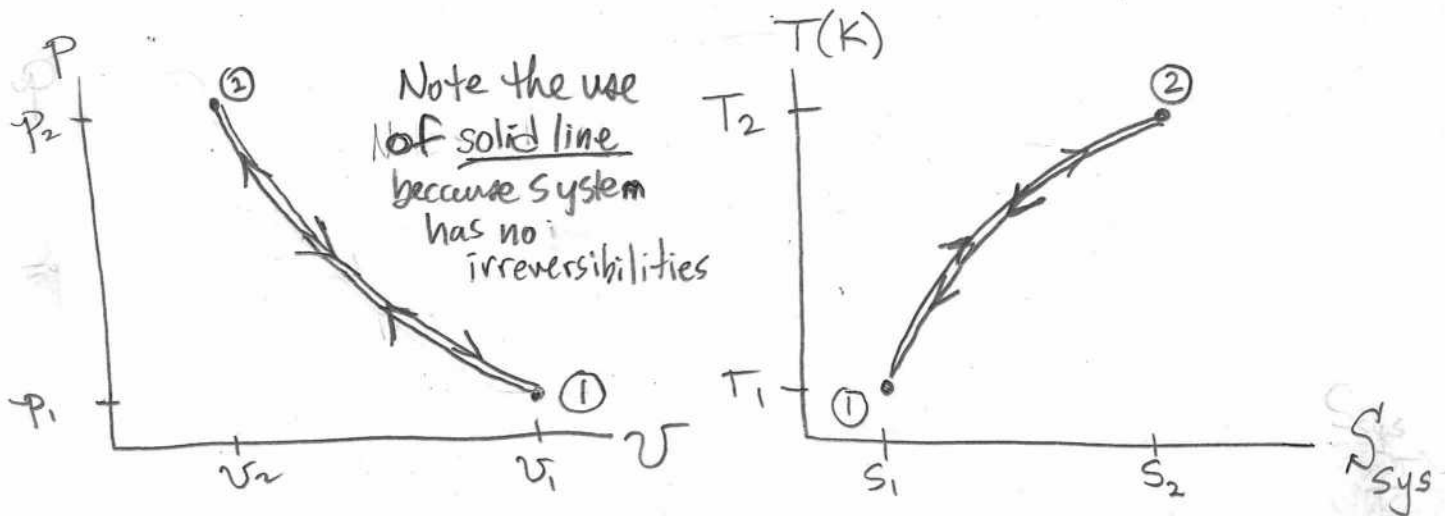
- Note that as $\Delta T \rightarrow dT$, $Q_{\text{in}} \rightarrow \delta Q_{\text{in}}$ and the irreversibility (New Entropy generation) becomes infinitesimally small so that the heat transfer process is practically reversible.
- The notion of heat transfer when $\Delta T = 0$ is not possible in reality, but is useful as a theoretical tool.

Internally Reversible (but Externally Irreversible)

(14)

No Irreversibilities occur Inside the System Boundaries during this process, but Irreversibilities occur Outside the System

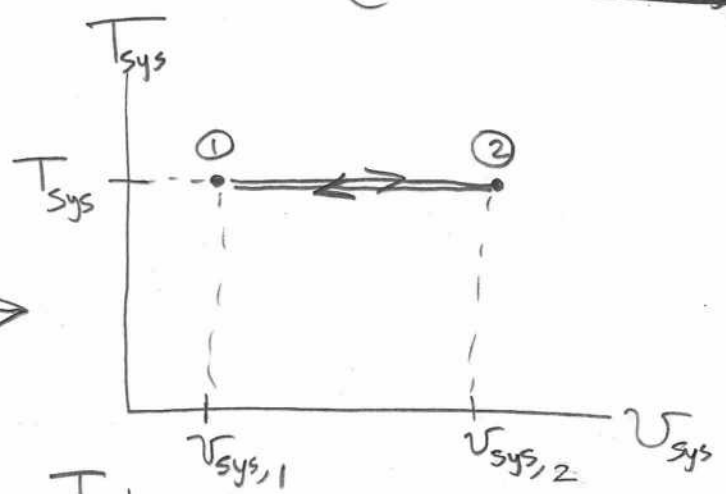
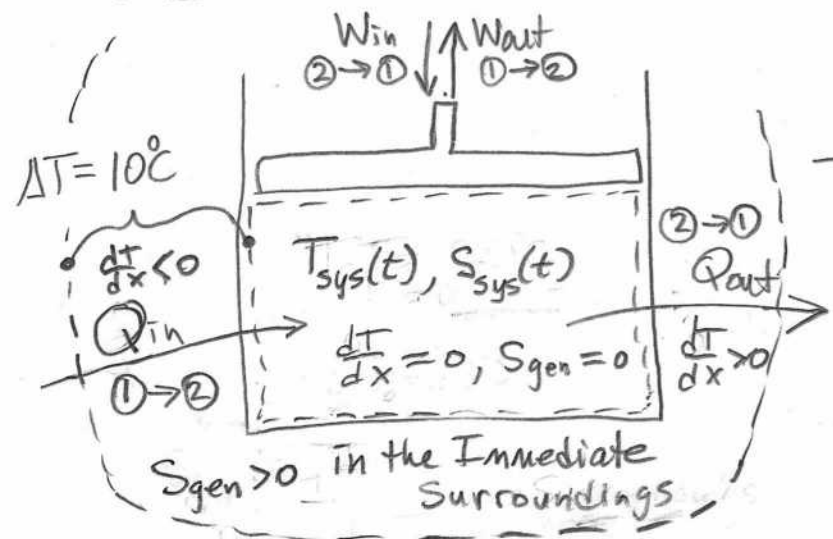
- The System process is quasi-equilibrium, without friction, chemical reactions, mixing, etc. occurring within the Control Volume Surface Boundaries.
- When the Process is reversed, the System goes back along exactly the same path.



- Note: External to the system the surroundings may be affected with new generation of Entropy, S_{gen}. (a "memory" or "Footprint" of the process)

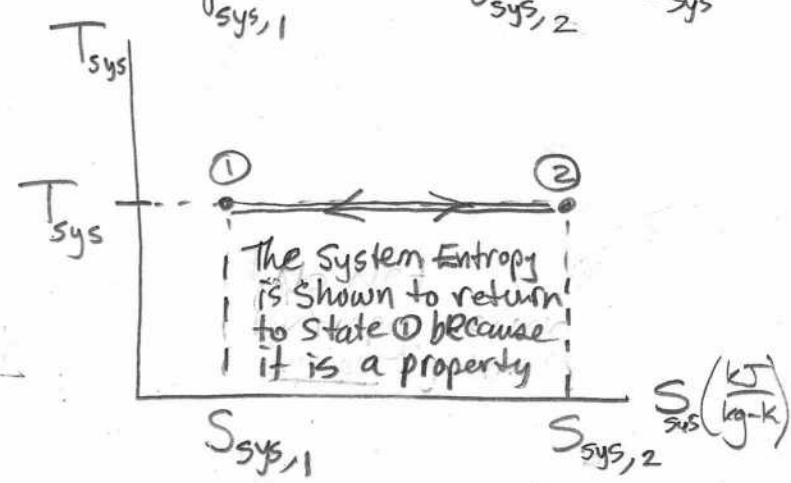
Examples

(1) Isothermal, Quasiequilibrium process (without Friction)

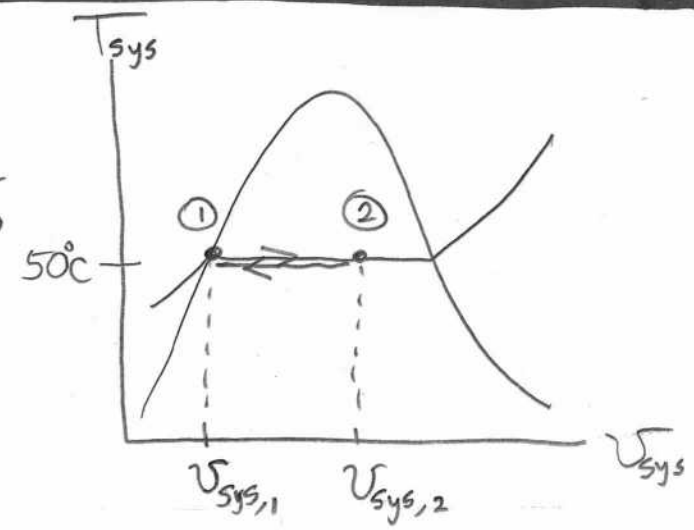
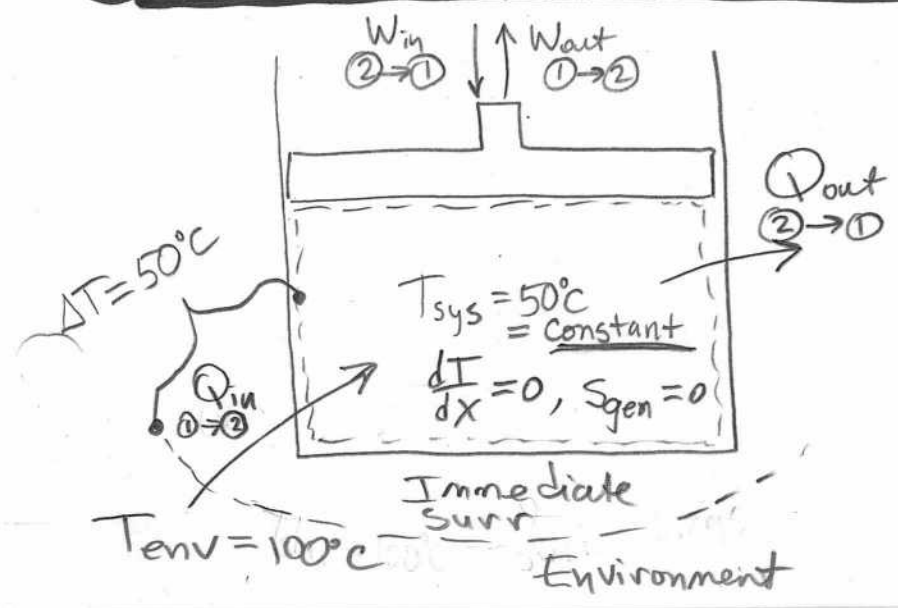


$S_{gen} = 0$ in Environment @ T_{env}

⇒ This process is externally irreversible within the immediate surroundings because $\frac{dT}{dx} \neq 0$ there.



(2) Heat transfer to a saturated mixture (no friction and quasiequilibrium)



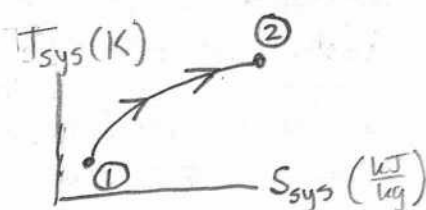
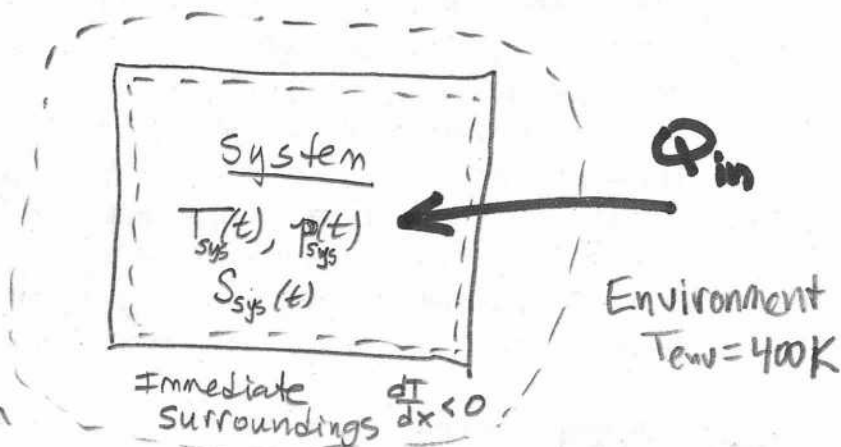
(3) Heat transfer Across the Immediate Surroundings Into a Device Having a Wall of zero thickness

- Consider a rigid, closed tank containing a gas (for simplicity) initially at $T_{sys} = 300K$ and The Environment Temperature $T_{env} = 400K = \text{constant}$:

- The walls of the tank have zero, or, infinitesimal thickness.

- Assume that the process of heat addition to the system is quasiequilibrium so that we can define a System properties at all times:

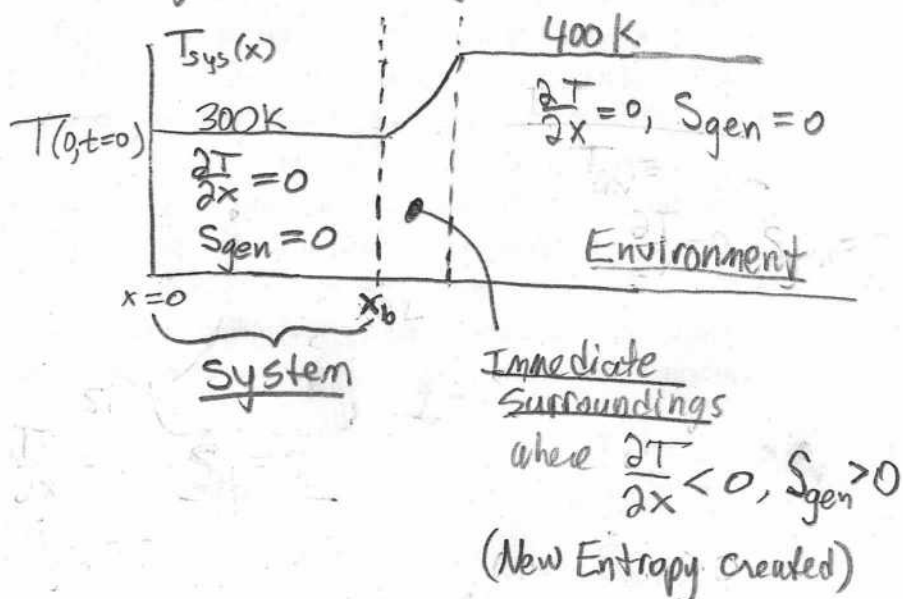
$$T_{sys}(t), P_{sys}(t), S_{sys}(t), U_{sys}(t), \text{etc.}$$



- Since the process is quasiequilibrium, note that the temperature is uniform throughout the system at all

$$\text{times: } \frac{\partial T_{sys}(t)}{\partial x} = 0$$

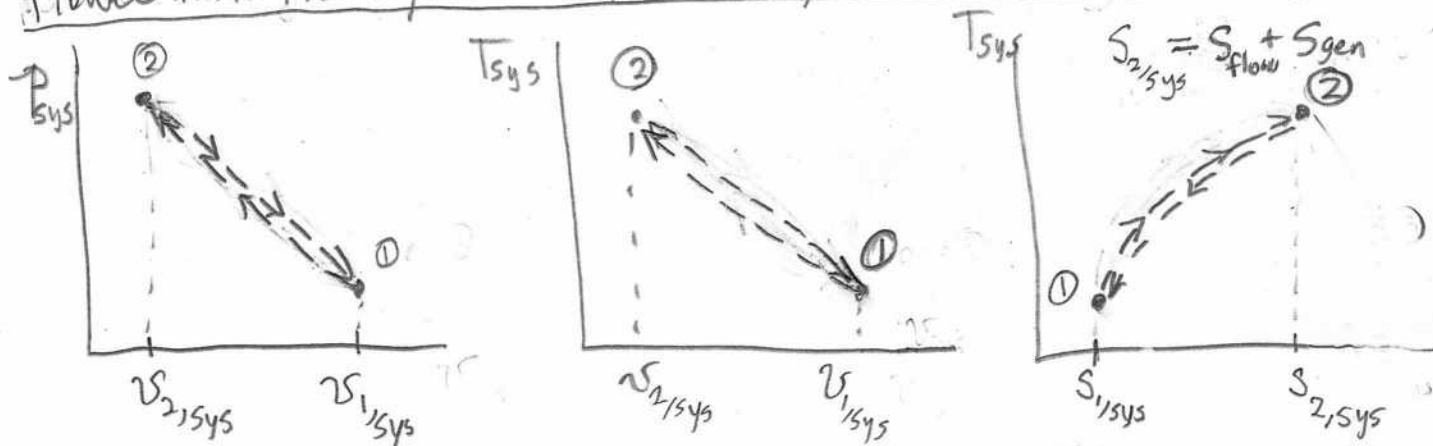
- The irreversibility of the Process occurs in the Immediate Surroundings where $\dot{S}_{gen} > 0$. This is outside the System Boundary so it is an Externally irreversible Process



Externally Reversible (but Internally Irreversible)

No irreversibilities occur Outside of the system in the Immediate Surroundings or the Environment during the process, but irreversibilities occur within the System.

- The process may even be quasiequilibrium if there are chemical reactions, slow diffusional mixing, or friction occurs within the System!
- The process may be an adiabatic free expansion (definitely non-equilibrium)
- Some Internally irreversible processes can be quasiequilibrium, but we have still used a dashed line on Property diagram to denote that Entropy Flowed into the system and/or was Generated within the System.

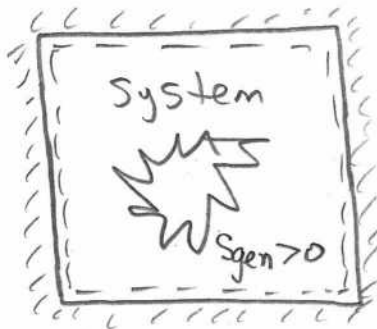


Note: S_{gen} inside the System caused the property s to always increase!

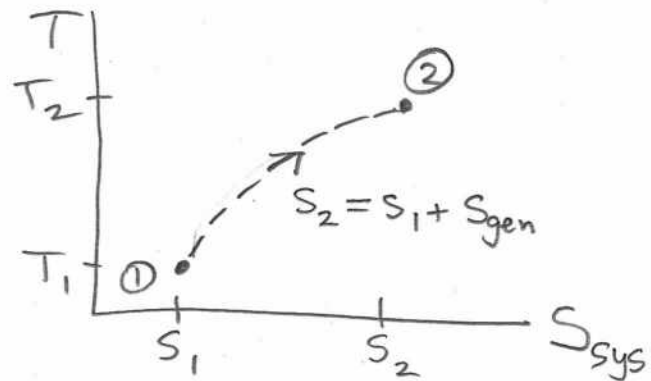
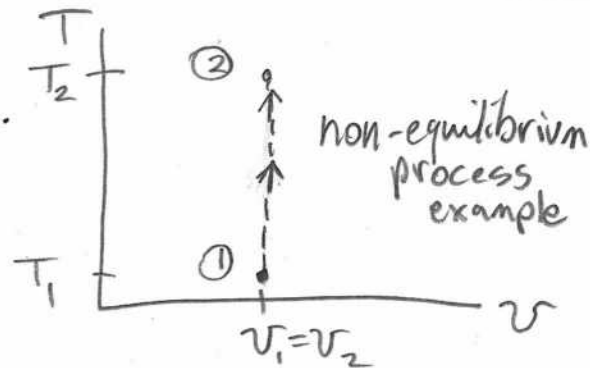
Examples

18

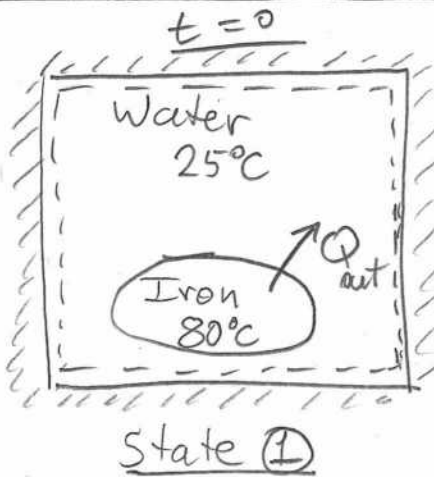
(1) Chemical Reactions in a well insulated Box



Environment
@ $T_{env} = 20^\circ\text{C}$



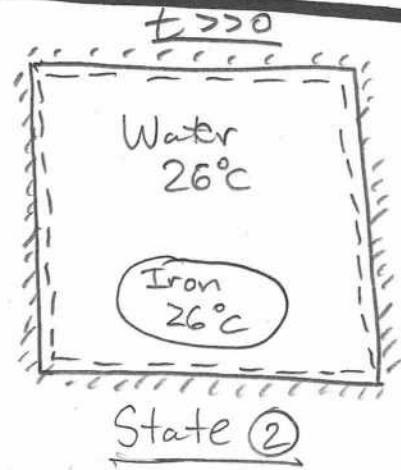
(2) Hot Iron Block / Water in well insulated Box



Environment
@ $T_{env} = 20^\circ\text{C}$

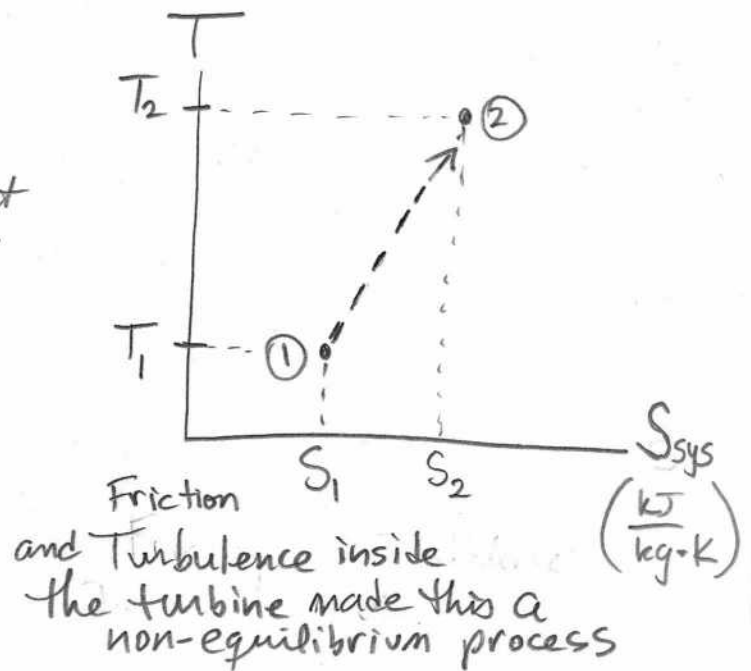
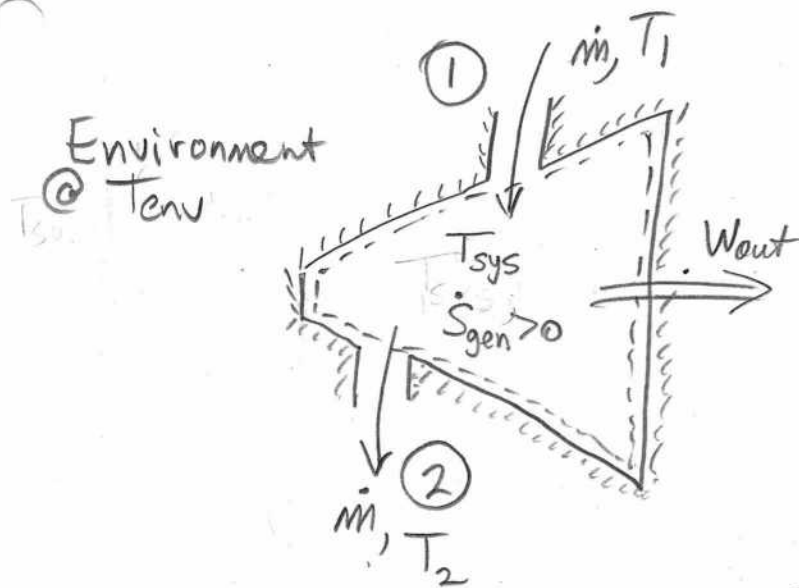
Non-Equilibrium
process
with
Heat transfer
through finite
 $\Delta T \neq 0$
Inside the
System

So, $S_{gen} > 0$

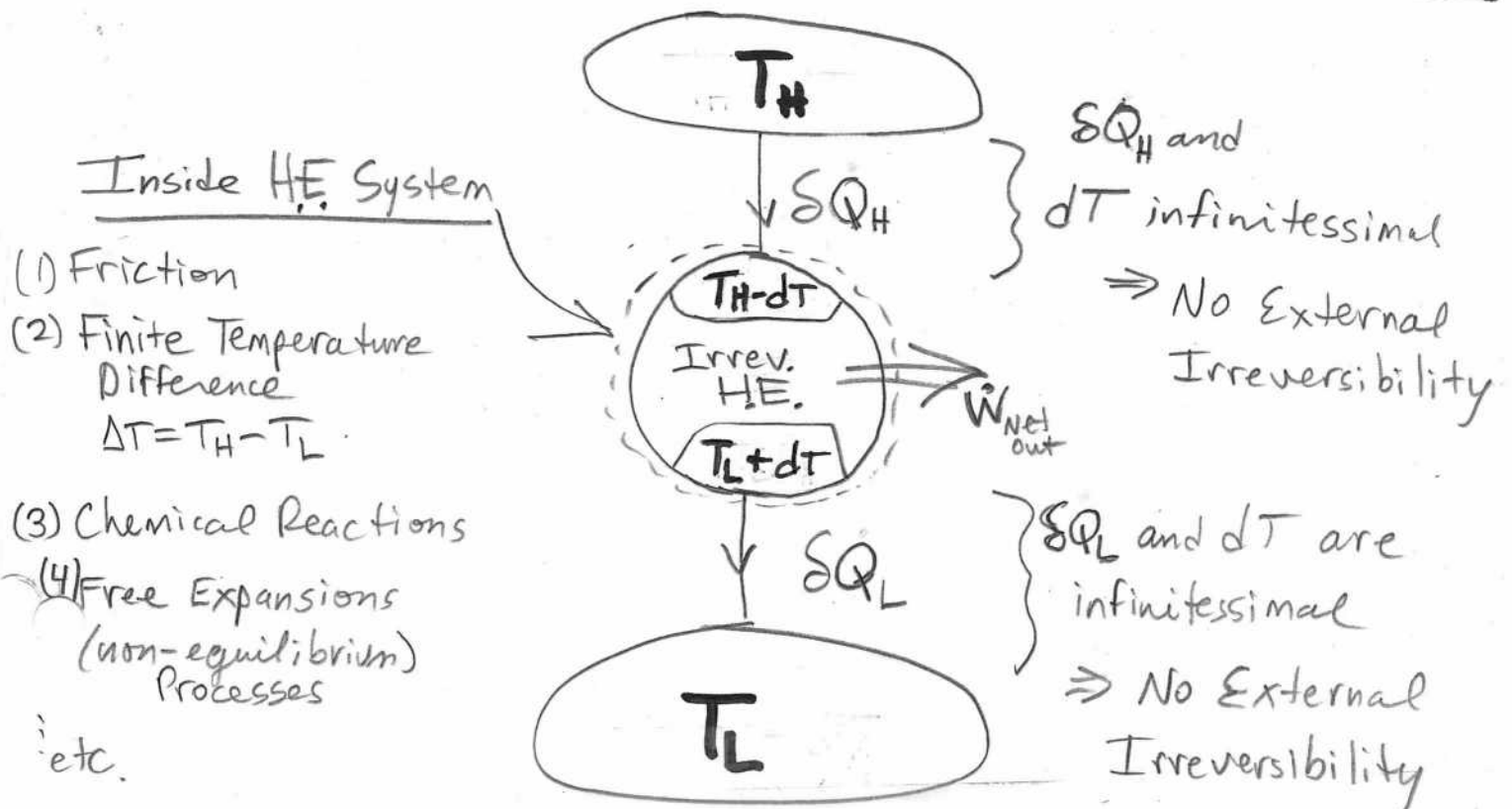


Environment
@ $T_{env} = 20^\circ\text{C}$

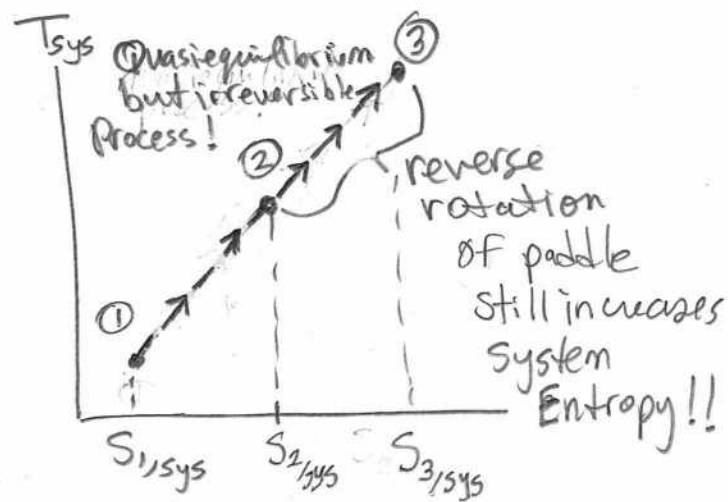
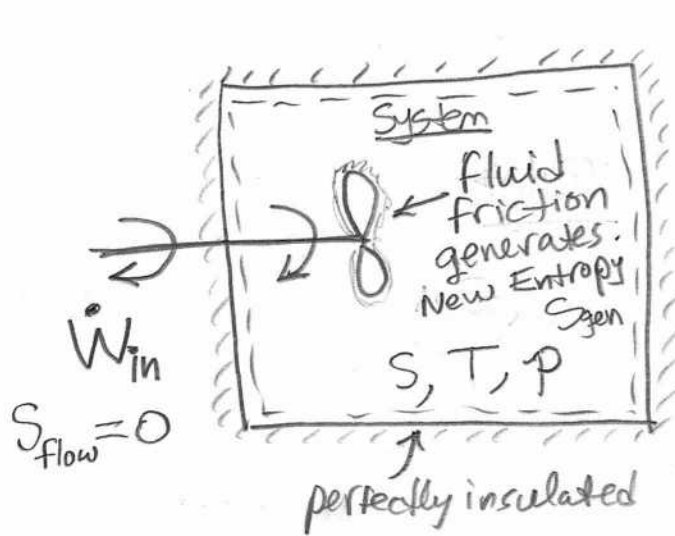
(3) Adiabatic Turbine with Friction & turbulence



(4) Irreversible Heat Engine with infinitesimal heat transfer at infinitesimal temperature difference

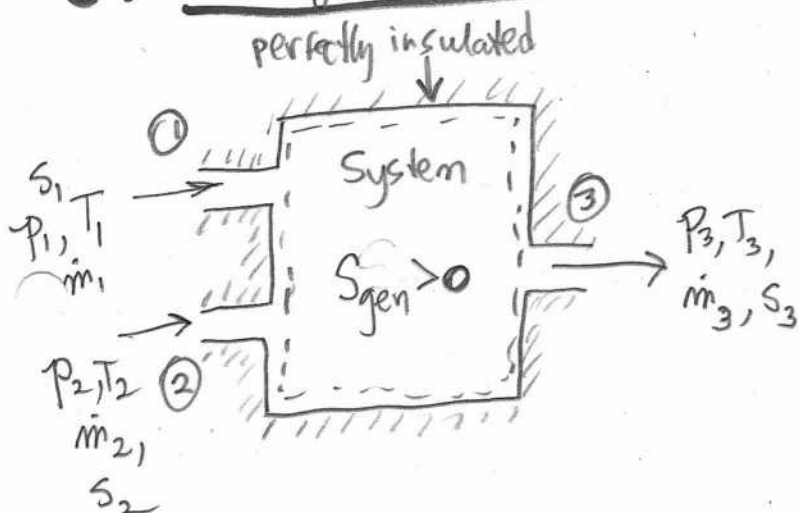


(5) Paddle wheel performing work on a perfectly insulated system, Quasiequilibrium Process (20)



- Even if the paddle wheel moves slowly so the process is quasiequilibrium, there is friction occurring at the interface between the paddle wheel and the gas/fluid!
- Reversing the rotation does not reverse the process! This only creates more new Entropy, S_{gen} , so the System Entropy still increases!

(6) Mixing of Fluid Streams in an Adiabatic Chamber



- The 2nd Law demands that $S_{gen} > 0$ in the chamber, so that

$$S_3 > S_1 + S_2$$

Reversible Process (both Internally and Externally Reversible)

- A process that is both Internally and Externally reversible
- No NEW Entropy, S_{gen} , is created anywhere
- Both the System and Surrounding Environment can be returned to their initial states without any Entropy generation, S_{gen} .

→ The Universe does not "know" that this process occurred because there is no memory of it in the form of new Entropy generation, S_{gen}

→ You just fooled Mother Nature!!

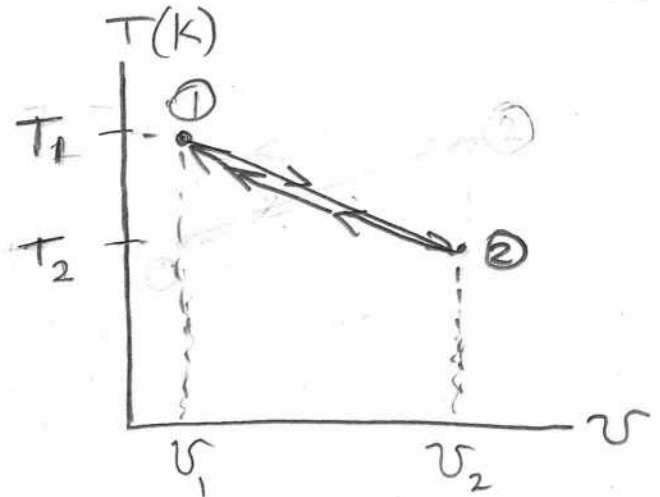
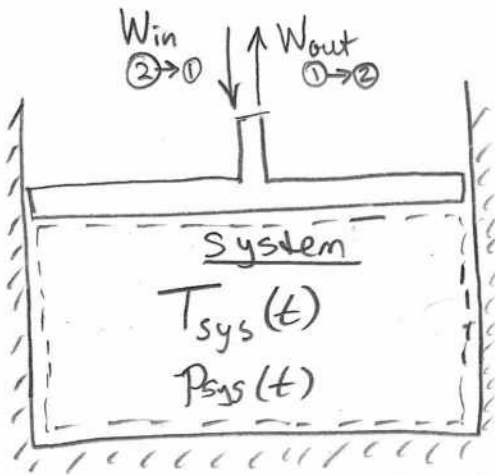


→ This is definitely a fictitious process but we use it as a tool for providing an upper limit to the efficiency of our engineering device design.

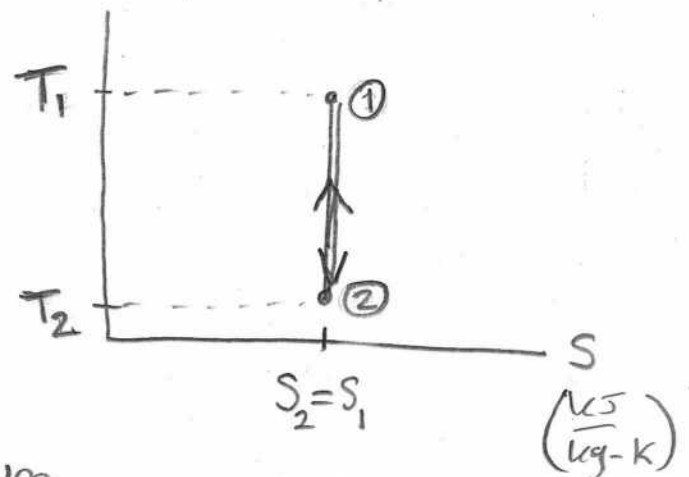
Examples

(22)

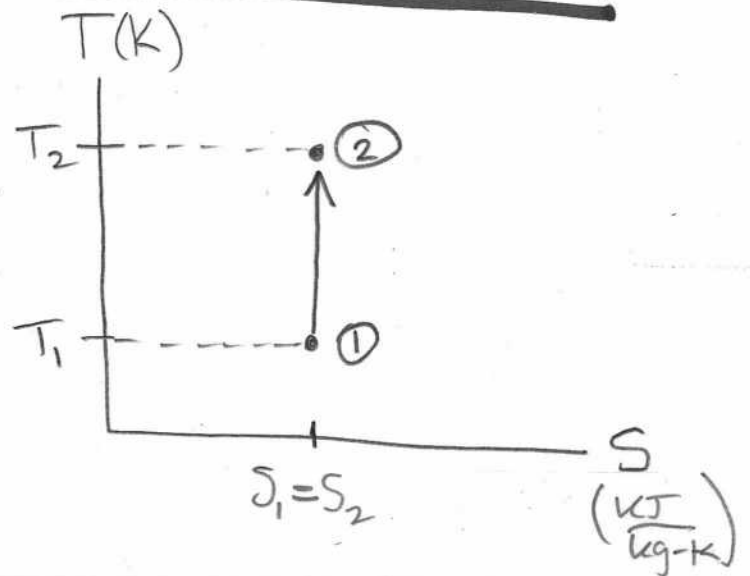
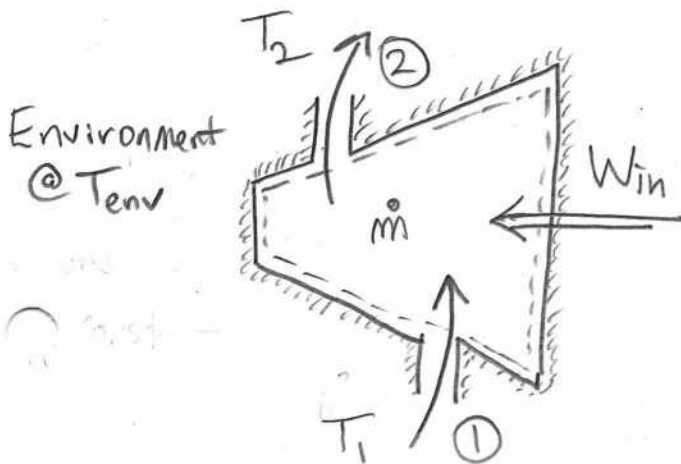
1) Adiabatic, No Friction, Quasi-Equilibrium, etc.
(This is called an **"Isentropic" Process**)



Environment
@ T_{env}



(2) Adiabatic, Frictionless
Compressor, Quasi-equilibrium
(This is again called an **"Isentropic" Process**)

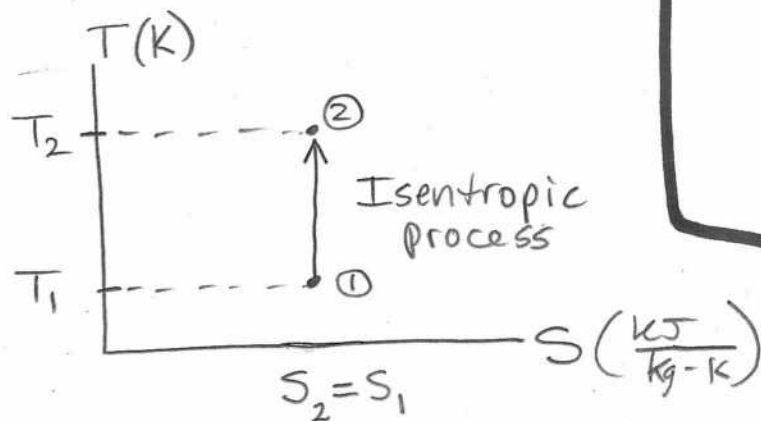


Isentropic Process

- A process that is both Internally and Externally Reversible
- Since it is adiabatic, the surroundings of the device are not affected by heat transfer, so it is also a reversible process

- $S_2 = S_1$

- $S_{gen} = 0$ Everywhere

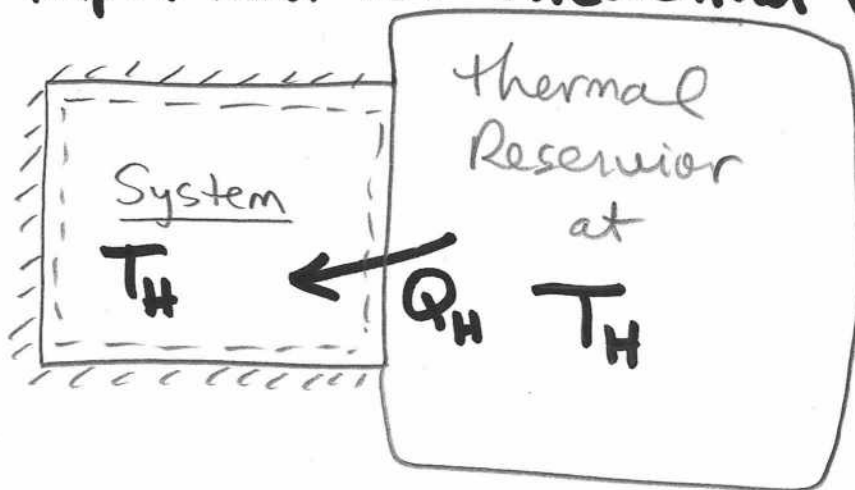


- We will be using isentropic processes (and relations) soon.

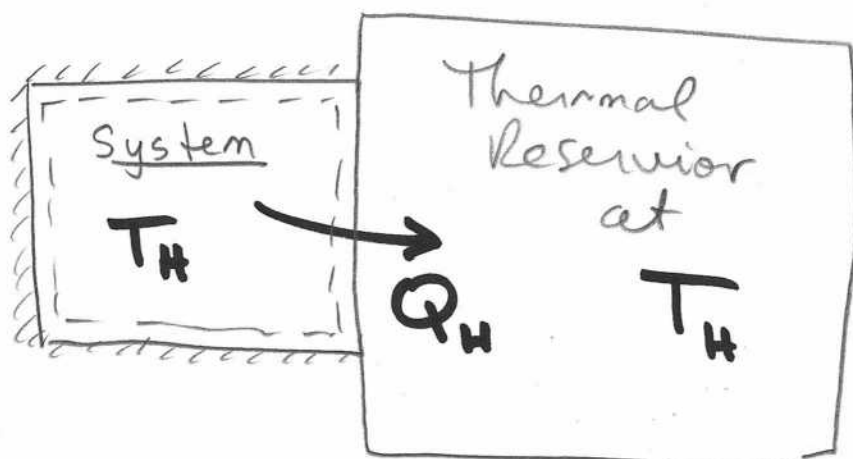
(3) Heat Transfer without a temperature difference

- This is absolutely impossible but is important for theoretical reasons!

Process
① → ②



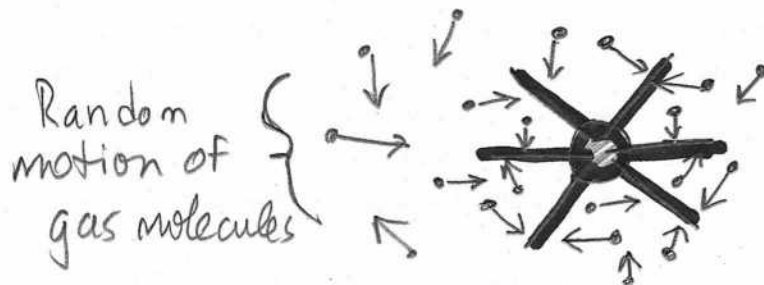
Process
② → ①



- We will use this in the development of the Carnot Cycle.
- The usefulness of this fictional Heat transfer is that it provides a "benchmark", or upper limit to our device performance.

Heat Engines and refrigerators re-organize only a Part of Random Thermal Energy

- Consider a paddle wheel immersed in a non-flowing gas:



Molecules hit the paddle wheel from all directions and the wheel, on average, remains motionless!

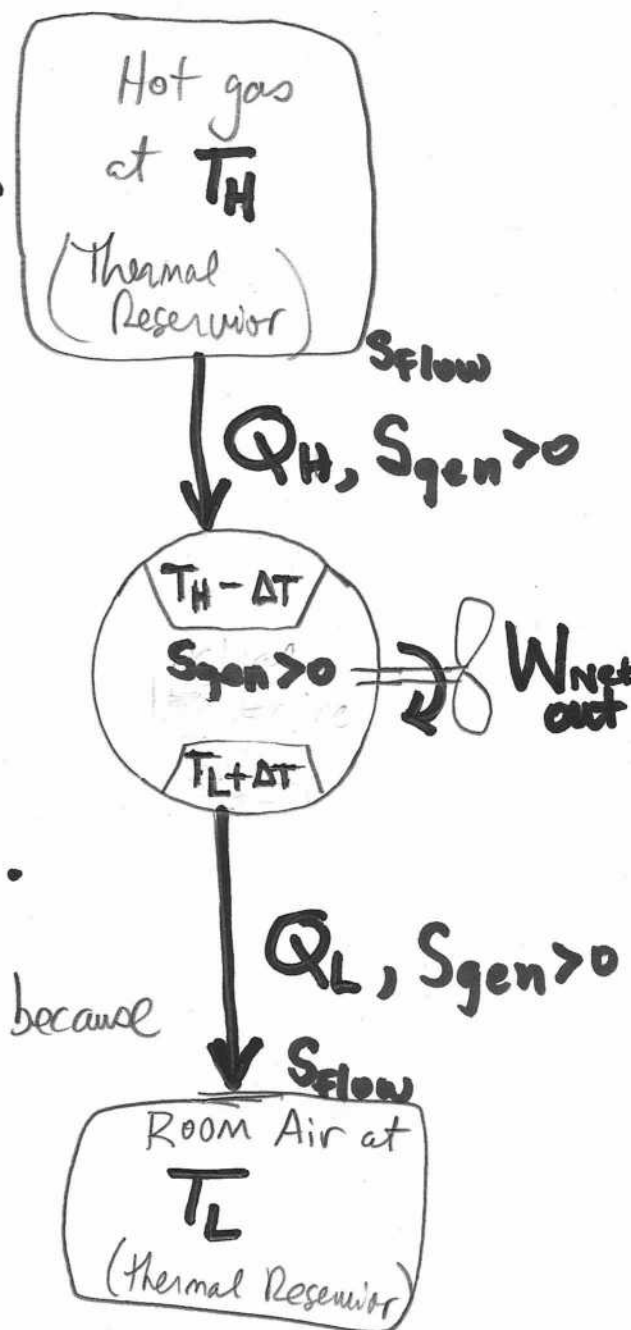
- You cannot extract work from a paddle wheel just "sitting" in a non-flowing gas!
- The 2nd Law states that disorganized energy cannot spontaneously, on its own, suddenly become organized and perform work output.
- In order to make the paddle wheel perform work output we need a device called a Heat Engine.
- Heat Engines can only convert a part of that disorganized thermal energy into ordered work output

⇒ The other part of the thermal energy that cannot be converted into work must be rejected as waste heat (thermal poop) at T_L which has a higher Entropy per unit mass than the initial energy at T_H

Bottom Line

26

Only a portion of this High T_H reservoir Thermal Energy can be re-organized and converted into useful net work output through the use of a Heat Engine.



⇒ Thermal Energy is "degraded" because of new Entropy generated, S_{gen} , during the process of:

- "Moving Q_H from the reservoir at T_H to the H.E. at $T_H - \Delta T$
- "Moving Q_L from the H.E. at $T_L + \Delta T$ to the room air at T_L
- "Converting thermal energy within the H.E. to work because of Friction, non-equilibrium processes, etc."

⇒ These interactions produce more disorder, or Entropy, and Q_H loses Ability to convert into Work

Lecture 23 | The Carnot Cycle: A Reversible Cycle That Achieves Maximum Thermal Efficiency

①

- We have learned that due to irreversibilities (Entropy production) within the System or Both the System + Surroundings, the ability of a device to achieve maximum efficiency is degraded.
- How do we know what the maximum possible efficiency is in order to say our design is "good" or "bad"?



Sadi Carnot, age 34
in 1830

⇒ Sadi Carnot derived a theoretical cycle that allows us to make this comparison (1823)

- Sadi devised the idea of closed cycle operation!

- Sadi Carnot published his work in a little (2)
book called "Reflections on the Motive Power of Fire" in 1824 but it didn't sell very well and was nearly forgotten.
- A famous chemist named Émile Clapeyron rediscovered this book years later and told Rudolf Clausius about it. With some help from Lord Kelvin, Clausius was able to discover the concept of Entropy in 1865 (Clausius' 9th Memoir).
- **The Carnot Cycle is purely fictional and cannot be achieved in Reality.** However, it provides us with a "benchmark" to assess the performance of an actual Heat Engine, Refrigerator, or Heat Pump. — **Independent of the details of the actual engine components!**
- The Carnot cycle is composed of 4 Reversible Processes:
 - (1) Isothermal Heat Addition & Expansion with Work output
 - (2) Isentropic Expansion with work output
 - (3) Isothermal Heat rejection & Compression with work input
 - (4) Isentropic Compression with work input

③

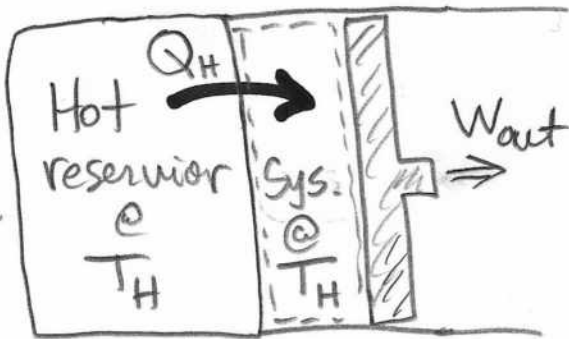
- Since each of these four processes making up the cycle are Reversible, there is no Net "footprint" left behind on either the System or Surroundings.
- **In fact, Carnot was very clever — he devised each process such that no irreversibilities (Entropy generated) occurred at all!**
 - ⇒ No net Entropy increase in the Universe occurs for the Carnot Cycle and so it provides the highest thermal efficiency any cyclical device can achieve.
- This makes sense — if there is no friction, it is a quasiequilibrium process, heat transfer occurs (unrealistically) without any ΔT , etc., then your device will operate perfectly from a thermodynamic viewpoint and it will have maximum efficiency!

The Carnot Cycle

④

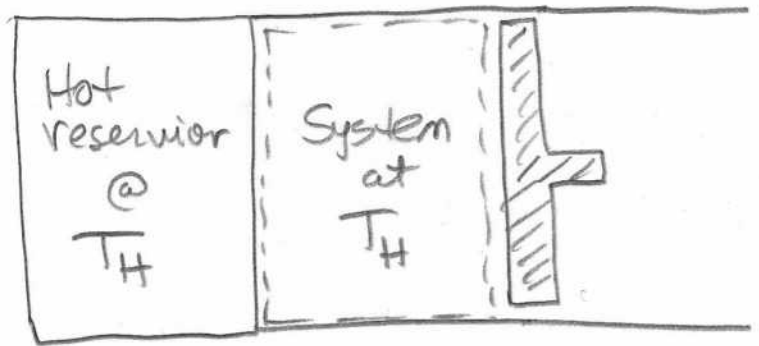
Step 1 Isothermal Heat Addition & Expansion with work output

State ①

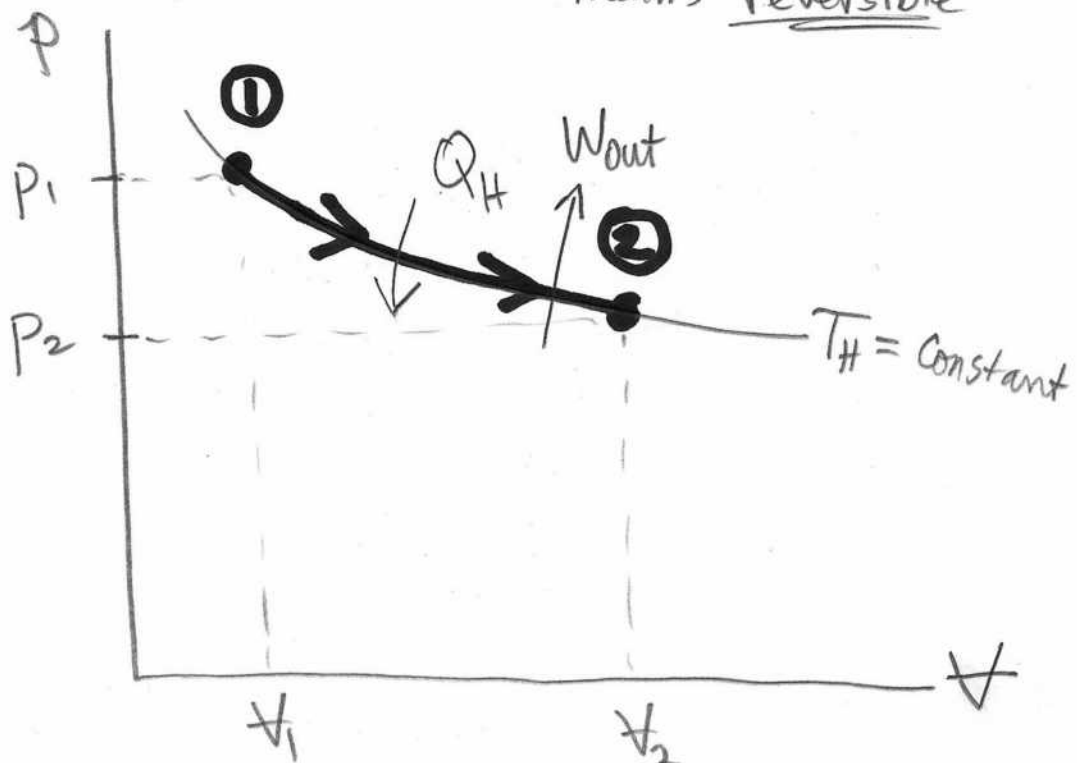


$\Delta T = 0$

State ②



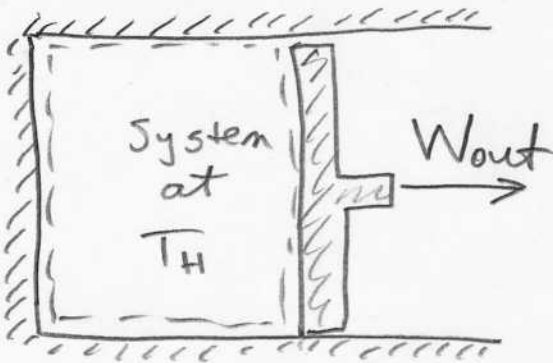
(the unrealistic assumptions here are reversible heat transfer → $\Delta T = 0$, no friction, quasiequilibrium process, etc. which means reversible)



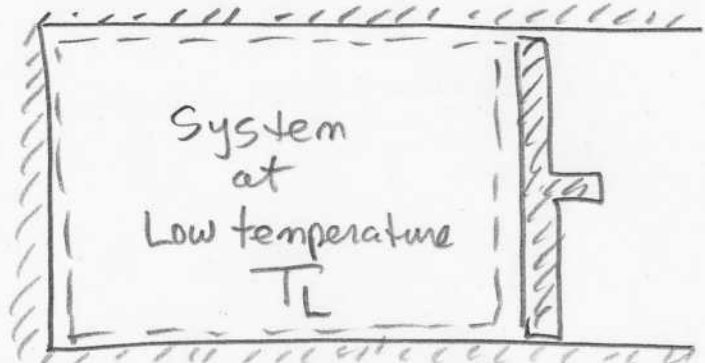
Step 2 Isentropic Expansion with work output

- The T_H reservoir is taken away and the cylinder is covered with perfect insulation
- Since the pressure inside the cylinder is greater than the surroundings, the piston still continues to perform work output.

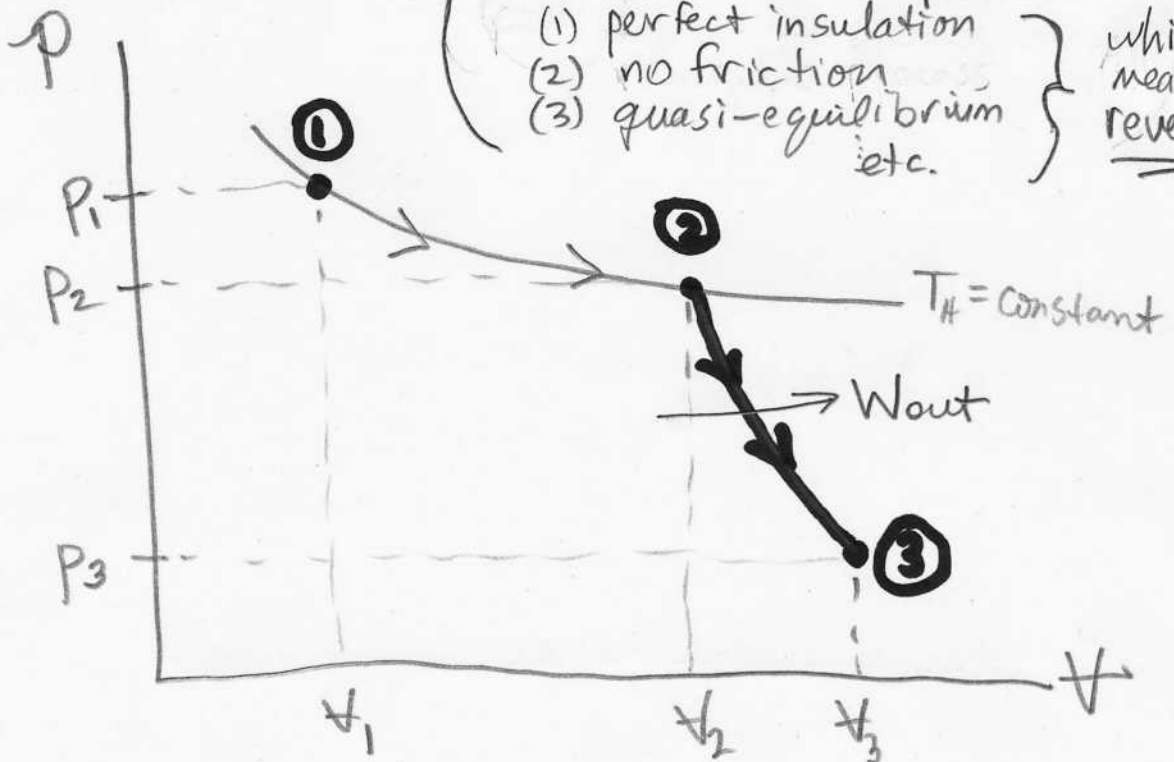
State ②



State ③



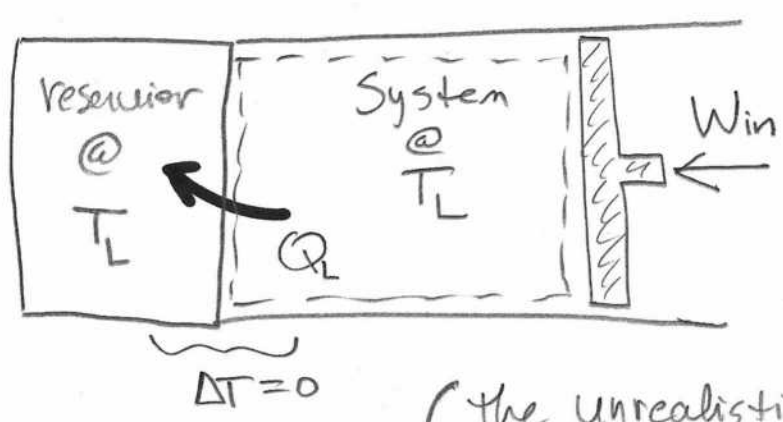
(the unrealistic assumptions are :
 (1) perfect insulation
 (2) no friction
 (3) quasi-equilibrium etc.) } which means reversible



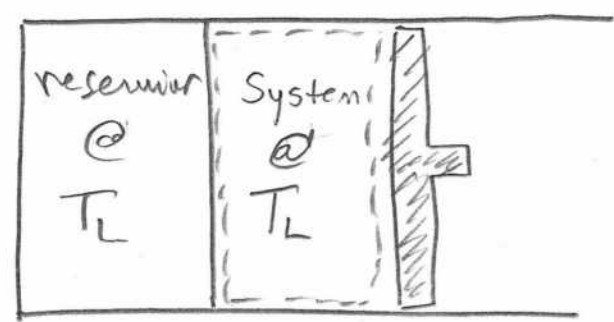
Step 3 Isothermal Heat rejection & Compression with Work input

- The perfect insulation is taken off the cylinder and a thermal reservoir at T_L is placed on the cylinder head.
- Work input is provided to the System through the piston.

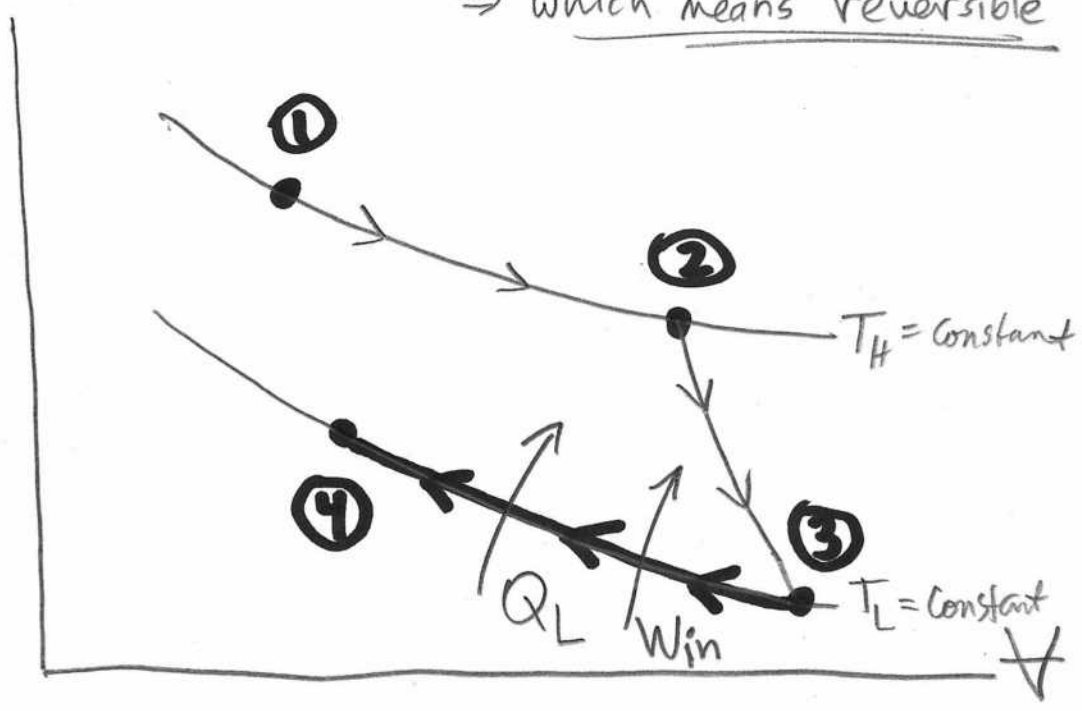
State ③



State ④

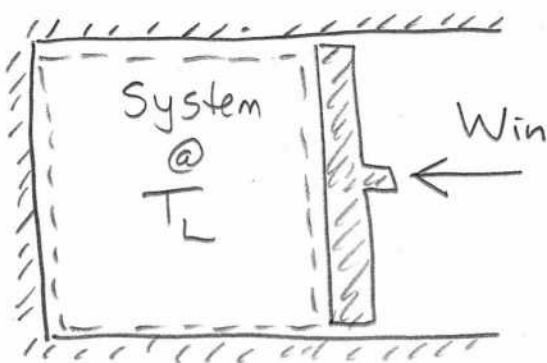
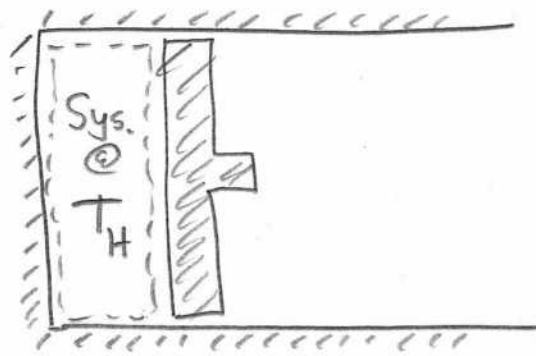


(the unrealistic assumptions here are reversible heat transfer — $\Delta T = 0$, no friction, quasiequilibrium process, etc.)
 \Rightarrow which means reversible



Step 4I sentropic Compression with Work Input

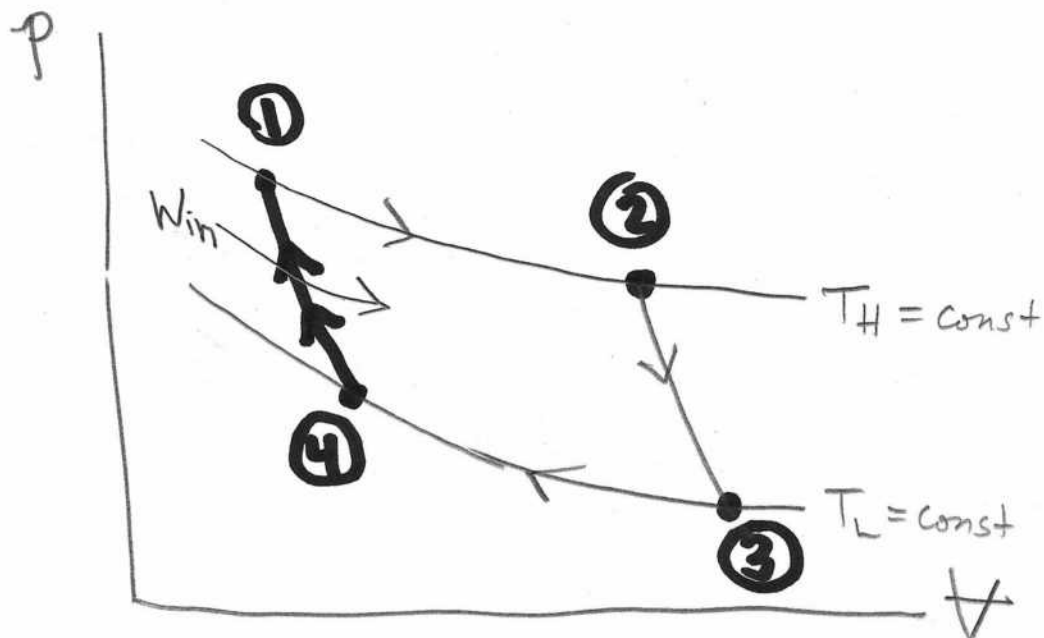
- The reservoir @ T_L is taken away and the cylinder is wrapped with perfect insulation again.
- Work input is provided to the System through the piston.

State ④State ①

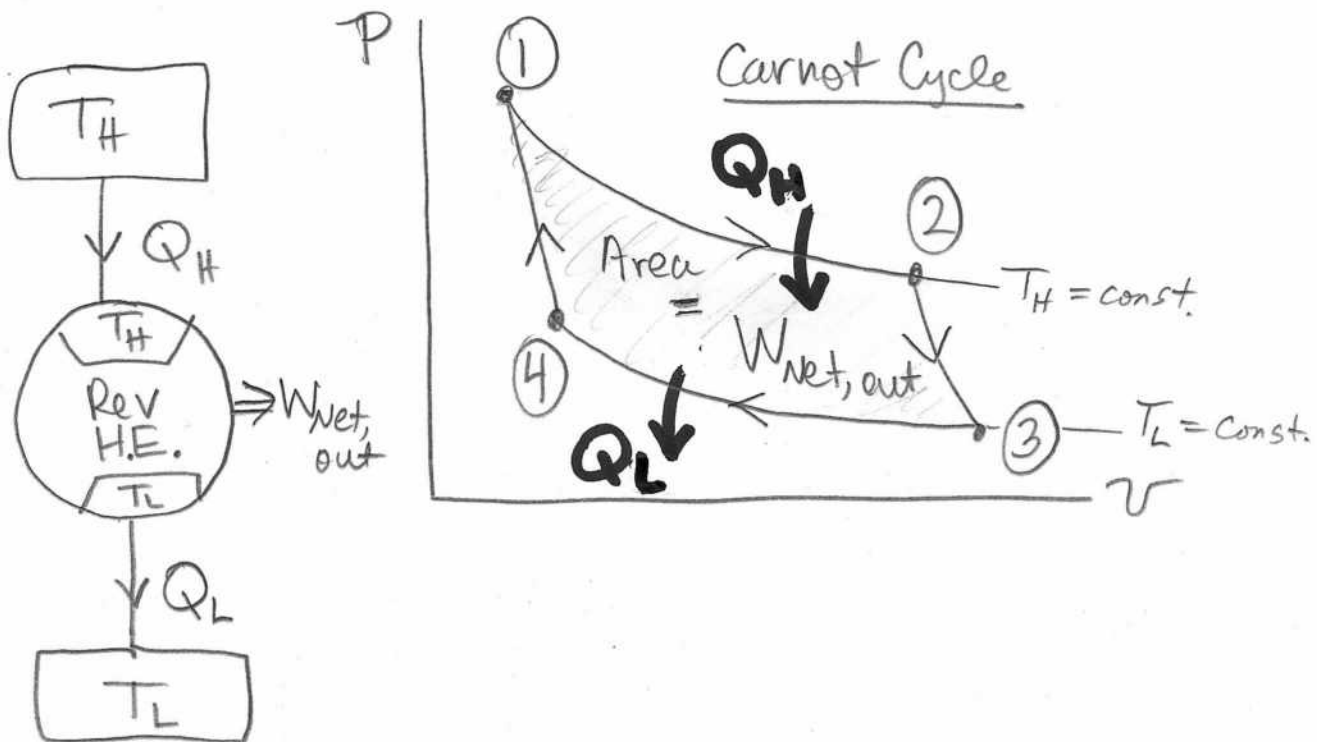
the unrealistic assumptions here are

- (1) perfect insulation
- (2) no friction
- (3) quasi-equilibrium etc.

which means
its reversible

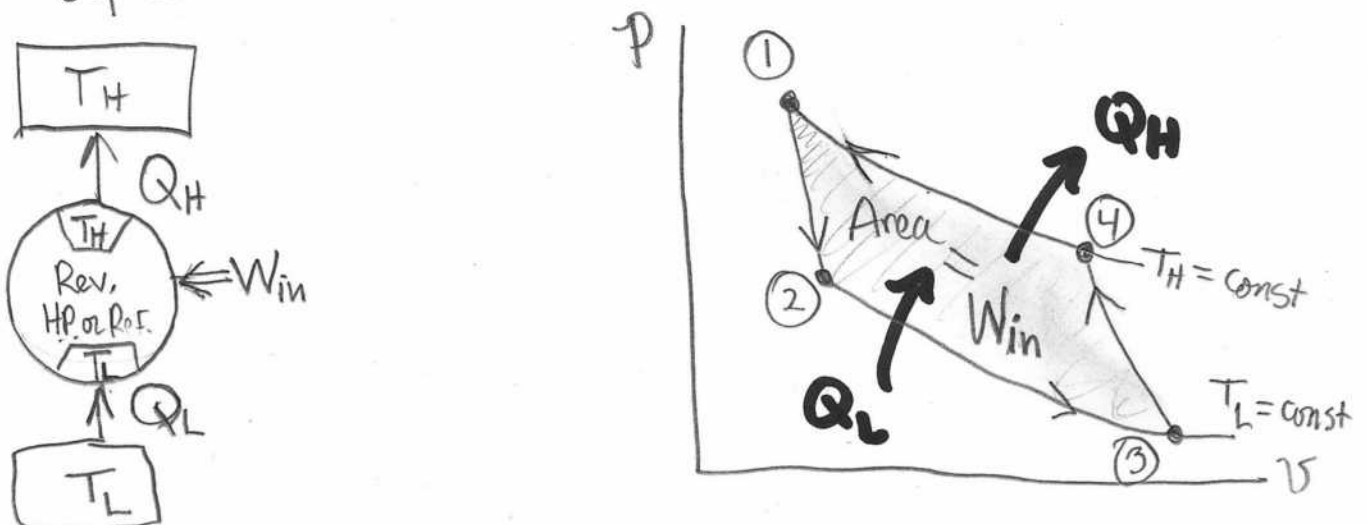


- Since the only work performed here is " $p dV$ " work, then the Area under a p - V diagram indicates the Net Work Output of the Cycle:



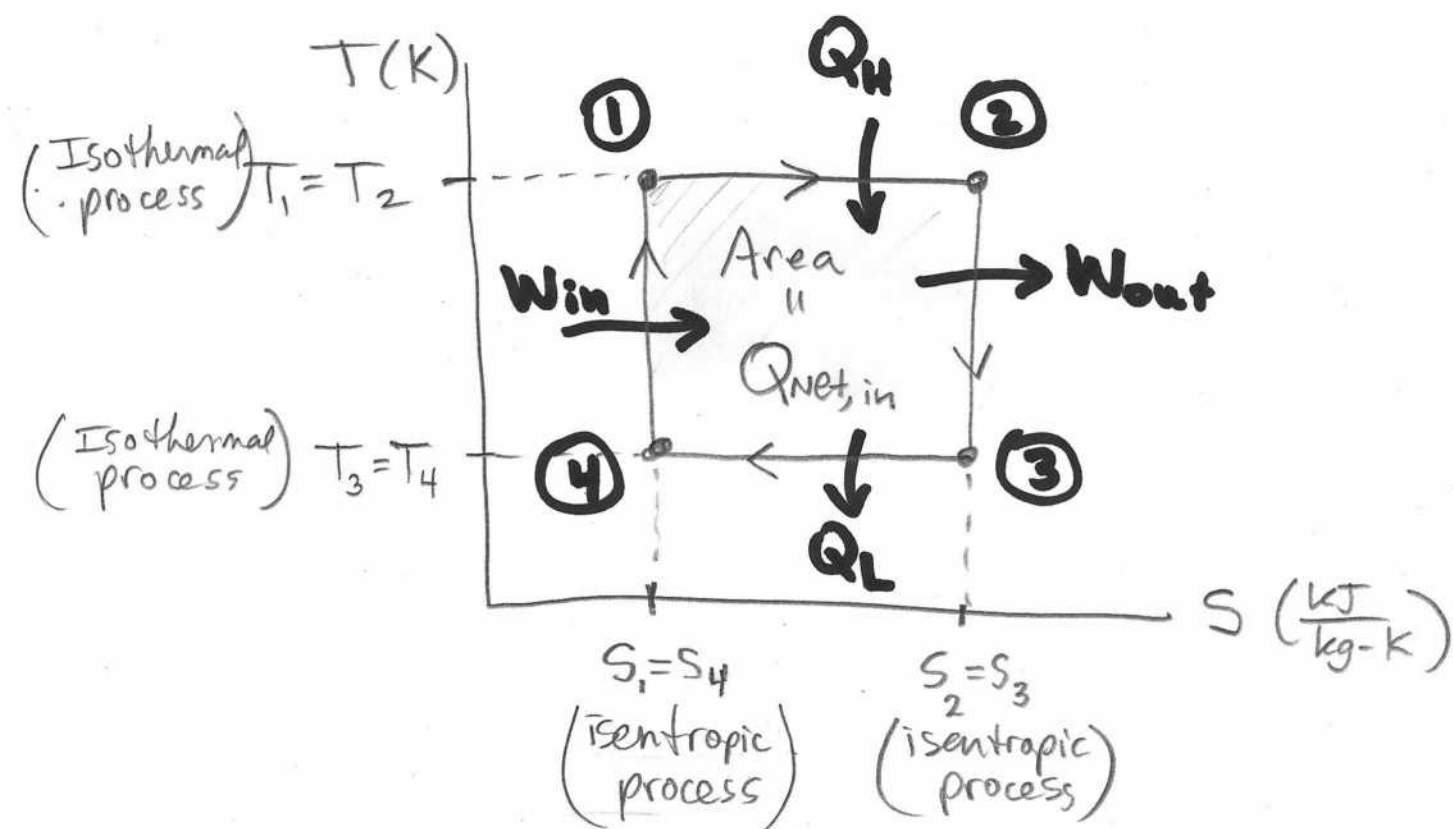
• Carnot Refrigerator/Heat Pump

The Processes, when performed in the opposite direction, lead to a Reversible Refrigeration/H.P. cycle:



Carnot Cycle Easy to visualize on a T-s diagram

- One neat thing about the T-s diagram is that it allows you to easily visualize the processes of the Carnot cycle:



⇒ The Carnot Cycle is a "Box" on a T-s diagram! Easy to remember!

- For a Carnot Refrigerator or Heat-Pump, all directions of processes & interactions are reversed on a T-s diagram.

The Carnot Principles

(10)

- The Carnot Principles pertain to the thermal efficiencies of reversible and irreversible (actual) Heat Engines (and Refrigerator/Heat Pumps) based on both the Kelvin-Planck and Clausius Statements of the 2nd Law of Thermodynamics.

The Carnot Principles

(1) $\eta_{th} < \eta_{th, rev}$ When both Heat Engines operate between the same two reservoir temperatures T_H and T_L

(2) The thermal efficiency of all reversible Heat Engines ($\eta_{th, rev}$) is the same when operating between the same two reservoir temperatures, T_H and T_L

$$\Rightarrow \eta_{th, rev} = f(T_H, T_L) \text{ only !}$$

$\Rightarrow \eta_{th, rev}$ is independent of what mechanisms and processes going on inside the H.E.

The Kelvin-Carnot Relation and Reversible Cycle (Carnot) Thermal Efficiency

Q: How do we actually calculate the thermal efficiency of any reversible Heat Engine or Refrigerator/Heat Pump?

A: We need to Consider a Reversible Heat Engine and derive a relationship between T_L , T_H , Q_L , and Q_H to obtain $\eta_{th, rev} = f(T_H, T_L)$.

• Lord Kelvin used Carnot's idea of a reversible cycle to obtain this relationship

• Because the Kelvin-Carnot Relation is so IMPORTANT, I will outline it here.

⇒ Note that this is where many students stumble in Thermo II because they don't know how or when to use the Kelvin-Carnot relation for computing reversible cycle efficiencies!

Brief Derivation of the Kelvin Carnot Relation 12

- The full derivation is in the textbook, but this will give you an idea of where it comes from.
- For any Heat Engine, irreversible or reversible,

$$\boxed{\eta_{th} = 1 - \frac{Q_L}{Q_H}}$$

- From The Carnot Principle #2, for any reversible Heat Engine,

$$\boxed{\eta_{th, rev} = f(T_H, T_L)} \text{ only}$$

$$\Rightarrow \eta_{th, rev} = 1 - \frac{Q_L}{Q_H} = f(T_H, T_L)$$

\Rightarrow A functional relationship exists between only Q_L, Q_H, T_L , and T_H for reversible H.E.'s

$$\Rightarrow \boxed{\left(\frac{Q_H}{Q_L}\right)_{rev} = f(T_H, T_L)}$$

- Using Reversible cycles, it can be shown that

$$\boxed{\left(\frac{Q_H}{Q_L}\right)_{rev} = \frac{\phi(T_H)}{\phi(T_L)}}$$

where ϕ is some function of temperature.

Q: What is the functional form of $\varphi(T)$?

Is it

(1) $\varphi(T) = Ae^{T/B}$?

(2) $\varphi(T) = \sqrt{a + bT}$?

(3) $\varphi(T) = a + bT + cT^2 + dT^3$?

A: The functional form of $\varphi(T)$ is completely arbitrary!

⇒ The only requirement that the 2nd Law places on reversible H.E.'s or Refrigerator/H.P.'s is that

$$\left(\frac{Q_H}{Q_L}\right)_{\text{rev}} = \frac{\varphi(T_H)}{\varphi(T_L)}$$

So the choice is yours!

• Lord Kelvin chose

$$\varphi(T) \equiv T$$

$$0 \leq T < \infty$$

T is in units of Kelvin

⇒

$$\left(\frac{Q_H}{Q_L}\right)_{\text{rev}} = \frac{T_H}{T_L}$$

The Kelvin-Carnot Relation

Memorize This!

Carnot Heat Engine Cycle Efficiency

Starting with

$$\eta_{th} = 1 - \frac{Q_L}{Q_H}$$

Any cycle
(Heat Engine)

and Using Kelvin's Relation for a Reversible Cycle,

$$\left(\frac{Q_L}{Q_H}\right)_{rev} = \frac{T_L}{T_H}$$

we find that the thermal efficiency of a reversible Heat Engine is

$$\eta_{th, rev} = 1 - \frac{T_L}{T_H}$$

Totally Reversible H.E.
cycle only!

Notice that Even the Carnot cycle cannot be 100% efficient because it must reject Heat to a cold thermal reservoir according to the 2nd Law of Thermo (see Lecture 20, page 10)

$$\eta_{th, rev} < 1 \Rightarrow$$

$\left. \begin{array}{l} T_L \neq 0 \\ Q_L \neq 0 \end{array} \right\}$ for a totally reversible cycle like the Carnot cycle

Carnot Refrigerator & Heat Pump Cycle Efficiency

- (1) A Carnot Refrigerator is simply a Carnot H.E. operated backwards.

For any refrigerator,

$$\text{COP}_R = \frac{1}{\frac{\dot{Q}_H}{\dot{Q}_L} - 1}$$

Using Kelvin's Relation

$$\left. \frac{\dot{Q}_H}{\dot{Q}_L} \right|_{\text{rev}} = \frac{T_H}{T_L},$$

So

$$\boxed{\text{COP}_{R,\text{rev}} = \frac{1}{\frac{T_H}{T_L} - 1}}$$

> 1

**Reversible
Carnot
Refrigerator**

- (2) A Carnot Heat Pump is simply a Carnot H.E. operated backwards.

For any Heat Pump,

$$\text{COP}_{\text{HP}} = \frac{1}{1 - \frac{\dot{Q}_L}{\dot{Q}_H}}$$

Using Kelvin's Relationship,

$$\left. \frac{\dot{Q}_H}{\dot{Q}_L} \right|_{\text{rev}} = \frac{T_H}{T_L}$$

$$\boxed{\text{COP}_{\text{HP},\text{rev}} = \frac{1}{1 - \frac{T_L}{T_H}}}$$

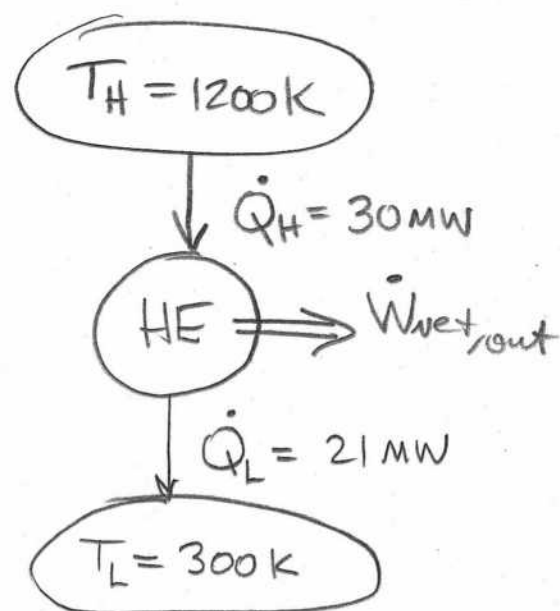
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**Reversible
Carnot
Heat Pump**

Example 1 Heat Engine Problem

You are an engineer in an electric-generation station. The boiler is at 1200K and the cooling water is 300K from a nearby river. The Heat transfer rate from the Furnace to the Boiler is 30 MW and the heat rejection rate to the river is 21 MW.

- Find**
- (a) η_{th}
 - (b) $\eta_{th, rev}$
 - (c) $\dot{W}_{net, out}$



Analysis

- (a) The actual thermal efficiency of this power plant is

$$\eta_{th} = 1 - \frac{\dot{Q}_L}{\dot{Q}_H} = 1 - \frac{21 \text{ MW}}{30 \text{ MW}}$$

$$\boxed{\eta_{th} = 30\%}$$

- (b) The Maximum efficiency that this plant can ever achieve using the given T_H and T_L is:

$$\eta_{th, rev} = 1 - \frac{T_L}{T_H} = 1 - \frac{300 \text{ K}}{1200 \text{ K}}$$

$$\boxed{\eta_{th, rev} = 75\%}$$

- (c) The Net Power Output of this plant is

$$\dot{W}_{net, out} = \dot{Q}_{in} - \dot{Q}_{out} = 30 - 21 \text{ MW} \Rightarrow \boxed{\dot{W}_{net, out} = 9 \text{ MW}}$$

Example 2 Refrigerator Problem

(17)

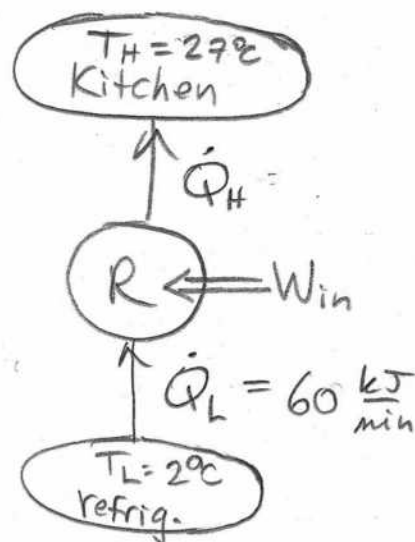
A household refrigerator with a $COP_R = 7.0$ removes heat from a refrigerated space at a rate of $60 \frac{kJ}{min}$.

The refrigerator cold space is at $5^\circ C$ while the kitchen is at $27^\circ C$.

Find (a) Electric Power Consumed by the refrigerator in Watts

(b) the rate of heat transfer to the kitchen air in kW

(c) the Maximum COP_R attainable for the given reservoir temperatures. (totally reversible cycle)



Analysis

(a) By Definition, $COP_R = \frac{\dot{Q}_L}{\dot{W}_{in}}$ so that

$$\dot{W}_{in} = \frac{\dot{Q}_L}{COP_R} = \frac{60 \frac{kJ}{min} \times \frac{1 min}{60 sec}}{7.0} = \frac{1 kW}{7}$$

$$\boxed{\dot{W}_{in} = 143 W}$$

(b) From a closed system Energy Balance for a cycle,

$$\dot{W}_{in} = \dot{Q}_H - \dot{Q}_L$$

so $\dot{Q}_H = \dot{W}_{in} + \dot{Q}_L = 143 W + 1000 W$

$$\boxed{\dot{Q}_H = 1143 W}$$

(c) $COP_{R,rev} = \frac{1}{T_H/T_L - 1} = \frac{1}{\frac{27+273}{2+273} - 1} \rightarrow \boxed{COP_{R,rev} = 11.0}$

Lecture 24

Examples of Reversible Cyclical Device Performance and The Degradation of Energy According to the 2nd Law

Example 1 Heat Engine Problem

You are an engineer in an electric-generation station. The boiler is at 1200K and the cooling water is 300K from a nearby river. The Heat transfer rate from the Furnace to the Boiler is 30 MW and the heat rejection rate to the river is 21 MW.

Find (a) η_{th}

(b) $\eta_{th, rev}$

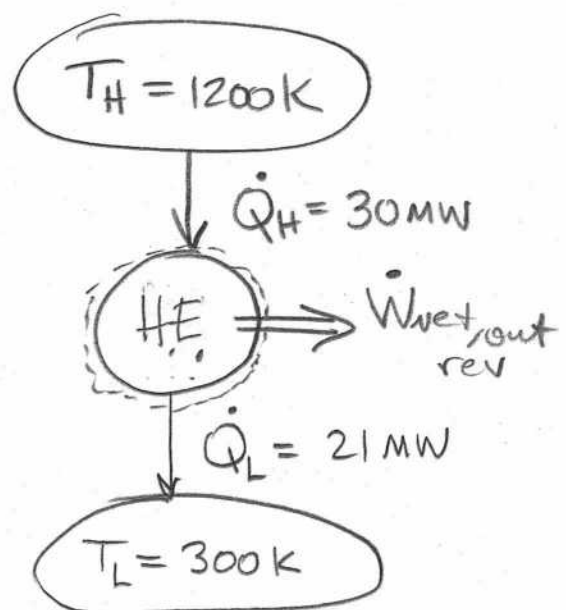
(c) $\dot{W}_{net, out, rev}$ (Maximum allowed for given T_L, T_H)

Analysis

(a) The actual thermal efficiency of this power plant is

$$\eta_{th} = 1 - \frac{\dot{Q}_L}{\dot{Q}_H} = 1 - \frac{21 \text{ MW}}{30 \text{ MW}}$$

$$\boxed{\eta_{th} = 30\%}$$



- (b) The Maximum efficiency that this plant can ever achieve using the given T_H and T_L is:

$$\eta_{th, rev} = 1 - \frac{T_L}{T_H} = 1 - \frac{300K}{1200K}$$

$$\eta_{th, rev} = 75\%$$

- (c) The Net Power Output of this plant is

$$\dot{W}_{net, out, rev} = \dot{Q}_{in} - \dot{Q}_{out} = 30 - 21 \text{ MW} \Rightarrow \dot{W}_{net, out, rev} = 9 \text{ MW}$$

• The degradation of \dot{Q}_H of Irreversible vs. Reversible H.E.

- Consider a single operational cycle period, Δt , such that the Heat input during that cycle is $Q_H = \dot{Q}_H \Delta t$ (MJ). This Q_H is the Area for both pie charts:

Note: % Waste Heat = $\frac{Q_H - Q_L}{Q_H}$
or % Waste Heat = $1 - \frac{Q_L}{Q_H} = \eta_{th}$

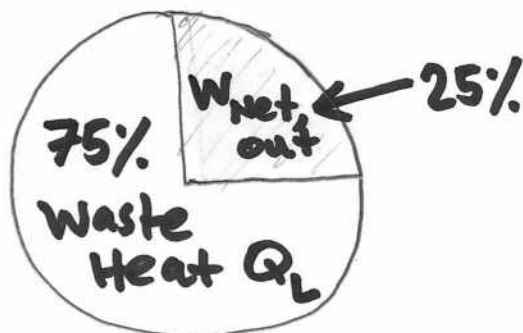
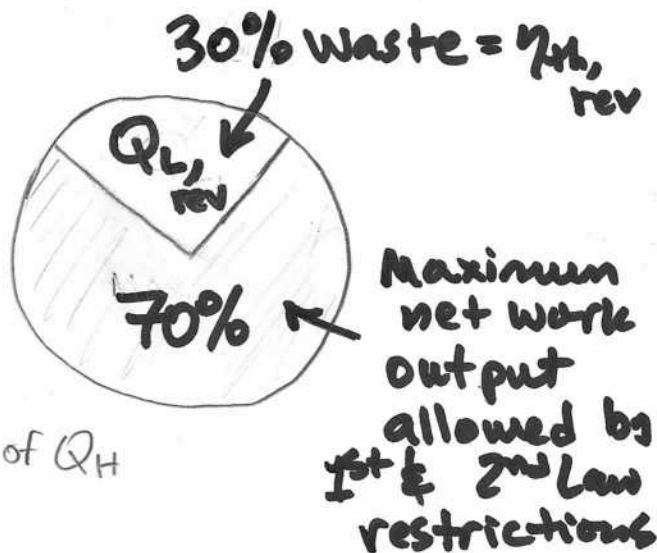
Reversible H.E.

- The 2nd Law required that 30% of Q_H be rejected \rightarrow 30% degradation of Q_H

Irreversible H.E.

- The 2nd Law required you to reject 75% of Q_H

$\Rightarrow Q_H$ degraded by 75% due to internal and external Irreversibilities



Example 2 Refrigerator Problem

(3)

A household refrigerator with a $COP_R = 2.8$ ($EER \approx 9.5$) removes heat from the cold space at a rate of $\dot{Q}_L = 30 \text{ kJ/min}$. The refrigerator cold space is maintained at 2°C , while the kitchen is at 27°C .

Find: (a) The electric power consumed by this refrigerator in Watts (W).

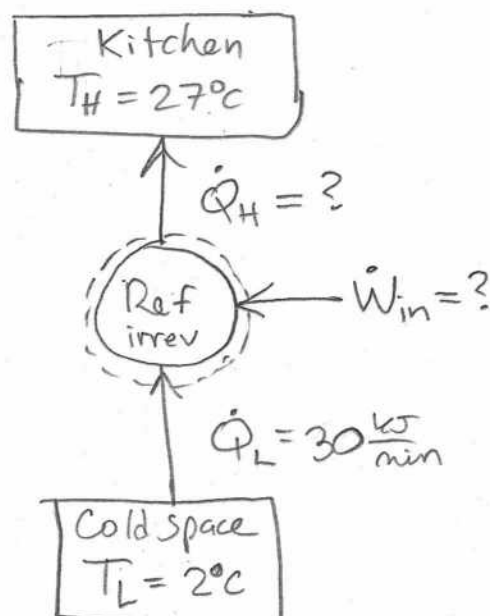
(b) The rate of heat rejection to the kitchen in Watts (W)

(c) The Maximum COP_R and EER allowed by the 2nd Law for the given T_L, T_H .

(d) The minimum work input required by the 2nd Law assuming that $\dot{Q}_L = 30 \frac{\text{kJ}}{\text{min}}$ for the given T_L & T_H .

(e) The minimum heat rejection allowed by the 2nd Law for given T_L & T_H .

Assume (1) Steady, cyclical operation



Analysis (a) \dot{W}_{in} (Watts)

- Since we only know \dot{Q}_L , we cannot use the Closed Sys. Energy balance to find \dot{W}_{in} .
- We have been given $COP_R = 2.8$ so we use the definition:

$$COP_R \equiv \frac{\dot{Q}_L}{\dot{W}_{in}}$$
$$\text{or } \dot{W}_{in} = \frac{\dot{Q}_L}{COP_R} = \frac{30 \frac{\text{kJ}}{\text{min}} \times \frac{1 \text{ min}}{60 \text{ sec}}}{2.8} = 0.178 \text{ kW} \times \frac{1000 \text{ W}}{\text{kW}}$$

$\dot{W}_{in} = 178 \text{ W}$

(b) $\dot{Q}_H (W)$

- We can now use the Closed System energy balance on the refrigerator:

$$\dot{W}_{\text{Net},\text{out}} = \dot{Q}_{\text{Net},\text{in}}$$

$$\dot{W}_{\text{out}} - \dot{W}_{\text{in}} = \dot{Q}_{\text{in}} - \dot{Q}_{\text{out}}$$

$$-\dot{W}_{\text{in}} = \dot{Q}_L - \dot{Q}_H$$

or, $\boxed{\dot{Q}_H = \dot{Q}_L + \dot{W}_{\text{in}}}$

$$\dot{Q}_H = \left(30 \frac{\text{kJ}}{\text{min}} \times \frac{1 \text{ min}}{60 \text{ sec}}\right) \times \frac{1000 \text{ J}}{\text{kJ}} + 178 \text{ W}$$

$$\dot{Q}_H = 500 \text{ W} + 178 \text{ W} \longrightarrow \boxed{\dot{Q}_H = 678 \text{ W}}$$

(Note: $\dot{Q}_L = 500 \text{ W}$)

(c) $\text{COP}_{R,\text{rev}}$ and EER_{rev}

- Since we do not have either $\dot{W}_{\text{in},\text{rev}}$ or $\dot{Q}_{L,\text{rev}}$, we must use the Kelvin-Carnot relation to find $\text{COP}_{R,\text{rev}}$:

$$\text{COP}_{R,\text{rev}} = \frac{1}{\frac{\dot{Q}_H}{\dot{Q}_L} - 1} \quad \text{and} \quad \left(\frac{\dot{Q}_H}{\dot{Q}_L}\right)_{\text{rev}} = \frac{T_H}{T_L}$$

Combining these expressions,

$$\text{COP}_{R,\text{rev}} = \frac{1}{\frac{T_H}{T_L} - 1} = \frac{1}{\left(\frac{27^\circ\text{C} + 273^\circ\text{C}}{2^\circ\text{C} + 273^\circ\text{C}}\right) - 1} = \frac{1}{\frac{300 \text{ K}}{275 \text{ K}} - 1}$$

$$\text{COP}_{R,\text{rev}} \approx \frac{1}{1.0909 - 1}$$

$$\boxed{\text{COP}_{R,\text{rev}} = 11}$$

$$\boxed{\text{EER}_{\text{rev}} \approx 37 \frac{\text{Btu}}{\text{Wh}}}$$

$$\text{EER}_{\text{rev}} \approx \text{COP}_{R,\text{rev}} \times 3.412 \frac{\text{Btu}}{\text{Wh}} = 11 \times 3.412 \frac{\text{Btu}}{\text{Wh}}$$

⑤

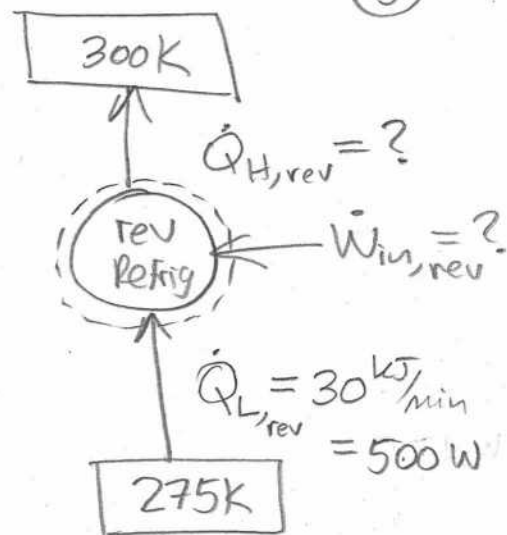
(d) $\dot{W}_{in,rev}$ (Watts)

- The most direct way to compute $\dot{W}_{in,rev}$ is to use the definition of COP_R :

$$COP_{R,rev} = \frac{\dot{Q}_{L,rev}}{\dot{W}_{in,rev}}$$

$$\dot{W}_{in,rev} = \frac{\dot{Q}_{L,rev}}{COP_{R,rev}} = \frac{500W}{11.0} = 0.045 kW \times \frac{1000W}{1kW}$$

$$\dot{W}_{in,rev} = 45W$$



(e) $\dot{Q}_{H,rev}$ (W)

- Using a closed system energy balance on the refrigerator,

$$\dot{Q}_{H,rev} = \dot{Q}_{L,rev} + \dot{W}_{in,rev}$$

$$\dot{Q}_{H,rev} = 500W + 45W \rightarrow \dot{Q}_{H,rev} = 545W$$

Summary

	T_H (K)	T_L (K)	\dot{Q}_L (W)	\dot{Q}_H (W)	\dot{W}_{in} (W)	COP_R	EER (Btu/Wh)
Irreversible	303	275	500	678	178	2.8	9.5
Reversible				545	45	11	37

Actual heat rejection is 1.2x higher than minimum allowed

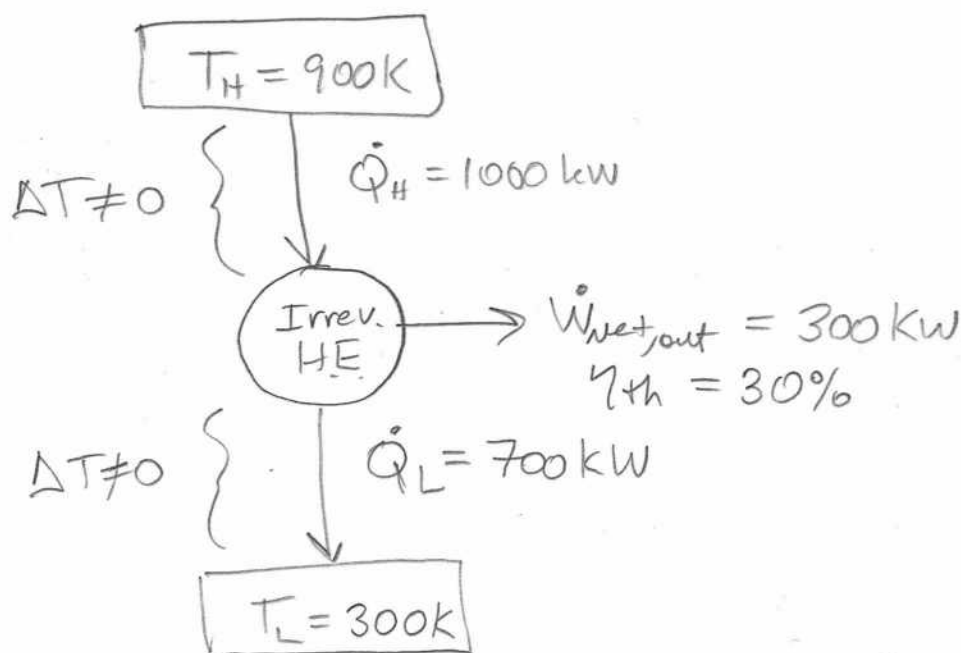
Actual work required is 4x the minimum required!

Example 3

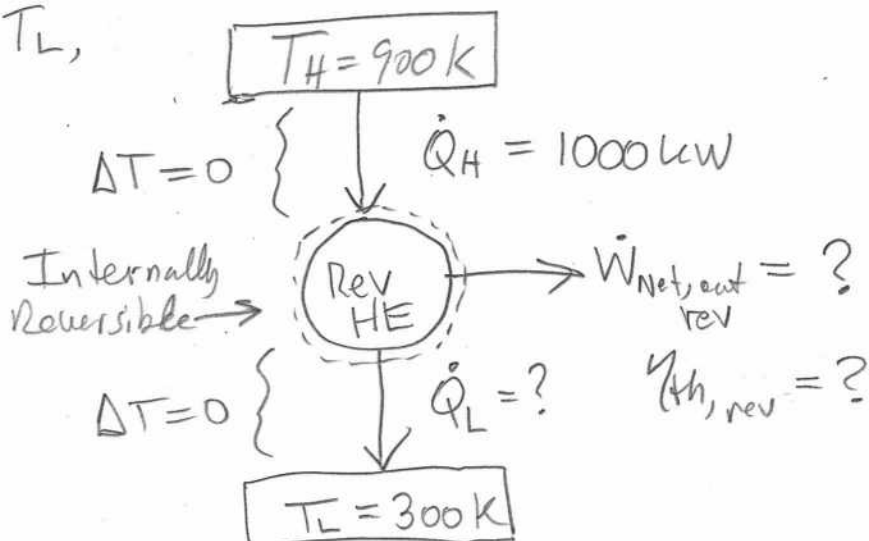
Reversible H.E. Compared to an Irreversible H.E. For same T_H, T_L

(6)

- Recall the Heat Engine Example 1 from Lecture 21:



- In that Example, the HE was assumed to be both Externally and Internally Irreversible and so $\eta_{\text{th}} = 30\%$ for $T_H = 900\text{ K}$ and $T_L = 300\text{ K}$.
- Let us now find the Maximum thermal efficiency allowed by the 2nd Law by assuming that it is Reversible for the same T_H, T_L , and \dot{Q}_H .

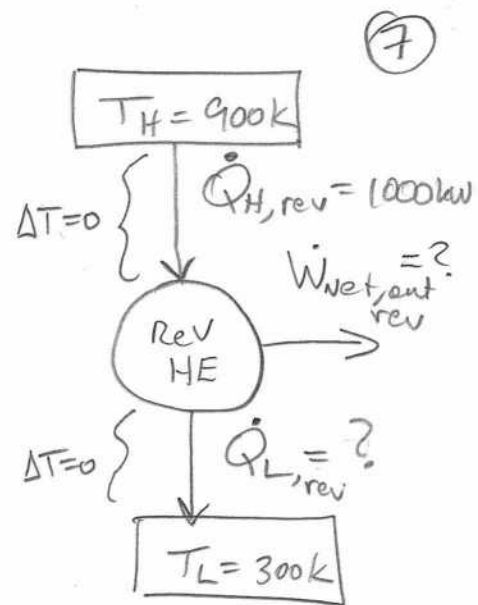


Find: (a) The minimum heat rejection rate required by the 2nd Law.

(b) The Maximum net work output allowed by the 2nd Law.

(c) The Maximum thermal efficiency allowed by the 2nd Law for the given T_L and T_H .

Assume: (1) Steady cyclical operation
(2) Totally reversible H.E.



Analysis

(a) $\dot{Q}_{L,rev}$

- We want to find the minimum heat rejection rate $\dot{Q}_{L,rev}$ given $\dot{Q}_{H,rev}$ for the specified T_H and T_L .

\Rightarrow The Kelvin-Carnot relationship relates these four quantities for reversible Heat Engines:

$$\left(\frac{\dot{Q}_H}{\dot{Q}_L} \right)_{rev} = \frac{T_H}{T_L}$$

$$\Rightarrow \dot{Q}_{L,rev} = \dot{Q}_{H,rev} \left(\frac{T_L}{T_H} \right)$$

$$\dot{Q}_{L,rev} = (1000\text{ kW}) \left(\frac{300\text{ K}}{900\text{ K}} \right) \rightarrow$$

$$\boxed{\dot{Q}_{L,rev} = 333\text{ kW}}$$

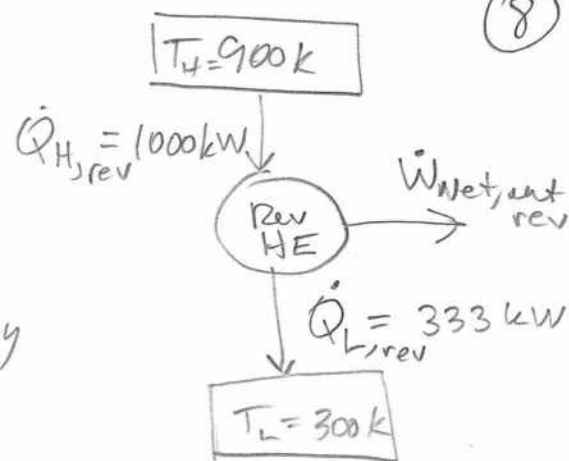
(b) $\dot{W}_{net,out,rev} = \max \dot{W}_{net,out}$

- Now that $\dot{Q}_{L,rev}$ is known, we have enough information to use the Closed System Energy Balance to find $\dot{W}_{net,out,rev}$:

$$\dot{W}_{net,out,rev} = \dot{Q}_{H,rev} - \dot{Q}_{L,rev}$$

$$\dot{W}_{net,out,rev} = 1000 \text{ kW} - 333 \text{ kW}$$

$$\Rightarrow \boxed{\dot{W}_{net,out,max} = 667 \text{ kW}}$$



(c) $\eta_{th,rev}$

- There are two ways of finding $\eta_{th,rev}$:

Approach 1

$$\eta_{th,rev} = \frac{\dot{W}_{net,out,rev}}{\dot{Q}_{H,rev}} = \frac{667 \text{ kW}}{1000 \text{ kW}}$$

$$\boxed{\eta_{th,rev} = 66.7\%}$$

Approach 2

- Use the Kelvin-Carnot relation

$$\eta_{th,rev} = 1 - \frac{\dot{Q}_{L,rev}}{\dot{Q}_{H,rev}} = 1 - \frac{T_L}{T_H}$$

$$\eta_{th,rev} = 1 - \frac{300 \text{ K}}{900 \text{ K}}$$

$$\boxed{\eta_{th,rev} = 66.7\%}$$

Summary

	T_H (K)	T_L (K)	\dot{Q}_H (kW)	\dot{Q}_L (kW)	\dot{W}_{net} (kW)	η_{th} (%)
Irreversible	900	300	1000	700	300	30
Reversible				333	667	67

9

• The Degradation of \dot{Q}_H For Irreversible vs. Reversible Heat Engine

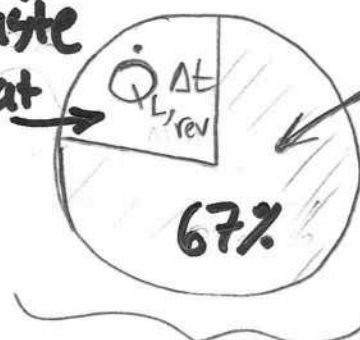
- For a single operational cycle period, Δt , the Heat input (kJ) for both reversible or irreversible H.E. is $Q_H = \dot{Q}_H \Delta t = \text{Area of both pie charts} :$

Reversible H.E.

for $T_H = 900\text{K}$
 $T_L = 300\text{K}$

Q_H degraded 33%

33%
Waste
Heat



Maximum
net work
output
allowed
by the
2nd Law

This is the theoretical
best you can do based
upon restrictions of the
1st and 2nd Laws

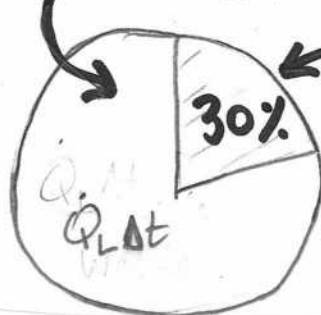
Irreversible H.E.

for $T_H = 900\text{K}$
 $T_L = 300\text{K}$

$W_{out} = 300\text{kJ}$

Q_H degraded 70%

70% Waste Heat

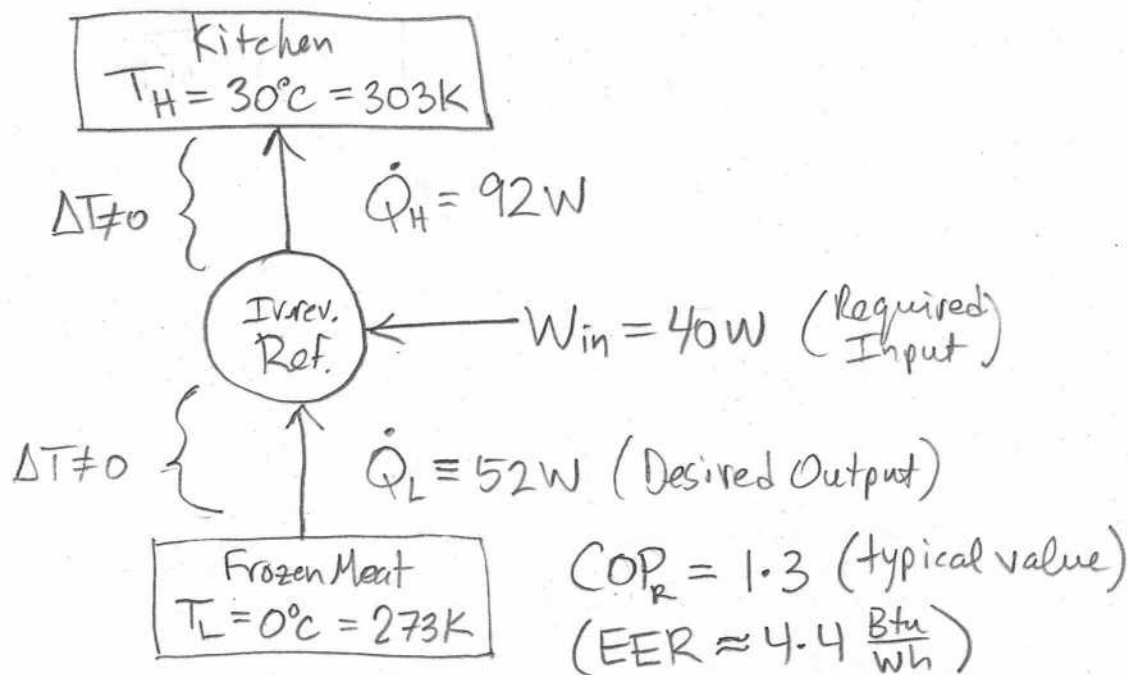


Real world net
Work Output

- The 2nd Law required you to reject 33% of $\dot{Q}_H \Delta t = Q_H (\text{kJ})$ as waste heat to the thermal reservoir in the case of a reversible H.E. Q_H was degraded by 33%.
- For the real H.E., you had to "pay more to Mother Nature" (a double penalty) by degrading Q_H 70% for the same T_H and T_L

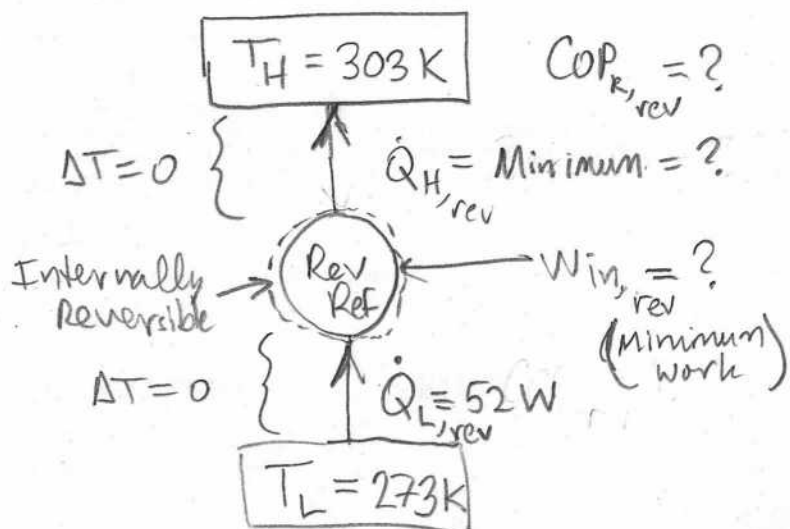
Example 4 Reversible Refrigerator compared to an Irreversible Refrigerator for same T_H, T_L

- Recall the Refrigerator Example 4 from Lecture 21:

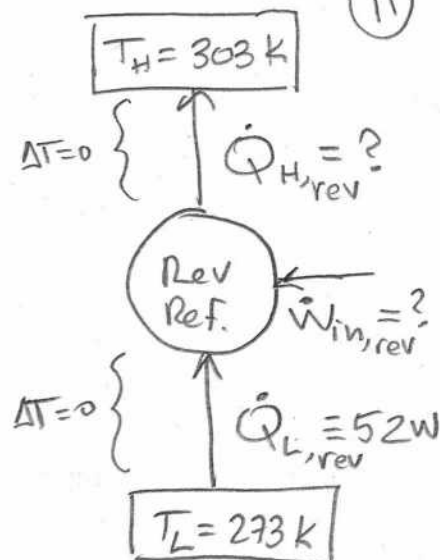


- In that Example, the Refrigerator/Freezer was assumed to be both Externally and Internally Irreversible so that $\dot{Q}_H = 92\text{W}$ with $W_{in} = 40\text{W}$ for a Desired Heat transfer rate $\dot{Q}_L = 52\text{W}$, giving $COP_R = 1.3$ (typical value for Freezer).

- Let us now Find the Maximum COP_R and EER Allowed by the 2nd Law by assuming that it is reversible for the same T_L, T_H , and \dot{Q}_L (Desired Output)



- Find:**
- (a) The Minimum heat rejection rate allowed by the 2nd Law
 - (b) The minimum work input required by the 2nd Law
 - (c) The Maximum $COP_{R,rev}$ allowed by the 2nd Law for this $T_H \neq T_L$.



- Assume:**
- (1) Steady cyclical operation
 - (2) Totally Reversible Refrigerator

Analysis

(a) $\dot{Q}_{H,rev}$

- We want to find the Minimum heat rejection rate $\dot{Q}_{H,rev}$ given the desired $\dot{Q}_{L,rev}$ for the specified T_H and T_L . (We want \dot{Q}_H to be minimum in order to make the cooling coil engineering design easier)
- We don't know the minimum work input required, so an energy balance will not help.

⇒ Use the Kelvin-Carnot Relationship!

$$\left(\frac{\dot{Q}_H}{\dot{Q}_L} \right)_{rev} = \frac{T_H}{T_L}$$

$$\Rightarrow \dot{Q}_{H,rev} = \dot{Q}_{L,rev} \left(\frac{T_H}{T_L} \right)$$

$$\dot{Q}_{H,rev} = (52 \text{ W}) \left(\frac{303 \text{ K}}{273 \text{ K}} \right) \longrightarrow$$

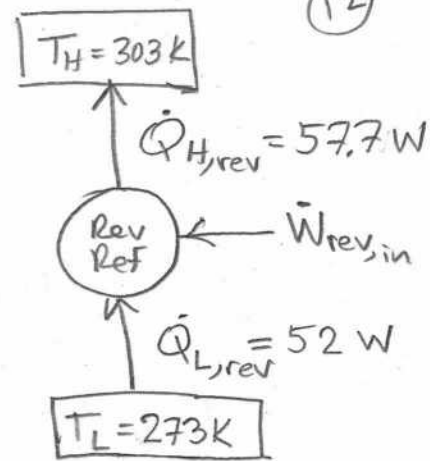
$$\boxed{\dot{Q}_{H,rev} = 57.7 \text{ W}}$$

(b) $\dot{W}_{in,rev} = \min \dot{W}_{in}$

- Now that $\dot{Q}_{H,rev}$ is known, we have enough information to use the Closed System Energy Balance to find $\dot{W}_{in,rev}$:

$$\dot{W}_{in,rev} = \dot{Q}_{H,rev} - \dot{Q}_{L,rev}$$

$$\dot{W}_{in,rev} = 57.7\text{W} - 52\text{W} \rightarrow \boxed{\dot{W}_{in,rev} = 5.7\text{W}}$$



(c) COP_{rev} and EER

- There are two ways of finding COP_{rev} :

Approach 1

$$COP_R \equiv \frac{\dot{Q}_{L,rev}}{\dot{W}_{in,rev}} = \frac{52\text{W}}{5.7\text{W}}$$

$$\boxed{COP_R = 9.1}$$

$$EER \approx COP_R \times 4.312 \frac{\text{Btu}}{\text{Wh}}$$

$$\boxed{EER \approx 39}$$

Approach 2

- Use the Kelvin-Carnot relation:

$$COP_{R,rev} = \frac{1}{\frac{\dot{Q}_{H,rev}}{\dot{Q}_{L,rev}}} = \frac{1}{\frac{T_H}{T_L} - 1}$$

$$COP_{R,rev} = \frac{1}{\frac{303\text{K}}{273\text{K}} - 1} \Rightarrow \boxed{COP_{R,rev} = 9.1}$$

Summary

	T_H (K)	T_L (K)	\dot{Q}_L (W)	\dot{Q}_H (W)	\dot{W}_{in} (W)	COP_R	EER (Btu/Wh)
Irreversible	303	273	52	92	40	1.3	4.4
Reversible				58	6	9.1	39

Minimum heat rejection allowed drops by $\approx 2\times$ \uparrow

Minimum work input required drops by $7\times$ \uparrow

Example 5 The degradation of Energy due to P.E. \rightarrow K.E. \rightarrow Thermal Energy Conversion processes

- Work is a more valuable form of Energy transfer to a system than Heat transfer (recall that there is no Entropy flow or New Entropy generation due to a Work interaction).
- 100% of a Work interaction can be converted to random thermal (Internal) Energy, U , and Mother Nature is very happy to help you waste all of your money!
- Only a fraction of random thermal (Internal) Energy, U , can be converted to Work, and Mother Nature sets restrictions, through both the 1st and 2nd Laws of Thermo (1st Law \Rightarrow Energy must be conserved, 2nd Law \Rightarrow dictates the direction and extent of process).

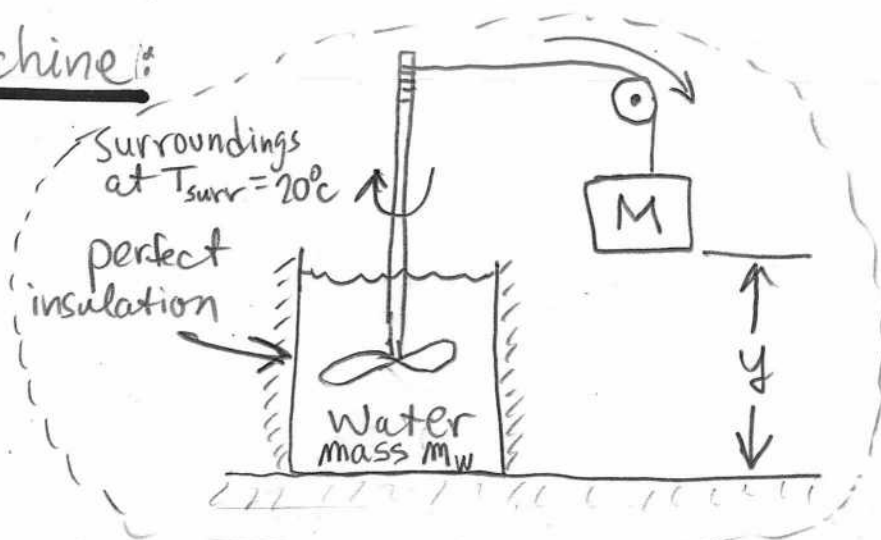
- Consider this Machine:

$$y = 20 \text{ m}$$

$$M = 500 \text{ kg}$$

$$m_w = 0.7 \text{ kg}$$

$$C_w = 4.18 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} @ 300 \text{ K}$$



- The water is initially at $T_1 = 20^\circ\text{C}$
- Now let the $M = 500\text{kg}$ mass fall 20m to the ground so that

$$\Delta PE = Mgy = (500\text{kg})(9.81\frac{\text{m}}{\text{s}^2})(20\text{m}) \Rightarrow \Delta PE = 98.1\text{kJ}$$
- Assuming frictionless, massless pulleys & ropes, i.e., no irreversibilities in the mechanical system (except for the paddle wheel which must be able to stir the water!), then this P.E. is converted into K.E. of the paddle wheel and hence, stirs the water of mass m_w .

- Taking the water as the Closed System,

$$\Delta E)_{\text{sys}} = \dot{Q}_{\text{net},\text{in}} - \dot{W}_{\text{net},\text{out}}$$

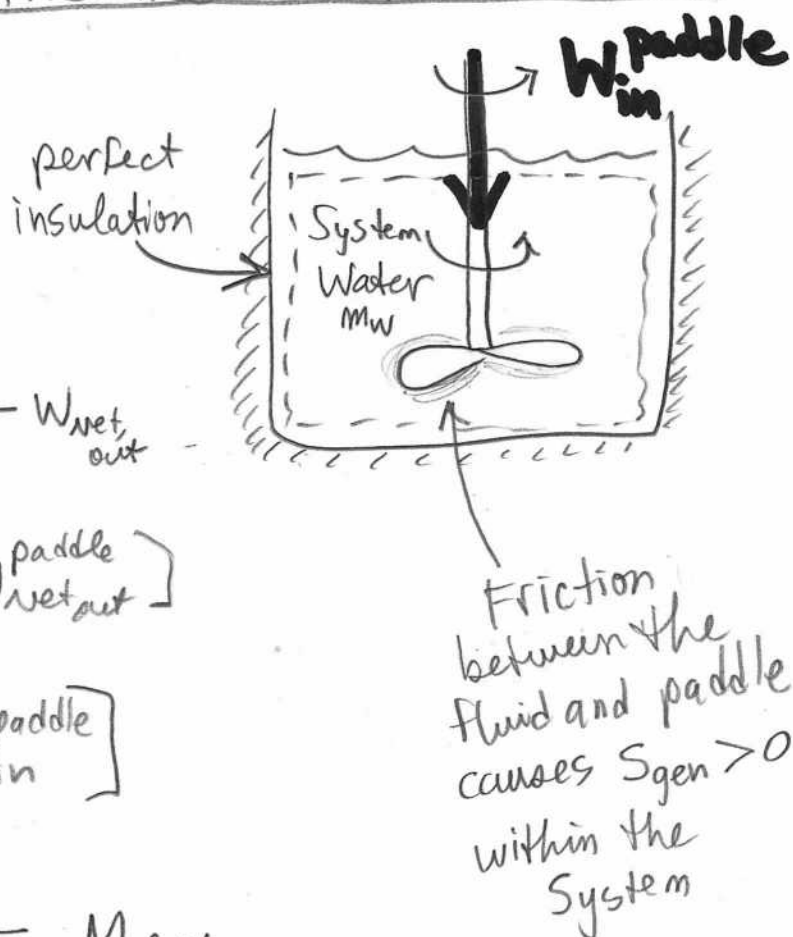
$$\Delta U)_{\text{sys}} + \Delta KE)_{\text{sys}} + \Delta PE)_{\text{sys}} = \dot{Q}_{\text{net},\text{in}} - \dot{W}_{\text{net},\text{out}}$$

$$\Delta U_{\text{sys}} = -[W_{\text{net},\text{out}}^{\text{paddle}} + W_{\text{net},\text{out}}^{\text{paddle}}]$$

$$\Delta U_{\text{sys}} = -[W_{\text{net},\text{out}}^{\text{paddle}} - W_{\text{in}}^{\text{paddle}}]$$

$$(U_2 - U_1)_{\text{sys}} = W_{\text{in}}^{\text{paddle}} = Mgy$$

$$m_w(U_2 - U_1)_{\text{sys}} = Mgy$$



$$(U_2 - U_1)_{\text{sys}} = \frac{M}{m_w} g y$$

$$C_w (T_2 - T_1)_{\text{sys}} = \frac{M}{m_w} g y$$

$$T_2 - T_1 = \frac{M g y}{m_w C_w}$$

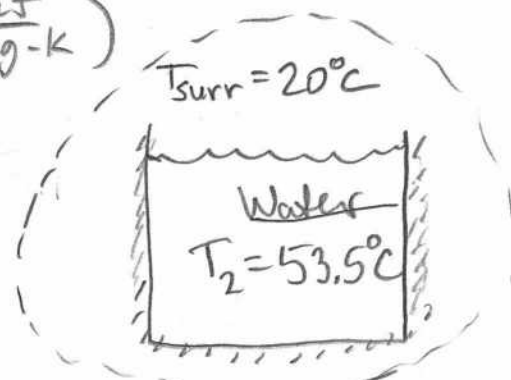
$$T_2 = T_1 + \frac{M g y}{m_w C_w}$$

Substituting values,

$$T_2 = 20^\circ\text{C} + \frac{(500\text{kg})(9.81\text{m/s}^2)(20\text{m}) \times \frac{1\text{kJ}}{1000\text{J}}}{(0.7\text{kg})(4.18\frac{\text{kJ}}{\text{kg}\cdot\text{K}})}$$

$$T_2 = 20^\circ\text{C} + 33.5^\circ\text{C}$$

$$T_2 = 53.5^\circ\text{C}$$

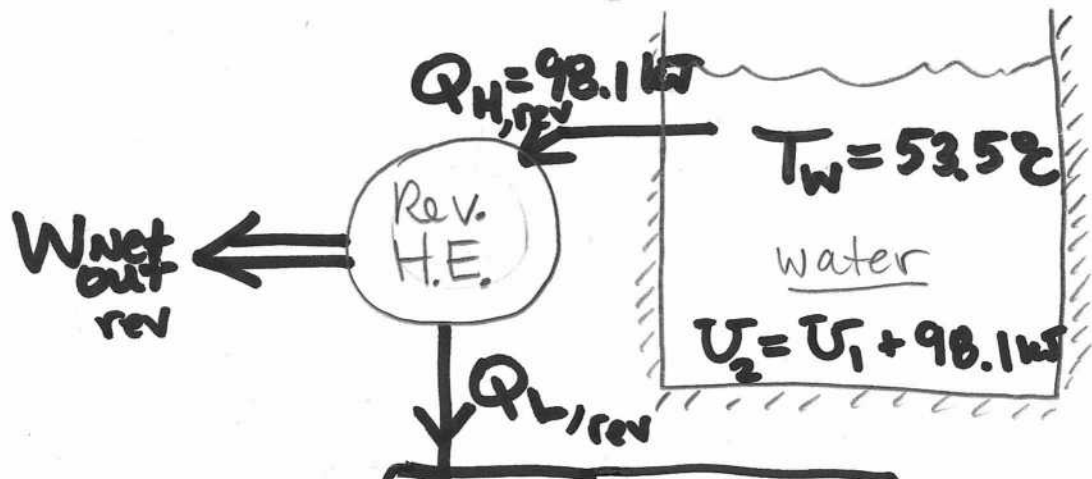


The change in the water's Internal Energy is

$$\Delta U_{\text{water}} = M g y = (500\text{kg})(9.81\text{m/s}^2)(20\text{m}) \times \frac{1\text{kJ}}{1000\text{J}}$$

$$\Delta U_{\text{water}} = 98.1\text{kJ}$$

- Now, let's see what happens when we try to extract this 98.1 kJ from the water by running it into a perfect, reversible Heat Engine, and reject the waste Heat to the surroundings at 20°C



- For any reversible H.E., **20°C Environment**

$$\eta_{th,rev} = 1 - \frac{T_L}{T_H} \quad \left[\begin{array}{l} 1^{st} \text{ and } 2^{nd} \text{ Laws of} \\ \text{Thermo included here} \end{array} \right]$$

$$\Rightarrow \eta_{th,rev} = 1 - \frac{(20^\circ\text{C} + 273)}{(53.5^\circ\text{C} + 273)}$$

$$\eta_{th,rev} = 1 - 0.8974$$

$$\eta_{th,rev} = 0.103$$

$$\eta_{th,rev} = 10.3\%$$

- For any H.E., $\eta_{th} = 1 - \frac{Q_{L,rev}}{Q_{H,rev}} \quad [1^{st} \text{ Law of Thermo only}]$

$$\Rightarrow Q_{L,rev} = (1 - \eta_{th,rev}) Q_H$$


$$Q_{L,rev} = (1 - 0.103)(98.1 \text{ kJ}) \Rightarrow Q_{L,rev} = 88 \text{ kJ}$$

- For any H.E., $W_{net,out,rev} = Q_{H,rev} - Q_{L,rev}$

$$W_{net,out,rev} = 98.1 \text{ kJ} - 88 \text{ kJ} \Rightarrow$$

$$W_{net,out,rev} = 10 \text{ kJ}$$

Bottom Line :

- The original 98.1 kJ of Potential Energy stored within Mass M was perfectly converted to Pure Work = 98.1 kJ
- The pure Work of 98.1 kJ was then 100% converted into 98.1 kJ of Internal Random Energy of the water. (1st Law)
- When we used a perfectly reversible Heat Engine to try to extract 100% of the originally pure 98.1 kJ of pure potential Energy from the degraded, random Internal Thermal Energy in the water, we failed!!
 - ⇒ The 2nd Law allowed us to convert, at best, only 10% of the 98.1 kJ random Internal thermal energy (sensible) into Work output!  Mother Nature is laughing at you!!
 - $Q_L = 88 \text{ kJ}$ of Wasted Energy to _{rev} perform this conversion at $T_{\text{surr}} = 20^\circ\text{C}$
 - ⇒ **The Original 98 kJ was degraded 90%**

Lecture 25 • The Clausius Inequality and Entropy

- The Entropy Balance Egn. for the System only
- The Increase of Entropy Principle

(I) The Clausius Inequality and Entropy

- From 1855 to 1865, Dr. Rudolf Clausius was performing research on what caused irreversibilities in thermodynamic processes and in cyclical processes such as Heat Engines and refrigerators.
- Initially, Dr. Clausius based his work on that of Sadi Carnot, Lord Kelvin, and others.
- Based on his experimental and theoretical work, Clausius began to realize that the ratio $\frac{Q_{net,in}}{T}$ had an important role to play in irreversible processes within a system and in the associated surroundings.
- In 1865, Dr. Clausius published his results that expressed the consequences of irreversibilities (Internally to the System) on cyclical thermodynamic processes (Heat Engines, Refrigerators etc) with a single equation for any system under study:



$\oint \frac{\delta Q_{net,in}}{T} \leq 0$	The Clausius Inequality
--	--------------------------------

- $\delta Q_{net,in}$ = infinitesimal net heat addition to the system during the cyclical process
- T = Temperature (K) evaluated at the System boundary position where $\delta Q_{net,in}$ heat transfer takes place. Note $T \neq \text{constant}$!

(2)

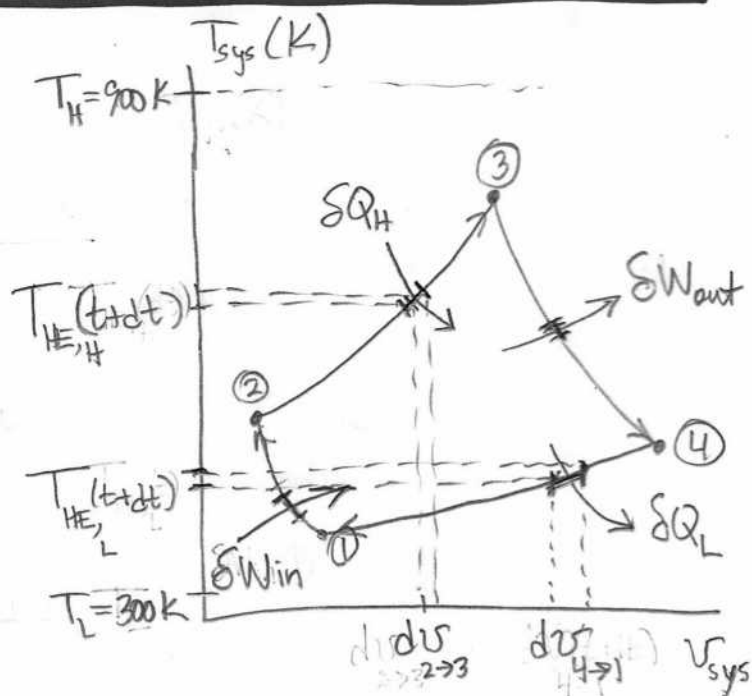
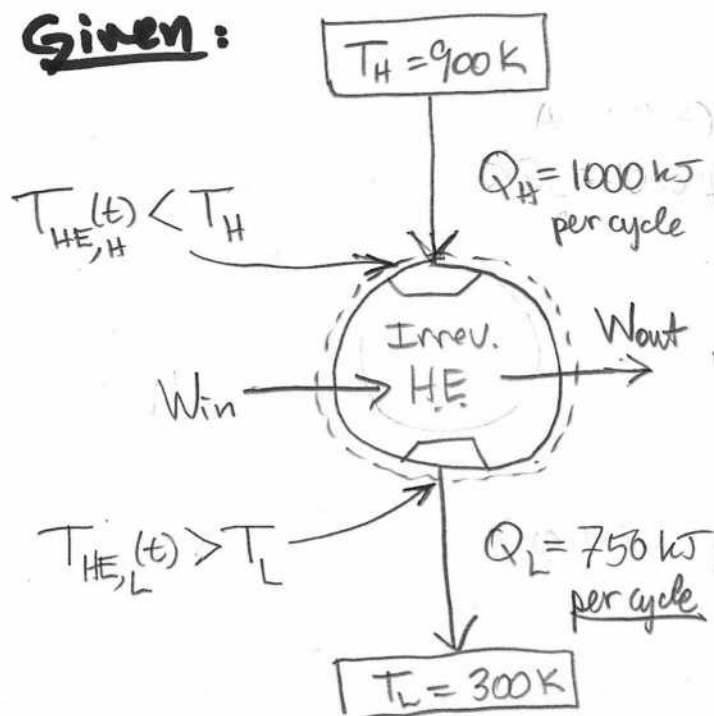
- The Clausius inequality is valid for Systems that are Internally irreversible, Internally reversible only, and Totally Reversible.
- For now we will focus on Systems with irreversibilities only and then, later, extend the Clausius inequality to processes that include both the System + Surroundings

(1) $\oint \frac{\delta Q_{\text{net},in}}{T} < 0$ for internally irreversible cycles

(2) $\oint \frac{\delta Q_{\text{net},in}}{T} = 0$ for internally reversible or totally reversible cycles

Example 1 Evaluating $\oint \frac{\delta Q_{\text{net},in}}{T}$ for an Internally Irreversible H.E.

Given:



(Note: Although this device is internally irreversible, I did not use dashed lines for clarity)

(3)

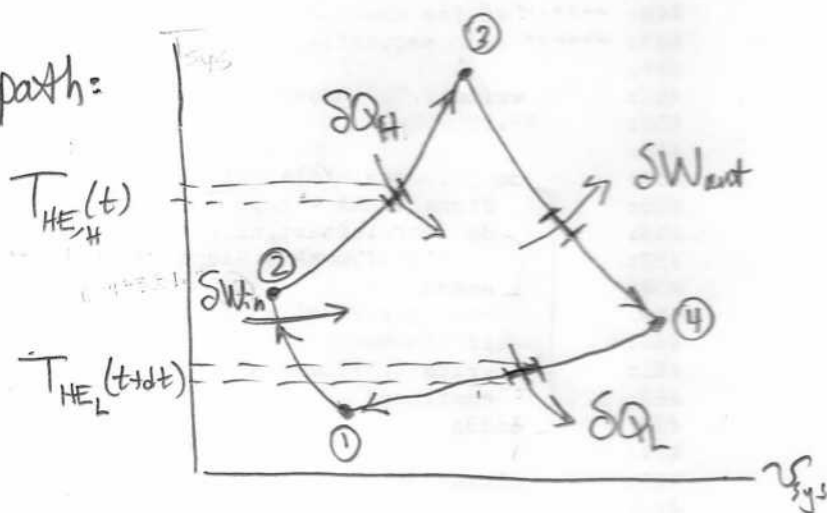
Analysis

- Let us integrate along each path over the total cycle to see if $\oint \frac{\delta Q_{\text{net},in}}{T} < 0$ and what that value is.

- Recall that along a path:

$$\boxed{W = \int_1^2 \delta W_{\text{out}}$$

$$Q = \int_1^2 \delta Q$$



- Integrating Along Each Process path,

$$\oint \frac{\delta Q_{\text{net},in}}{T(t)} = \int_1^2 \frac{\delta Q_{\text{net},in}}{T(t)} + \int_2^3 \frac{\delta Q_{\text{net},in}}{T_{HE,H}(t)} + \int_3^4 \frac{\delta Q_{\text{net},in}}{T_{HE,L}(t)} + \int_4^1 \frac{\delta Q_{\text{net},in}}{T_{HE,L}(t)}$$

\swarrow No heat transfer \swarrow No heat transfer

$$\oint \frac{\delta Q_{\text{net},in}}{T(t)} = \int_2^3 \frac{\delta Q_{\text{net},in}}{T_{HE,H}(t)} + \int_4^1 \frac{\delta Q_{\text{net},in}}{T_{HE,L}(t)}$$

- At this point, we are stuck because we have no analytic relationship connecting the time/path dependent δQ to the time/path dependent properties $T_{HE,H}(t)$ and $T_{HE,L}(t)$!
- Let us make an Engineering approximation:

$$T_{2 \rightarrow 3} \approx T_H = \text{constant} \quad \text{and} \quad T_{4 \rightarrow 1} \approx T_L = \text{constant}$$

$$\oint \frac{\delta Q}{T} \approx \frac{1}{T_H} \int_2^3 [\delta Q_{\text{in}} - \delta Q_{\text{out}}] + \frac{1}{T_L} \int_4^1 [\delta Q_{\text{in}} - \delta Q_{\text{out}}]$$

$$\boxed{\oint \frac{\delta Q}{T} \approx \frac{Q_H}{T_H} - \frac{Q_L}{T_L}}$$

$$\oint \frac{\delta Q}{T} \approx \frac{1000 \text{ kJ}}{900 \text{ K}} - \frac{750 \text{ kJ}}{300 \text{ K}}$$

$$\approx 1.111 \frac{\text{kJ}}{\text{K}} - 2.5 \frac{\text{kJ}}{\text{K}}$$

$$\boxed{\oint \frac{\delta Q_{\text{net},in}}{T} \approx -1.39 \frac{\text{kJ}}{\text{K}}}$$

yes, less than zero

Q: What does $\oint \frac{\delta Q}{T}$ mean?

(4)

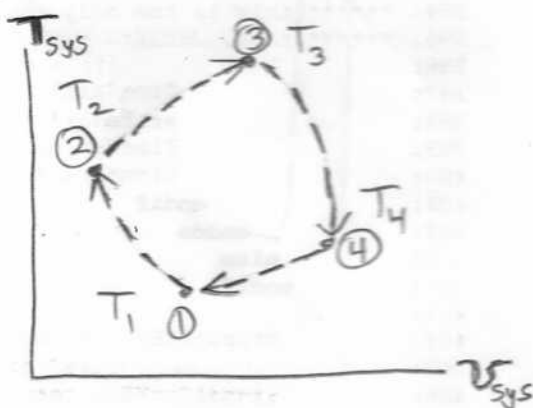
A: • Recall that properties of a System can be expressed as an exact differential = dT, dp, dV, dS, dU, dH , etc

- For a system undergoing a quasi-equilibrium cycle of processes (But it can be Internally Irreversible) we can define System properties along each point of the process path so that

$$\oint dT = 0 \quad \Rightarrow dp = 0$$

$$\oint dp = 0 \quad \text{etc.}$$

$$\oint dU = 0$$



- For Example, let's integrate T over the process path:

$$\oint dT = \int_1^2 dT + \int_2^3 dT + \int_3^4 dT + \int_4^1 dT$$

$$\oint dT = [T_2 - T_1] + [T_3 - T_2] + [T_4 - T_3] + [T_1 - T_4]$$

re-arranging,

$$\oint dT = (\cancel{T_2} - \cancel{T_2}) + (\cancel{T_1} - \cancel{T_1}) + (\cancel{T_3} - \cancel{T_3}) + (\cancel{T_4} - \cancel{T_4})$$

$$\Rightarrow \oint dT = 0 \quad \text{since } T \text{ is a property of the System!}$$

- Now consider the Clausius inequality for an internally reversible or reversible cyclical thermodynamic system:

$$\oint \left(\frac{\delta Q_{\text{net, in}}}{T} \right)_{\text{int rev}} = 0$$

- If $\oint \left(\frac{\delta Q}{T} \right)_{\text{int, rev}} = 0$ (recall $\oint dT = 0$) (5)

Then what kind of differential must $\left(\frac{\delta Q}{T} \right)_{\text{int, rev}}$ be?

⇒ A perfect differential!

- Clausius recognized this and gave it the letter S :

$$dS_{\text{sys}} \equiv \left(\frac{\delta Q}{T} \right)_{\text{int, rev}} \quad (1)$$

- If $\oint dS = 0$

Then what does that say about the physical nature of S ?

⇒ S_{sys} must be a property of the System!

- Clausius discovered this and in 1865 he defined it as

$$S_{\text{sys}} \equiv \text{Entropy, a new property of the System}$$

- S has units of $\frac{\text{kJ}}{\text{K}}$, and is an Extensive Property of the system:

$$S_{\text{sys}} = \sum_{i=1}^N S_i$$

S_1	S_2	S_3	S_4	
	etc			

← Subsystem

← System

- Lower case $s_{\text{sys}} \equiv \frac{S_{\text{sys}}}{m_{\text{sys}}} \left(\frac{\text{kJ}}{\text{kg-K}} \right)$

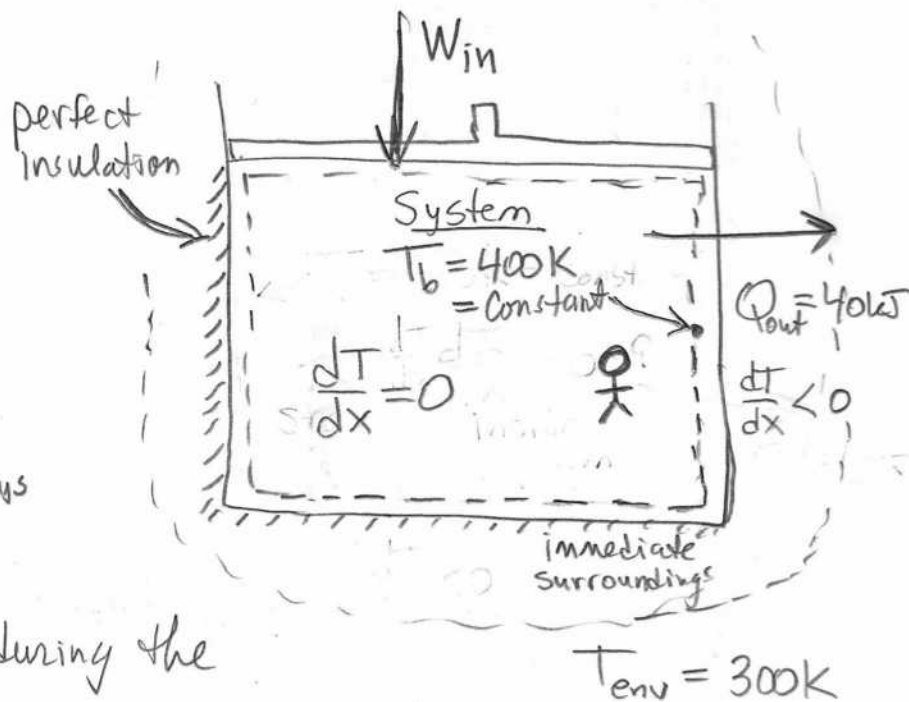
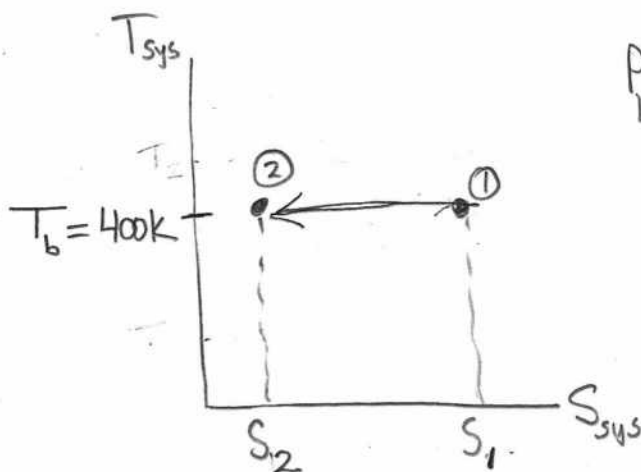
and is an Intensive property of the System.

Example 2

The System Entropy change for an Internally Reversible Process

⑥

- Consider the piston-cylinder system undergoing a quasi-equilibrium, internally reversible process from State ① → State ②



- Here, $Q_{\text{out}} = 40\text{kJ}$ during the entire process
- $T_{\text{sys}} = T_b = 400\text{K} = \text{Constant}$ during the process, Evaluated at the System boundary where Q_{out} occurs
- Stickman is placed inside the System because that is where we want to know where the Entropy is changing. Stickman measures the temperature of the boundary, T_b , where the Heat transfer occurs (along only one boundary)
- Since the process is internally reversible because $\left. \frac{dT}{dx} \right|_{\text{sys}} = 0$ and there is no paddle wheel causing friction,

$$dS_{\text{system}} = \left(\frac{SQ_{\text{net, in}}}{T} \right)_{\text{int rev}}$$

- Integrating $dS_{sys} = \frac{\delta Q_{net,in}}{T}$ from ① \rightarrow ②, ⑦

$$\int_{S_1}^{S_2} dS_{sys} = \int_1^2 \frac{\delta Q_{net,in}}{T} \Big|_{int \ rev}$$

$$(S_2 - S_1) = \Delta S_{sys} = \int_1^2 \frac{\delta Q_{net,in}}{T_b} = \frac{1}{T_b} \int_1^2 \delta Q_{net,in} = \frac{\cancel{Q_{in}} - Q_{out}}{T_b}$$

$$\boxed{\Delta S_{sys} = -\frac{Q_{out}}{T_b}}$$

- Plugging in values,

$$\Delta S_{sys} = -\frac{(40 \text{ kJ})}{(400 \text{ K})} \rightarrow \boxed{\Delta S_{sys} = -0.1 \frac{\text{kJ}}{\text{K}}}$$

- \Rightarrow The Entropy of the system decreased as thermal energy was transferred out of the system (Q_{out}) in a reversible way (No new Entropy generated inside the system during this process)
- \Rightarrow New Entropy was created outside the System due to the temperature gradient in the immediate surroundings,
- \Rightarrow Entropy "flowed", or was "transferred" out of the System at the physical location where heat transfer occurred — this is the Entropy flow
- $\Rightarrow \Delta S_{sys} = -0.1 \frac{\text{kJ}}{\text{K}} \equiv S_{net,in}^{flow} = \int_1^2 \frac{\delta Q_{net,in}}{T} \Big|_{rev \ process}$
- $\Rightarrow S_{net,in}^{flow} \equiv \cancel{S_{in}^{flow}} - S_{out}^{flow} = -0.1 \frac{\text{kJ}}{\text{K}} \rightarrow \boxed{S_{out}^{flow} = 0.1 \frac{\text{kJ}}{\text{K}}}$

(8)

(II) The Entropy Balance Equation for Systems

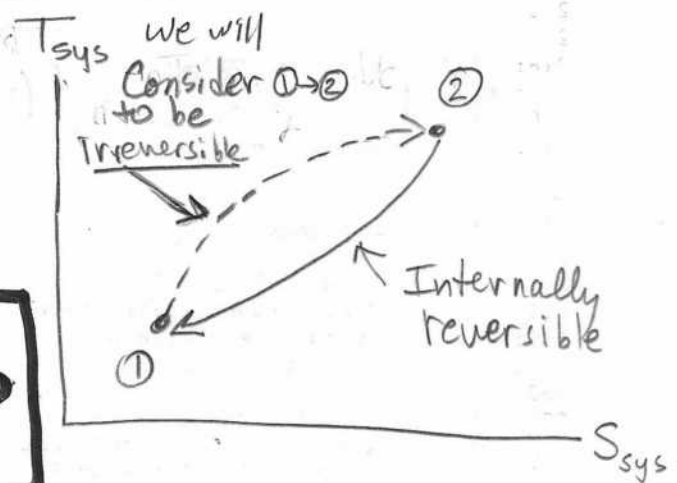
- The Clausius inequality holds strictly for Cyclical Systems that are internally irreversible

$$\oint \frac{\delta Q_{\text{net},in}}{T} \Big|_{\text{int irrev}} < 0 \quad (2)$$

- We don't want to deal with external irreversibilities right now, so consider the two-process cycle where path ①→② is either internally irreversible or Reversible (No external irrev.) and the path from ②→① is internally reversible

- Let us integrate $\oint \frac{\delta Q_{\text{net},in}}{T}$ over these two process paths, keeping the \leq sign for now:

$$(3) \quad \oint \frac{\delta Q_{\text{net},in}}{T} = \int_1^2 \frac{\delta Q_{\text{net},in}}{T} \Big|_{\text{int irrev}} + \int_2^1 \frac{\delta Q_{\text{net},in}}{T} \Big|_{\text{int rev}} \leq 0$$



- Recall Equation (1) above for internally reversible processes:

$$dS_{\text{sys}} = \frac{\delta Q}{T} \Big|_{\text{int rev}} \quad (1)$$

- The integral $\int_2^1 \frac{\delta Q}{T} \Big|_{\text{int rev}} = \int_2^1 dS_{\text{sys}} = (S_1 - S_2)_{\text{system}} = -\Delta S_{\text{sys}}$
- Substituting the integral result of Equation (1) into Equation (3),

$$\int_1^2 \frac{\delta Q_{\text{net},in}}{T} \Big|_{\text{int irrev}} + (S_1 - S_2)_{\text{sys}} \leq 0$$

- Re-arranging, we have

$$(S_2 - S_1)_{sys} \geq \int_1^2 \frac{\delta Q_{net,in}}{T} \Big|_{int \text{ irrev}} \quad (4)$$

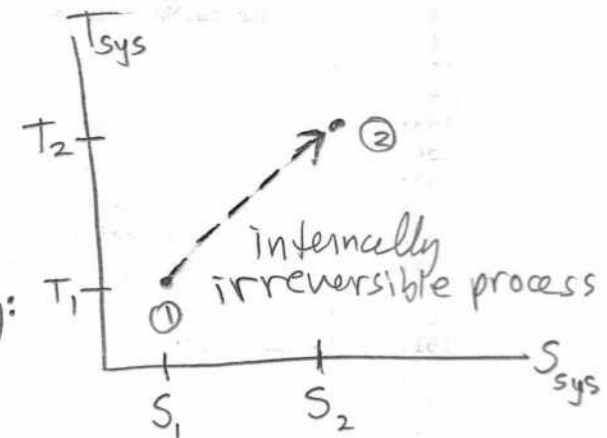
- In differential form,

$$ds_{sys} \geq \frac{\delta Q_{net,in}}{T} \Big|_{int \text{ irrev}} \quad (5)$$

What do these Equations Mean?!

Q: What do Equations (4) and/or (5) mean?

A: If we have a process from ① → ② that is internally irreversible, then Equations (4) and (5) have an Inequality only:



$$(S_2 - S_1)_{sys} > \int_1^2 \frac{\delta Q_{net,in}}{T} \Big|_{int \text{ irrev}}$$

internally Irreversible
Entropy flow, $S_{net,in}$, into or out of the System in direction of $Q_{net,in}$

$$S_{net,in}^{flow} \equiv S_{in}^{flow} - S_{out}^{flow} = \int_1^2 \frac{\delta Q_{net,in}}{T} \Big|_{int \text{ irrev}}$$

(6)

Entropy "flow" or "Transfer"

- In General $S_{net,in}^{flow}$ is Irreversible but later we will model the system so that it becomes Reversible or Make Approximations

Answer, continued;

(10)

- If we have a process from ① → ② that is Internally Irreversible, then what does this mean?

$$(S_2 - S_1)_{\text{sys}} > \int_1^2 \frac{\delta Q_{\text{net}, \text{in}}}{T} \equiv S_{\text{Net}, \text{in}}^{\text{flow}} = S_{\text{in}}^{\text{flow}} - S_{\text{out}}^{\text{flow}}$$

- This means that some other kind of irreversibility, in addition to $S_{\text{Net}, \text{in}}^{\text{flow}}$, which is irreversible, is adding to the already existing System Entropy during the process ① → ②!

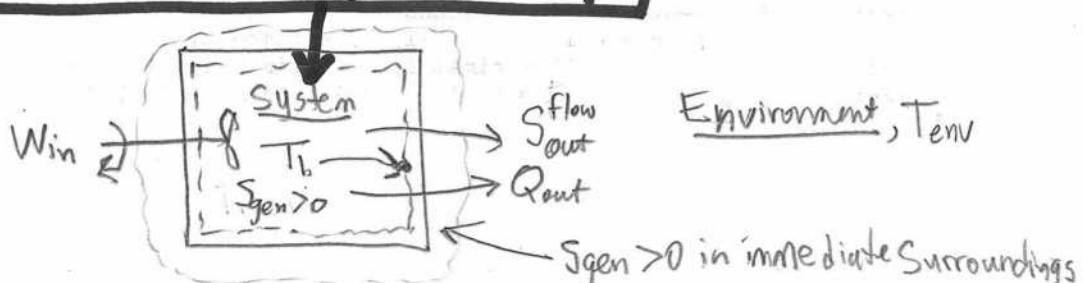
Q • What is this additional Entropy that is irreversible and adds to the System Entropy during the process ① → ②?

Q • What, mathematically, will turn the ">" sign into an "=" sign?

⇒ Add S_{gen} to right hand side!
of Eqn.(7)

$$(S_2 - S_1)_{\text{sys}} = \int_1^2 \frac{\delta Q_{\text{net}, \text{in}}}{T} + S_{\text{gen}, \text{sys}} \quad (8)$$

The Entropy Balance Equation for The System Only



Example 3

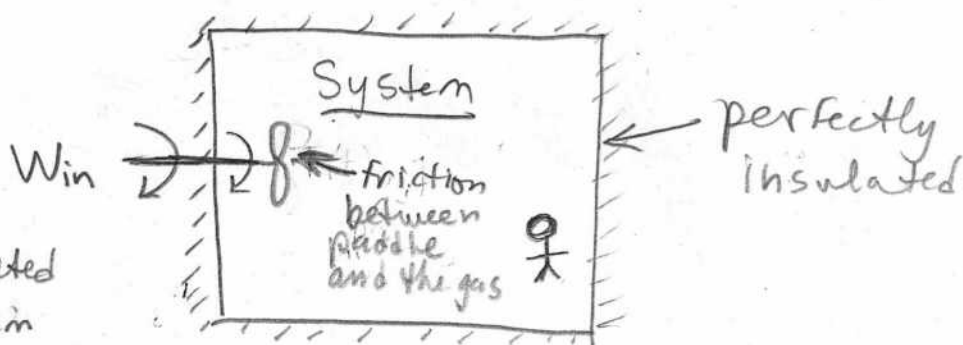
Paddle wheel performing work on a perfectly insulated tank containing an Ideal gas

(11)

Given:

Find: The New Entropy Generated within the System

due to the paddle wheel work on the system



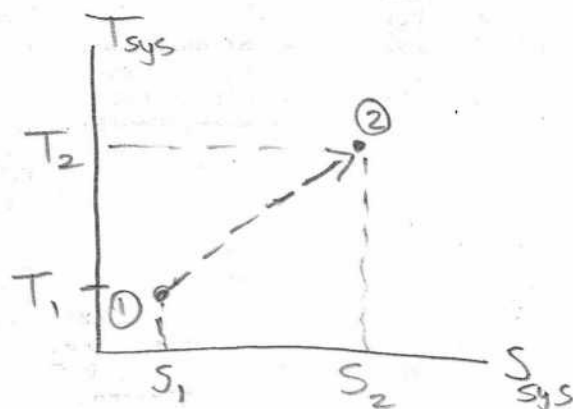
Analysis: The System Entropy

Balance has $S_{gen,sys}$:

$$S_2 - S_1 = \Delta S_{sys} = \int_1^2 \frac{\delta Q_{net,in}}{T} + S_{gen,sys}$$

$S_{flow,net,in} = 0$

int irrev



$$S_{gen,sys} = S_2(T_2, v_2) - S_1(T_1, v_1)$$

- In order to compute $S_2(T_2, v_2)$ and $S_1(T_1, v_1)$ we need to learn more about how the property S_{sys} is related to other thermodynamic properties such as T, v, p, u .
- In this case the New Entropy generated, S_{gen} , within the system was due only to the friction between the paddle wheel and the gas.
- Also, to complete the problem, we need to solve the Closed System Energy Balance to relate W_{in} to $\Delta U = C_v \Delta T$.

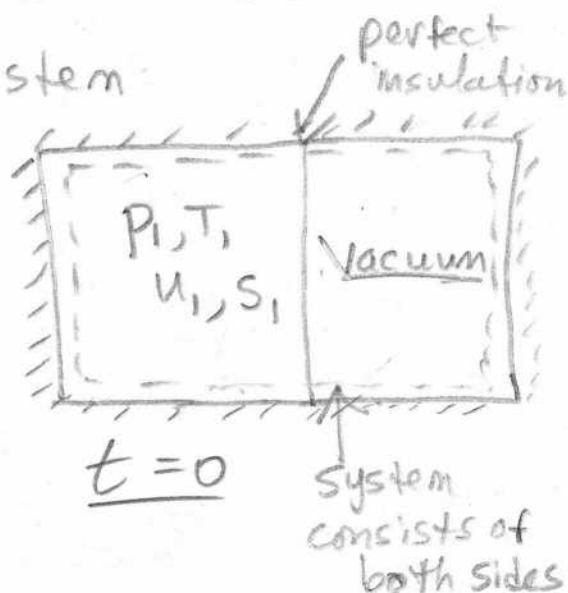
(III) The Increase of Entropy Principle

12

- Consider a completely isolated System

- The Entropy balance Equation for a system only states

$$\Delta S_{sys} = \underbrace{\int_1^2 \frac{\dot{S}_{net,in}}{T}}_{\dot{S}_{flow,net,in} (irrev.)} + S_{gen,sys}$$

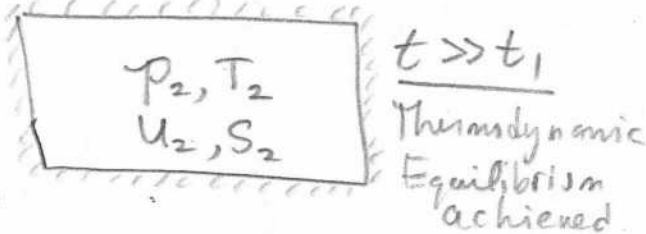


- When the partition is taken away, we have a non-equilibrium process without heat transfer

$$\Delta S_{sys} = \cancel{\dot{S}_{flow,net,in}} + S_{gen,sys}$$

or, as we saw from Equations (4) and (5),

$(S_2 - S_1)_{sys} \geq 0$	(4)
$dS_{sys} \geq 0$	(5)



Q: what do Egn.(4) & (5) mean for a spontaneous process that is occurring in an isolated system?

Answer:

The Increase of Entropy Principle

(1) The Entropy of an isolated system during a spontaneous process ALWAYS increases: $\Delta S_{\text{sys}} > 0$ ($S_{\text{gen}} > 0$)

(2) If the Isolated system should somehow undergo a reversible process, then $\Delta S_{\text{sys}} = 0$ ($S_{\text{gen}} = 0$)

- For a spontaneous process within an isolated system, Nature tries to Maximize Entropy for the Least amount of Energy!

The 2nd Law says \Rightarrow Nature is a Lazy Slob!

- The Increase of Entropy Principle Summary:

$$S_{\text{gen}} \begin{cases} > 0 & \text{Irreversible Process} \\ = 0 & \text{Reversible Process} \\ < 0 & \text{Impossible Process} \end{cases}$$

$$\Delta S_{\text{sys}} = \int_1^2 \frac{\delta Q_{\text{net, in}}}{T} + S_{\text{gen}}^{\text{int irrev}}$$

- The Increase of Entropy principle helps you determine whether a process is Irreversible, Reversible, or Impossible based on whether $S_{\text{gen}} > 0$, $S_{\text{gen}} = 0$, or $S_{\text{gen}} < 0$

Conclusions From Increase of Entropy Principle

- (1) While the Energy of the Universe is constant, the Entropy of the Universe ALWAYS increases (Clausius concluded this in his 1865 paper)

⇒ $S_{gen} > 0$ in the real world situations

- (2) Entropy is not conserved, like Energy is.

⇒ There is no such thing as "Conservation of Entropy"
Except in idealized (fictional) processes

- (3) The performance of Engineering Systems is degraded by the presence of irreversibilities either inside or even only outside the system.

⇒ S_{gen} is a measure of the magnitudes of the irreversibilities present during that process.

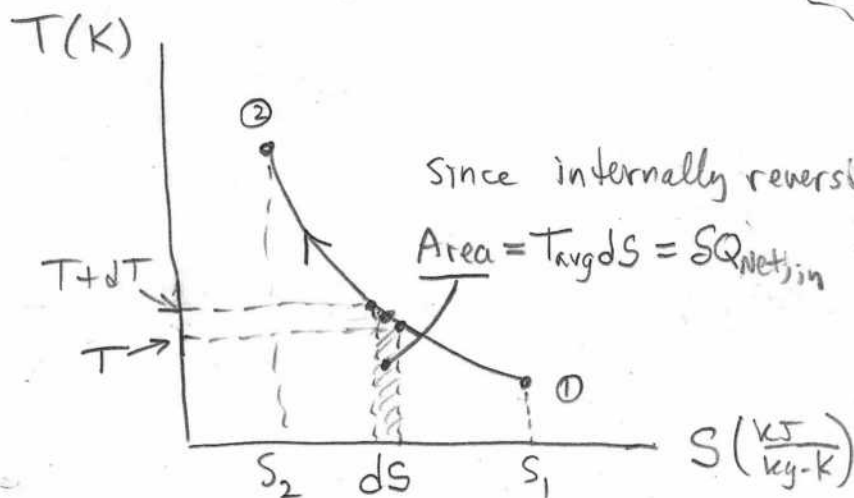
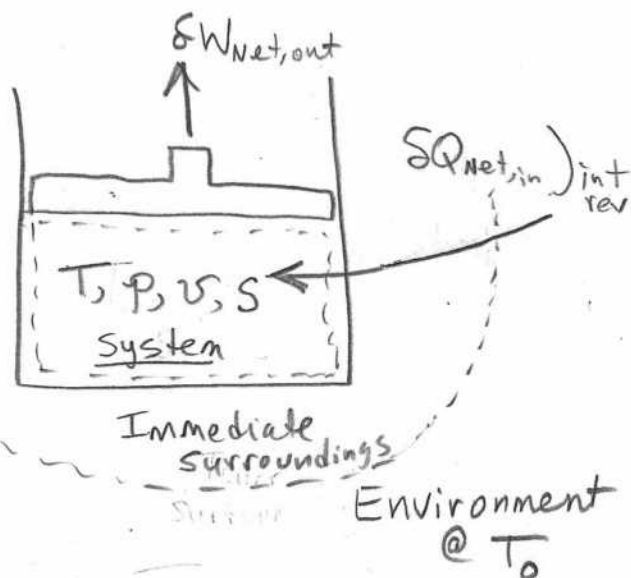
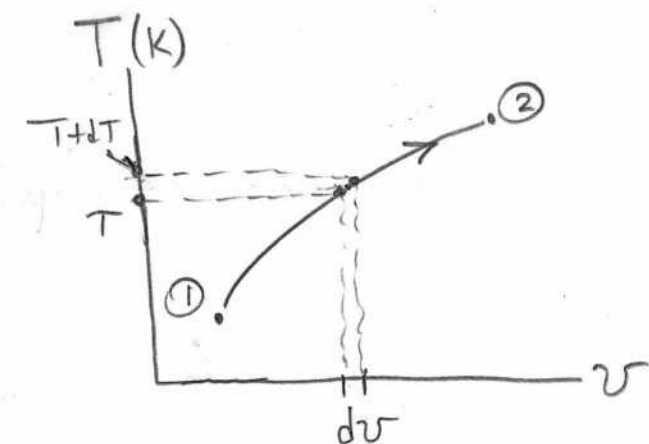
⇒ S_{gen} is used to establish criteria for the performance of engineering devices

Lecture 26 Calculating System Entropy Property Changes Between Two States

Consider a piston-cylinder device in which Heat, Q_{in} , is added and the system is allowed to expand in an internally reversible process:

- (1) quasi-equilibrium process
- (2) No friction at piston-cylinder walls
- (3) No chemical reactions in the System
etc.

Note: We add infinitesimal units of heat, $\delta Q_{int, rev}$, at each infinitesimal step of the quasi-equilibrium process from state ① to state ②



From the Closed System Energy Balance on the System,

$$\Delta E)_{\text{sys}} = \delta E_{\text{in}} - \delta E_{\text{out}}$$

$$dU)_{\text{sys}} + \underbrace{\Delta KE}_0 + \underbrace{\Delta PE}_0 = \delta Q_{\text{Net, in}})_{\text{int rev}} - \delta W_{\text{Net, out}}$$

$$\boxed{dU_{\text{sys}} = \delta Q_{\text{Net, in}})_{\text{int rev}} - \delta W_{\text{Net, out}}} \quad (1)$$

- For an Internally Reversible Process, Clausius showed us that $dS_{\text{sys}} = \delta Q/T)_{\text{int rev}}$, or

$$\boxed{\delta Q)_{\text{int rev}} = T ds_{\text{sys}}} \quad (2)$$

- Recall that p dV Work is

$$\boxed{\delta W_{\text{Net, out}} = p dV} \quad (3)$$

Combining Equations (1), (2), and (3),

$$T ds_{\text{sys}} = dU_{\text{sys}} + p dV$$

on a per-unit mass basis,

$$T ds_{\text{sys}} = du_{\text{sys}} + p dv$$

dividing by T,

$$\boxed{ds = \frac{du}{T} + \frac{p dv}{T}} \quad \text{1st Gibbs Equation Closed Systems}$$

③

For an Open System, we want to use Enthalpy, h :

$$h_{sys} = u_{sys} + p v$$

$$dh = du + p dv + v dp$$

$$\Rightarrow du = dh - p dv - v dp$$

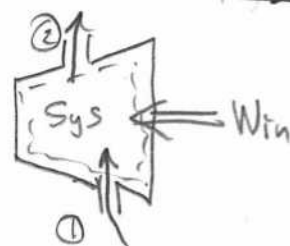
Substituting into $T ds = du_{sys} + p dv$ above,

$$T ds = (dh - p dv - v dp) + p dv$$

$$T ds = dh_{sys} - v dp$$

Dividing by T ,

$ds = \frac{dh}{T} - \frac{v dp}{T}$	2nd Gibbs Equation Open Systems
--------------------------------------	---



Important:

- (1) There are no inexact differentials
- (2) We initially said the process was internally reversible only, but all of the variables in Gibb's Equations are properties, which are independent of path.

$ds = \frac{du}{T} + \frac{p dv}{T}$	Gibb's 1
$ds = \frac{dh}{T} - \frac{v dp}{T}$	Gibb's 2

so they must be valid
for both Internally
Reversible & Internally
Irreversible
processes ① → ② !!

I System Entropy Change for a Liquid or Solid

- liquids & solids are \approx incompressible, so $dv \approx 0$

- For a closed system liquid or solid at Temperature T

$$C_p(T) = C_v(T) \equiv C(T)$$

so $du = C(T) dT$

- Integrating Gibb's 1st Eqn. between states ① \rightarrow ②,

$$ds = \frac{du}{T} + \frac{pdv}{T}$$

$$ds \approx \frac{du}{T} = \frac{C(T) dT}{T}$$

$$\int_{s_1}^{s_2} ds \approx \int_{T_1}^{T_2} \frac{C(T) dT}{T}$$

or

$$(s_2 - s_1)_{sys} = \Delta s_{sys} = \int_{T_1}^{T_2} \frac{C(T) dT}{T}$$

Δs_{sys}
Liquid
or
Solid

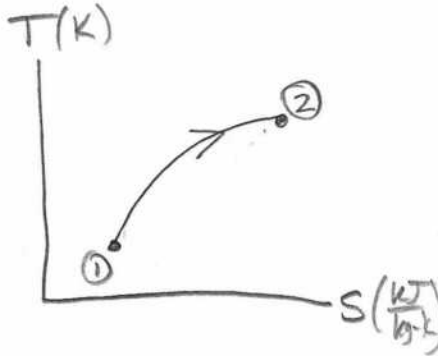
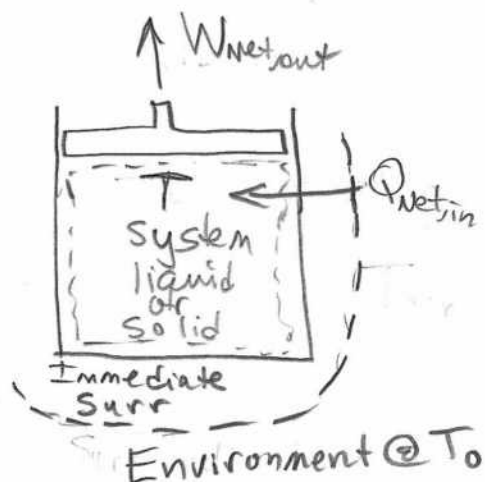
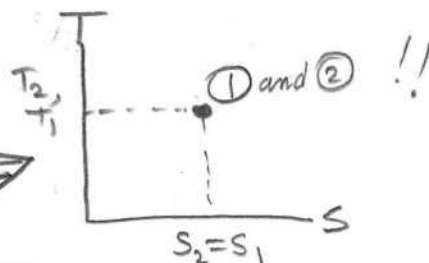
IF $C = \text{Constant} = C_{avg}$, \Rightarrow

$$(s_2 - s_1)_{sys} = C_{avg} \ln(T_2/T_1)$$

Isentropic process

$$s_2 = s_1 \Rightarrow \ln(T_2/T_1) \equiv 0$$

$$T_2 = T_1$$



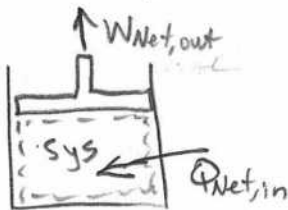
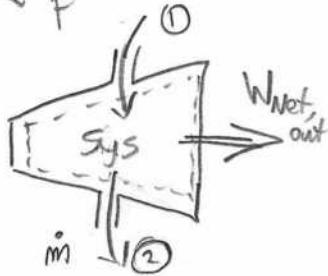
II) System Entropy Change for an Ideal Gas

$$PV = RT$$

$$C_v = C_v(T)$$

$$C_p = C_p(T)$$

- There are two expressions for the Entropy change of a System.
- Which one you use depends on what situation you have (Open or Closed system).

Gibb's 1 st Egn.	Gibb's 2 nd Egn.
$ds = \frac{du}{T} + \frac{pdr}{T}$	$ds = \frac{dh}{T} - \frac{vdp}{T}$
<p>Substitute $PV=RT$ into these</p> $ds = \frac{du}{T} + R \frac{dr}{r}$	$ds = \frac{dh}{T} - R \frac{dp}{p}$
<p>Now substitute</p> $du = C_v(T) dT$	<p>Now substitute</p> $dh = C_p(T) dT$
$ds = C_v(T) \frac{dT}{T} + R \frac{dr}{r}$	$ds = C_p(T) \frac{dT}{T} - R \frac{dp}{p}$
	
<p>Now Integrate between states ① → ②</p> $(S_2 - S_1)_{sys} = \int_{T_1}^{T_2} C_v(T) \frac{dT}{T} + R \ln\left(\frac{r_2}{r_1}\right)$	$(S_2 - S_1)_{sys} = \int_{T_1}^{T_2} C_p(T) \frac{dT}{T} - R \ln\left(\frac{P_2}{P_1}\right)$

⑥ A) Ideal Gas, Average Constant Specific Heats

$$(S_2 - S_1)_{\text{sys}} = C_{v,\text{avg}} \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right)$$

$$(S_2 - S_1)_{\text{sys}} = C_{p,\text{avg}} \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{p_2}{p_1}\right)$$

$$\left(\frac{\text{kJ}}{\text{kg} \cdot \text{K}}\right)$$

Isentropic Process: $S_2 \equiv S_1$

For an isentropic process, $(S_2 - S_1)_{\text{sys}} = 0 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$, so

$$0 = C_{v,\text{avg}} \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right)$$

or

$$\ln\left(\frac{T_2}{T_1}\right) = \ln\left(\frac{v_1}{v_2}\right)^{R/C_v}$$

For an Ideal gas, recall that

$$R = C_p - C_v$$

$$K \equiv C_p/C_v$$

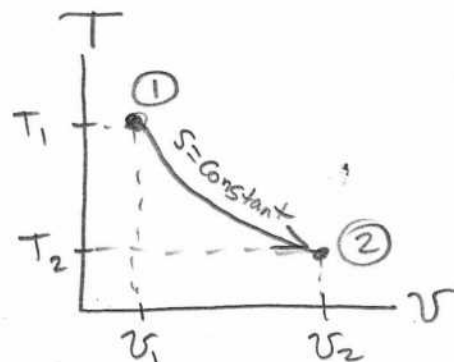
so

$$\ln\left(\frac{T_2}{T_1}\right) = \ln\left(\frac{v_1}{v_2}\right)^{\frac{C_p - C_v}{C_v}} = \ln\left(\frac{v_1}{v_2}\right)^{K-1}$$

or

$$\ln\left(\frac{T_2}{T_1}\right) = \ln\left(\frac{v_1}{v_2}\right)^{K-1}$$

$$\left(\frac{T_2}{T_1}\right)_{S=\text{const}} = \left(\frac{v_1}{v_2}\right)^{K-1}$$



You can obtain other "Isentropic Relations" using the 2nd Gibb's equation. (7)

Isentropic Relations for Ideal Gas, Constant C_p, C_v

$$\left(\frac{T_2}{T_1}\right)_{s=\text{const}} = \left(\frac{v_1}{v_2}\right)^{k-1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}$$

$$\text{or } \left(\frac{p_2}{p_1}\right)_{s=\text{const}} = \left(\frac{v_1}{v_2}\right)^k$$

$$k \equiv \frac{C_{p,\text{avg}}}{C_{v,\text{avg}}} = \text{Constant!}$$

- Note: (1) This is the polytropic relations for an Ideal gas with $n = k$!
- (2) $n \equiv k$ corresponds to another special case of a polytropic process
- (3) This $n \equiv k$ process is Isentropic, meaning that it is Adiabatic, no friction, and quasi-equilibrium.
- (4) Isentropic processes, where $S_1 \equiv S_2$, are totally reversible!

Example 1

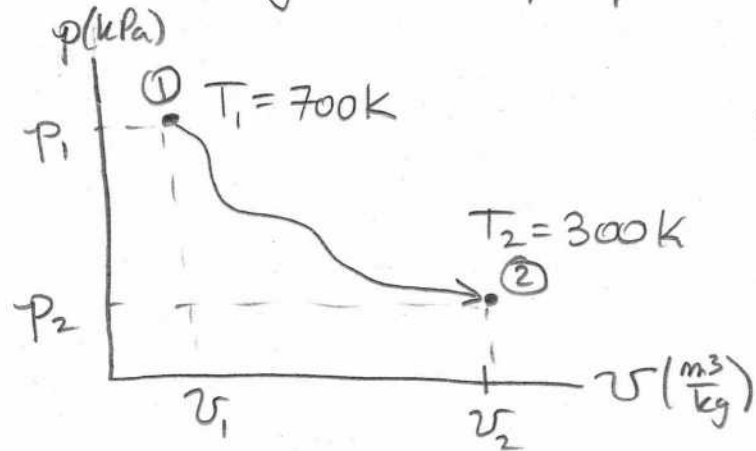
System Entropy change for an Ideal gas moving through an Open System

(8)

Given: Helium behaves as an Ideal gas, undergoing this process:

$$\text{State ①} \begin{cases} p_1 = 2000 \text{ kPa} \\ T_1 = 700 \text{ K} \end{cases}$$

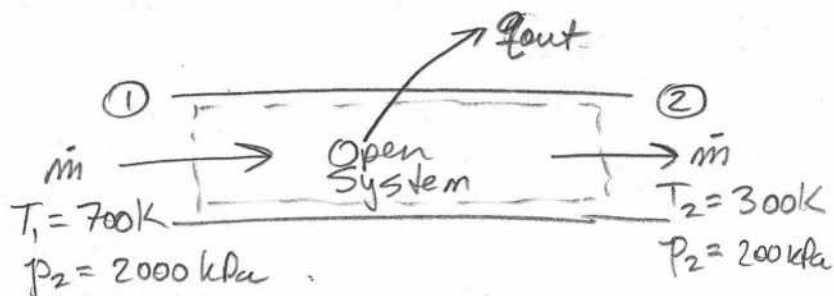
$$\text{State ②} \begin{cases} p_2 = 200 \text{ kPa} \\ T_2 = 300 \text{ K} \end{cases}$$



Assume: (1) Helium has constant, temperature averaged specific heats
(2) The System is Open, so we must use $C_p = C_{p,avg} = \text{const.}$

Analysis:

- For an Ideal Gas with an Open System, and assuming $C_p = C_{p,avg}$,



$$\Delta S_{\text{sys}} = C_{p,avg} \ln \left[\frac{T_2}{T_1} \right] - R_{\text{He}} \ln \left[\frac{p_2}{p_1} \right] \quad \left(\frac{\text{kJ}}{\text{kg-K}} \right)$$

- Helium is a monatomic gas, so the specific heats are independent of temperature! From Table 2a,

$$C_{p,\text{He}} = 5.1926 \frac{\text{kJ}}{\text{kg-K}} \quad \text{and} \quad R_{\text{He}} = 2.0769 \frac{\text{kJ}}{\text{kg-K}}$$

- The System Entropy change is

$$\Delta S_{\text{sys}} = \left(5.1926 \frac{\text{kJ}}{\text{kg-K}} \right) \ln \left[\frac{300 \text{ K}}{700 \text{ K}} \right] - \left(2.0769 \frac{\text{kJ}}{\text{kg-K}} \right) \ln \left[\frac{200 \text{ kPa}}{2000 \text{ kPa}} \right]$$

$$\Delta S_{\text{sys}} = (S_2 - S_1)_{\text{sys}} = -4.3997 \frac{\text{kJ}}{\text{kg-K}} + 4.7822 \frac{\text{kJ}}{\text{kg-K}}$$

$$(S_2 - S_1)_{\text{sys}} = 0.3825 \frac{\text{kJ}}{\text{kg-K}}$$

9

B) Ideal Gas, Variable Specific Heats

$$(S_2 - S_1)_{sys} = \int_{T_1}^{T_2} C_v(T) \frac{dT}{T} + R \ln\left(\frac{v_2}{v_1}\right)$$

$$(S_2 - S_1)_{sys} = \int_{T_1}^{T_2} C_p(T) \frac{dT}{T} - R \ln\left(\frac{p_2}{p_1}\right)$$

- How do we solve problems to find $(S_2 - S_1)_{sys}$ with variable specific heats?!

⇒ Use Ideal Gas Tables A-17 to A-25

- Look at Table A-17, for example,

T K	h $\frac{kJ}{kg}$	P _r	u $\frac{kJ}{kg}$	u _r	S ^o ($\frac{kJ}{kg \cdot K}$)
...					
310	---	---	---	---	1.73498
315	---	---	---	---	1.75106
320	---	---	---	---	1.76690

$$S^o \equiv \int_{T=0K}^T C_p(T) \frac{dT}{T} \quad \left(\frac{kJ}{kg \cdot K}\right)$$

Table's reference Temperature $T = 0K$, $S(0K) = 0 \frac{kJ}{kg \cdot K}$

- We use the tabulated values of $S^\circ(T)$ so that we can easily compute ΔS_{sys} :

(10)

$$(S_2 - S_1)_{sys} = S_2^\circ - S_1^\circ + R \ln(v_2/v_1) \quad \left(\frac{kJ}{kg \cdot K} \right)$$

$$(S_2 - S_1)_{sys} = S_2^\circ - S_1^\circ - R \ln(P_2/P_1)$$

For gases other than 'Air', the units are on a molar basis:

$$(\bar{S}_2 - \bar{S}_1) = \bar{S}_2^\circ - \bar{S}_1^\circ - R \ln(P_2/P_1) \quad \left(\frac{kJ}{kmol \cdot K} \right)$$

where R = Universal Gas Constant

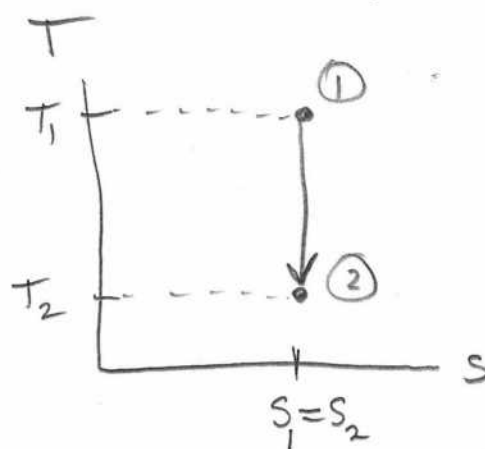
\Rightarrow to convert from $\frac{kJ}{kmol \cdot K}$ you must divide by the substance's molecular mass, M ($\frac{kg}{kmol}$)

Isentropic Process with Variable Specific Heats

- For an Isentropic process,

$$S_2 = S_1$$

- Setting the above equations equal to zero and solving then is tedious so,



\Rightarrow Use Table A-17

For an Isentropic Process with variable c_p, c_v , we define:

$$P_r \equiv e^{\frac{s_0}{R}} \equiv \text{"Relative Pressure" (unitless)}$$

$$v_r \equiv \text{"Relative Specific Volume" (unitless)}$$

such that at two different states ① & ②,

$$\left. \frac{P_2}{P_1} \right|_{s=\text{const}} = \frac{P_{r2}}{P_{r1}}$$

$$\left. \frac{v_2}{v_1} \right|_{s=\text{const}} = \frac{v_{r2}}{v_{r1}}$$

Isentropic Process
Variable Specific Heats

Example 2

Say we know P_1, T_1 at State ① and P_2 at State ② for Air

Given: $T_1 = 320 \text{ K}$
 $P_1 = 100 \text{ kPa}$
 $P_2 = 400 \text{ kPa}$

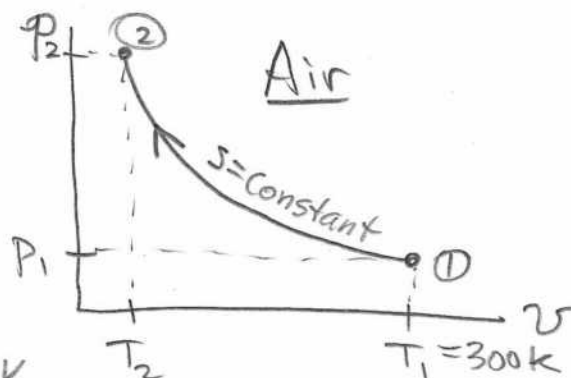
Find: T_2

From Table A-17, at $T_1 = 320 \text{ K}$,

$$P_{r1} = 1.7375$$

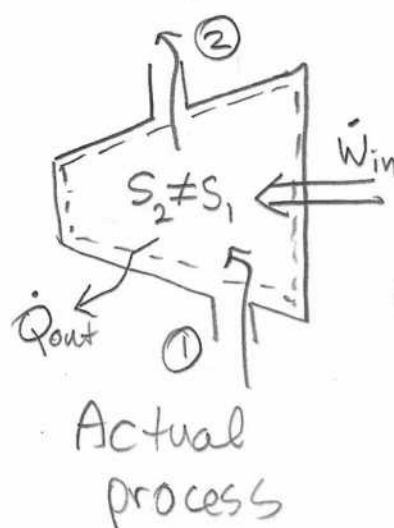
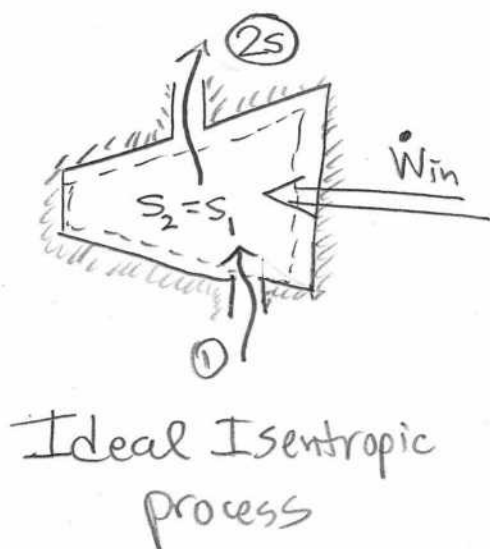
$$\text{Then } P_{r2} = P_{r1} \left(\frac{P_2}{P_1} \right) = (1.7375) \left(\frac{400 \text{ kPa}}{100 \text{ kPa}} \right) = 6.95$$

$$T_2 = 470 \text{ K}$$

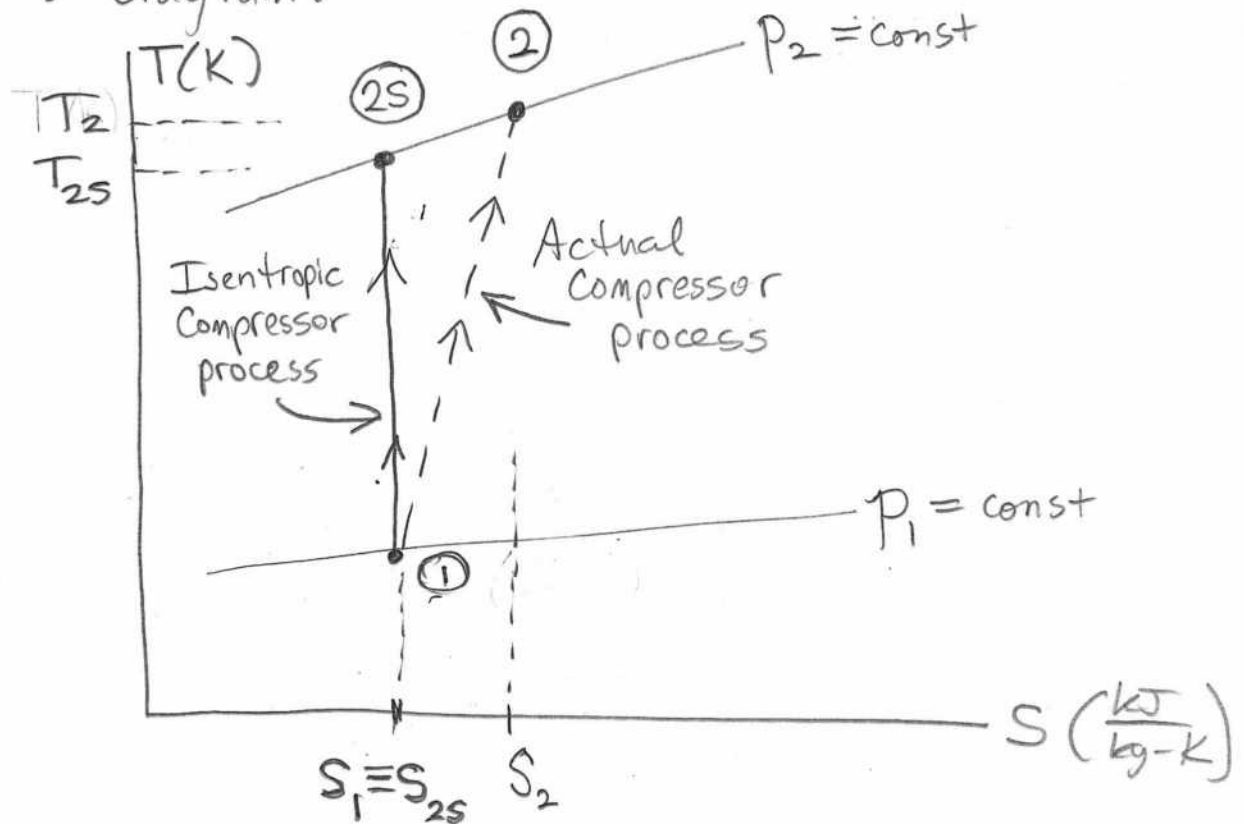


Isentropic Efficiencies of Steady Flow Engineering Devices

- Irreversibilities inherently accompany all actual processes \Rightarrow this degrades the performance of your device
- In the presence of irreversibilities, we need to be able to compute the degradation in performance of the device in an easy way.
- The standard that we compare the actual device process to is an Isentropic process
 \Rightarrow The Device is assumed to operate with an ideal, reversible process:



- For example, the Isentropic and Actual Compressor processes are best shown on a T-s diagram:



- For a Compressor, the "Isentropic Compressor Efficiency" is defined

$$\eta_c \equiv \frac{\text{Isentropic work input}}{\text{Actual compressor work input}}$$

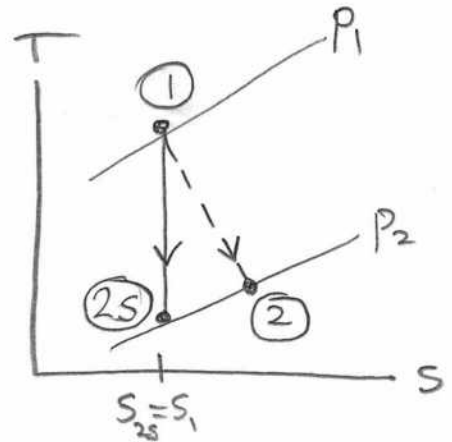
- When $\Delta KE)_{sys} \approx 0$, $\Delta PE)_{sys} \approx 0$,

$$\eta_c \approx \frac{h_{2s} - h_1}{h_2 - h_1}$$

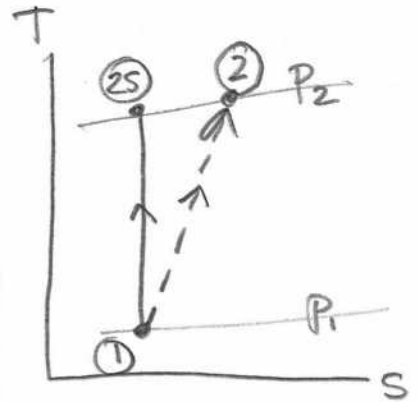
$$0 \leq \eta_c \leq 1$$

(1) Turbines

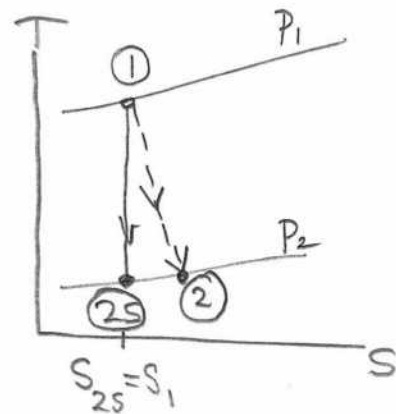
$$\eta_T \approx \frac{h_1 - h_2}{h_1 - h_{2s}}$$

(2) Compressors

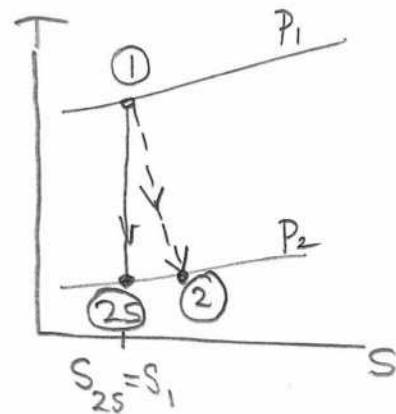
$$\eta_c \approx \frac{h_{2s} - h_1}{h_2 - h_1}$$

(3) Pumps

$$\eta_P \approx \frac{v(P_2 - P_1)}{h_2 - h_1}$$

(4) Nozzles

$$\eta_N \approx \frac{h_1 - h_2}{h_1 - h_{2s}}$$



Note: (1) Sometimes you are given the η and must find either h_2 , h_{2s} , or maybe h_1 .

(2) Sometimes you must find η based on the states.

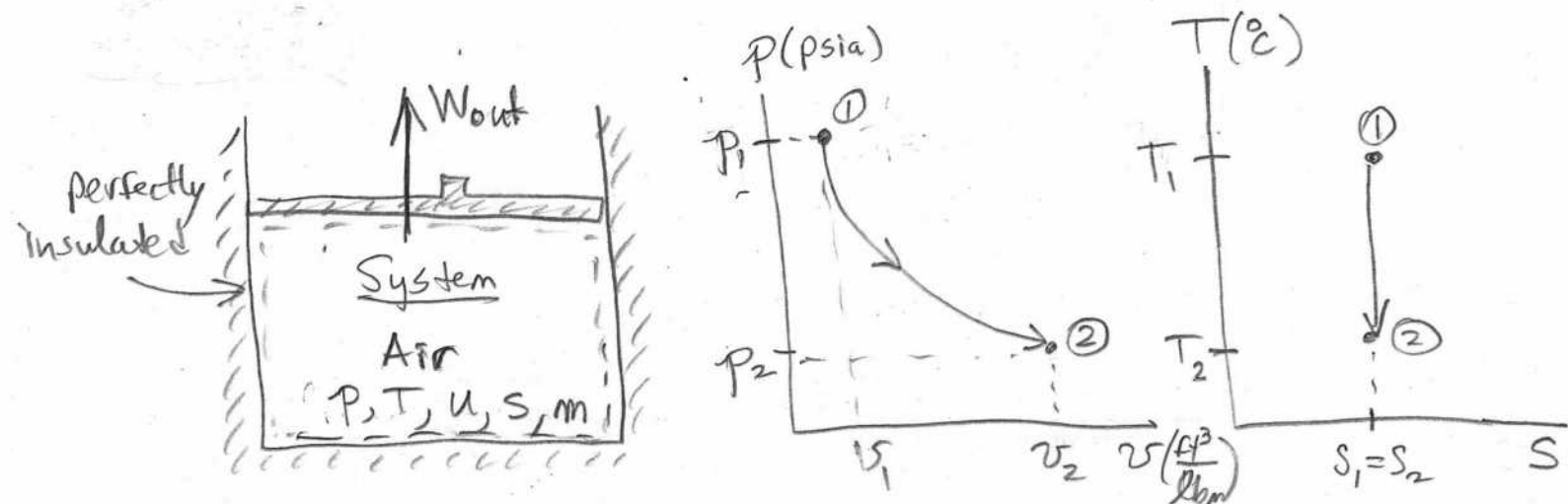
Example 3 Isentropic Expansion of Air as an Ideal Gas assuming Constant, then variable specific heats

Given: Air is expanded Isentropically in a closed system performing Work output.

Assume: (1) Air behaves as an Ideal Gas

(2) Specific heats are constant and so $k = \frac{C_p}{C_v} = \text{constant}$

(3) Adiabatic, quasiequilibrium process without any internal irreversibilities (no friction)



$$\text{State ①} \begin{cases} T_1 = 500^{\circ}\text{C} \\ P_1 = 100 \text{ psia} \end{cases}$$

$$\text{State ②} \begin{cases} T_2 = ? \\ P_2 = 20 \text{ psia} \end{cases}$$

Find: T_2 for two cases:

(a) $C_v = C_{v,\text{avg}} = \text{const}, C_p = C_{p,\text{avg}} = \text{constant} \Rightarrow \underline{k = \text{Constant}}$

(b) Variable specific heats: $C_v = C_v(T), C_p = C_p(T) \Rightarrow \underline{k = k(T)}$

Analysis

(16)

(a) $C_v = \text{const}$, $C_p = \text{const}$, $k = \text{const}$

- For an Ideal gas undergoing an Isentropic process with constant specific heats, we can use the isentropic relations immediately since we know P_1 , P_2 , and T_1 :

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}}$$

$$\boxed{T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}}}$$

- The problem, though, is we do not know the average value of $k_{\text{avg}}(T_{\text{avg}})$ because T_2 is unknown!

Q: What do we do?

A: Guess T_2 , compute T_{avg} , look up $k_{\text{avg}}(T_{\text{avg}})$ in Table A-2Eb, compute T_2 . We repeat this iterative process until T_2 doesn't change with successive iterations.

- I am going to guess that $T_2 = 100^\circ\text{F}$ so that

$$T_{\text{avg}} = \frac{1}{2} [T_1 + T_2] = \frac{1}{2} [500^\circ\text{F} + 100^\circ\text{F}]$$

$$\boxed{T_{\text{avg}} = 300^\circ\text{F}}$$

- From Table A-2Eb, $k_{\text{avg}}(300^\circ\text{F}) = 1.394$

$$\bullet \text{ Then } T_2 = (500^\circ\text{F} + 460\text{R}) \left[\frac{20\text{psia}}{100\text{psia}} \right]^{\frac{0.394}{1.394}} \rightarrow$$

$$\boxed{T_2 = 609\text{R} = 149^\circ\text{F}}$$

- For greater accuracy, repeat the procedure using this new value of T_2 .

(b) $c_p = c_p(T)$, $c_v = c_v(T)$, $k = k(T)$, Ideal gas only

- The exact analysis for variable specific heats during an Isentropic process involves using the Relative pressure

$$\boxed{\left(\frac{P_2}{P_1}\right)_{S=\text{const}} = \frac{P_{r_2}}{P_{r_1}}}$$

and the Ideal gas Tables (for air use Table A-17E) to find the corresponding values of S_1° and S_2° :

$$\boxed{(S_2 - S_1)_{\text{sys}} = (S_1^\circ - S_2^\circ) - R \ln\left(\frac{P_2}{P_1}\right)} \quad \text{where} \quad \boxed{S^\circ = \int_{T_1}^{T_2} \frac{c_p(T)}{T} dT}$$

Isentropic process

$$\Rightarrow S_2^\circ - S_1^\circ = R \ln\left[\frac{P_2}{P_1}\right]$$

$$\ln\left[\frac{P_2}{P_1}\right] = \frac{S_2^\circ - S_1^\circ}{R}$$

$$\Rightarrow \boxed{\left(\frac{P_2}{P_1}\right)_{S=\text{const}} = e^{\frac{S_2^\circ - S_1^\circ}{R}} = e^{S_2^\circ/R} e^{-S_1^\circ/R} = \frac{e^{S_2^\circ/R}}{e^{S_1^\circ/R}} \equiv \frac{P_{r_2}}{P_{r_1}}}$$

- First, we compute $\left(\frac{P_2}{P_1}\right)_{S=\text{const}} = \frac{20 \text{ psia}}{100 \text{ psia}} \Rightarrow \boxed{\left(\frac{P_2}{P_1}\right)_{S=\text{const}} = 0.2}$

- Thus, $\boxed{\frac{P_{r_2}}{P_{r_1}} = 0.2}$

- We are given $T_1 = 560^\circ\text{F} + 460\text{R} \Rightarrow \boxed{T_1 = 960\text{R}}$
This corresponds to $\boxed{P_{r_1} = 10.61}$ in Table A-17E.

- $P_{r_2} = 0.2 P_{r_1} = (0.2)(10.61) \Rightarrow \boxed{P_{r_2} = 2.122}$

- Interpolating between 600R and 620R, $\boxed{T_2 \approx 609\text{R} = 149^\circ\text{F}}$ for the corresponding P_{r_2}

Example 4

Air as an Ideal Gas is Compressed Adiabatically in a steady flow compressor with Internal Irreversibilities (friction, turbulence, heat transfer from components to the air, etc.)

Given:

Air enters a perfectly insulated compressor which has an isentropic efficiency $\eta_c = 84\%$

State ①

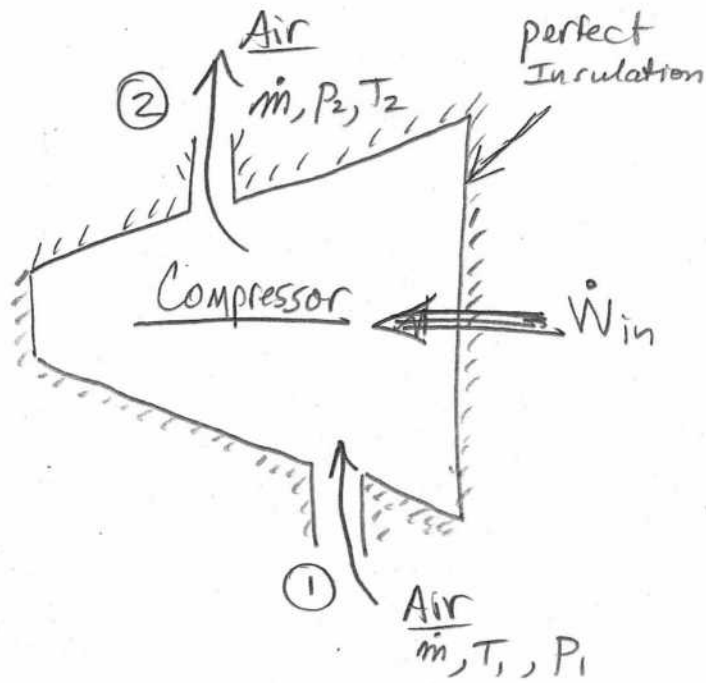
$$P_1 = 100 \text{ kPa}$$

$$T_1 = 17^\circ\text{C} = 290 \text{ K}$$

$$\dot{V}_1 = 2.4 \frac{\text{m}^3}{\text{s}}$$

State ②

$$T_2 = 257^\circ\text{C} = 530 \text{ K}$$



- Find:**
- (a) P_2 (kPa)
 - (b) \dot{W}_{in} (kW)
 - (c) ΔS_{sys} ($\frac{\text{kJ}}{\text{kg-K}}$)

Assume: (1) $\Delta KE)_{sys} = \Delta PE)_{sys} = 0$

(2) Steady flow

(3) Variable specific heats

(5) Perfectly insulated \Rightarrow Adiabatic process BUT this is not an Isentropic process ($\Delta S_{sys} \neq 0$) because there are Internal irreversibilities that make $\eta_c = 84\% < 100\%$

Analysis

(a) P_2 (kPa)

• Draw a picture!! \Rightarrow

• We want the actual exit pressure P_2 , but all we know is $T_2 = 530\text{K}$.

• Strategy: (1) Since we are given the Isentropic efficiency, this tells us to use an imaginary Isentropic process (see picture) to find P_2 since

$$P_2 = P_{2s}$$

through the equation $P_2 = P_{2s} = P_1 \left(\frac{P_{r2s}}{P_{r1}} \right)$

(2) But P_{r2s} is unknown because T_{2s} is unknown!!

\Rightarrow find T_{2s} using Table A-17E by finding the corresponding Enthalpy h_{2s}

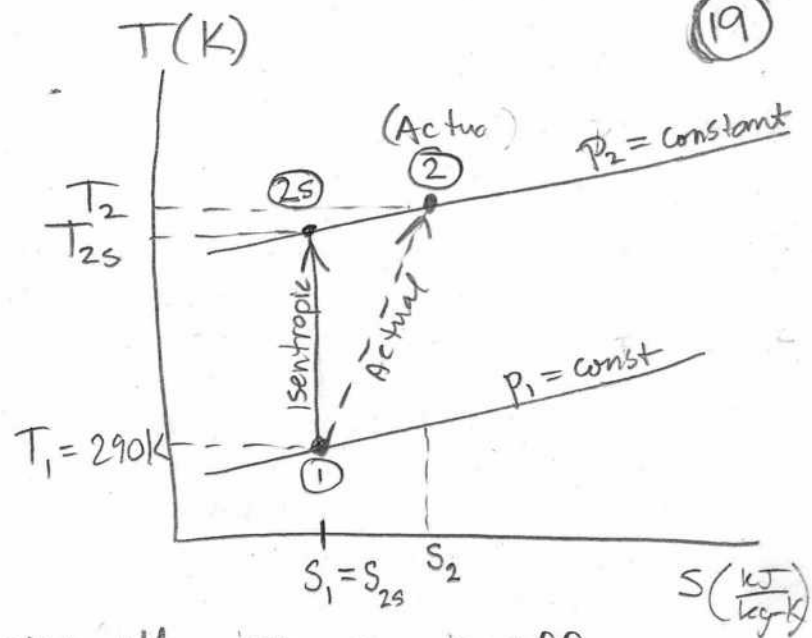
(3) But h_{2s} is unknown!

$$\Rightarrow \text{USE } \eta_c \equiv \frac{h_{2s} - h_2}{h_2 - h_1} = 0.84$$

$$h_{2s} = h_1 + 0.84(h_2 - h_1)$$

(4) find h_1 and h_2 using $T_1 = 290\text{K}$ and $T_2 = 530\text{K}$ in Table A-17E =

$$h_1 = 290.16 \frac{\text{kJ}}{\text{kg}}, \quad h_2 = 533.98 \frac{\text{kJ}}{\text{kg}}$$



- Now we can compute h_{2s} !

$$h_{2s} = 290.16 \frac{\text{kJ}}{\text{kg}} + (0.84)(533.98 \frac{\text{kJ}}{\text{kg}} - 290.16 \frac{\text{kJ}}{\text{kg}})$$

$$\boxed{h_{2s} = 495 \frac{\text{kJ}}{\text{kg}}}$$

- Now find the corresponding Relative Pressure $P_{r_{2s}}$ in Table A-17E:

$$\Rightarrow \boxed{P_{r_{2s}} \approx 7.951}$$

- Find P_{r_1} in Table A-17E at $T_1 = 290\text{K}$: $\boxed{P_{r_1} = 1.2311}$

- Now solve for P_2 : $P_2 = P_{2s} = (100\text{kPa}) \left(\frac{7.951}{1.2311} \right)$

$$\boxed{P_2 \approx 646\text{kPa}}$$

(b) \dot{W}_{in} (kW)

- From an Open system Energy Balance Analysis,

$$\boxed{\dot{W}_{in} = \dot{m}(h_2 - h_1)}$$

- We have h_1, h_2 but need \dot{m} ($\dot{m}_1 = \dot{m}_2$, obviously)

- We are given the inlet volume flow rate $\boxed{\dot{V}_1 = 2.4 \frac{\text{m}^3}{\text{s}}}$

- Using mass continuity Equation and the Ideal Gas EOS Together:

$$(i) \text{ mass continuity : } \dot{m} = \frac{\dot{V}}{v}$$

$$(ii) \text{ Ideal gas EOS : } p v = R T$$

Substitute $v = \frac{RT}{p} \rightarrow$ mass continuity to get, at State 1,

$$\boxed{\dot{m}_1 = \frac{\dot{V}_1 P_1}{R T_1}}$$

- Substituting known values, ($R_{air} = 0.287 \frac{kJ}{kg \cdot K}$)

(21)

$$\dot{m} = \frac{(2.4 m^3/s)(100 kPa)}{(0.287 \frac{kJ}{kg \cdot K})(290 K)} \rightarrow \boxed{\dot{m} = 2.884 kg/s}$$

- Now we can find \dot{W}_{in} :

$$\dot{W}_{in} = \dot{m}(h_2 - h_1) = (2.884 \frac{kg}{s})(533.98 \frac{kJ}{kg} - 290.16 \frac{kJ}{kg})$$

$$\boxed{\dot{W}_{in} = 703 kW}$$

(c) $\Delta S_{sys} (\frac{kJ}{kg \cdot K})$

- For an Ideal gas with variable specific heats,

$$\Delta S_{sys} = (S_2 - S_1)_{sys} = (S_2^0 - S_1^0) - R \ln\left(\frac{P_2}{P_1}\right)$$

- From Table A-17E, for $T_1 = 290K$ and $T_2 = 530K$,

$$\begin{array}{l} S_1^0 = 1.66802 \frac{kJ}{kg \cdot K} \\ S_2^0 = 2.27967 \frac{kJ}{kg \cdot K} \end{array}$$

- With $P_1 = 100 kPa$ and $P_2 = 646 kPa$,

$$\Delta S_{sys} = (2.27967 \frac{kJ}{kg \cdot K} - 1.66802 \frac{kJ}{kg \cdot K}) - (0.287 \frac{kJ}{kg \cdot K}) \ln\left(\frac{646 kPa}{100 kPa}\right)$$

$$\Delta S_{sys} = 0.61165 \frac{kJ}{kg \cdot K} - 0.53543 \frac{kJ}{kg \cdot K}$$

$$\boxed{\Delta S_{sys} = 0.07622 \frac{kJ}{kg \cdot K}}$$

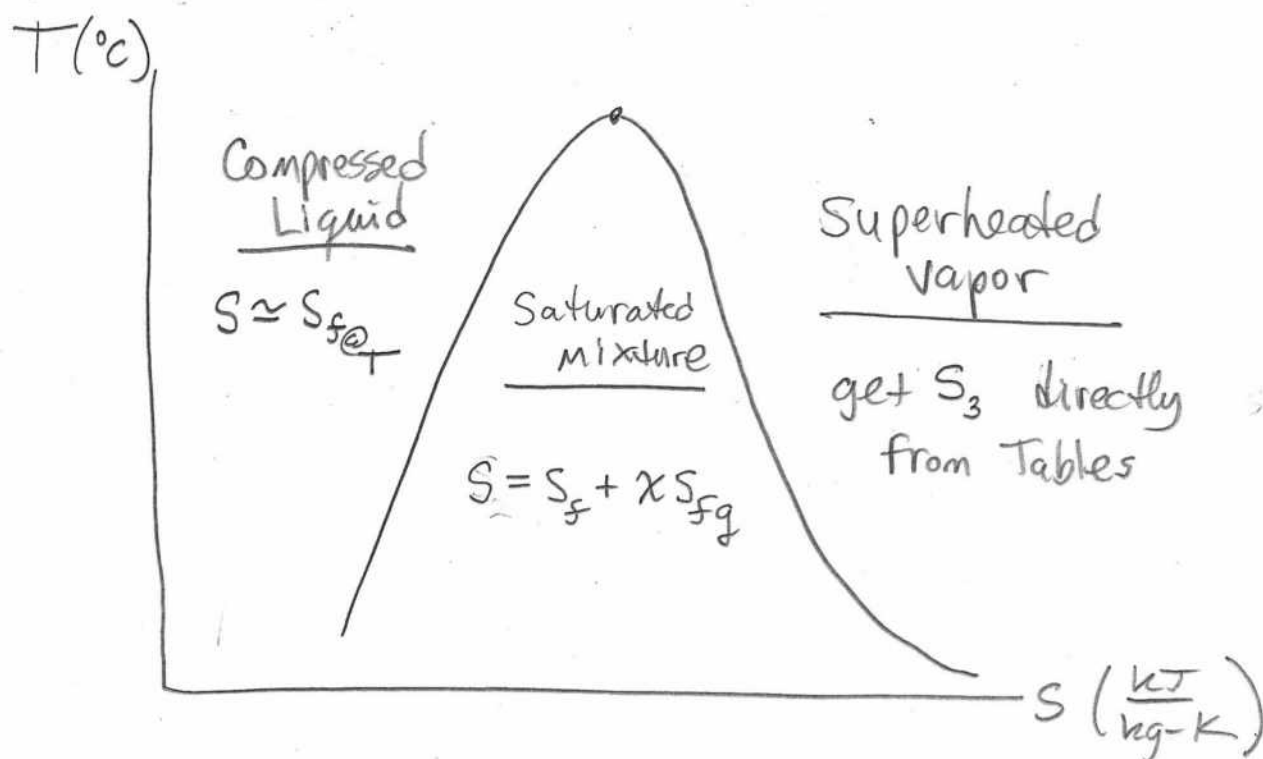
Note: The System Entropy increased because the inefficiencies within the Compressor — It was compressed adiabatically but not isentropically!

III System Entropy Change for Saturated Mixtures, Superheated Vapor, and Compressed Liquids

• Reference "Datum" for Entropy (Arbitrary)

- H₂O Tables : @ $T = 0.01^\circ\text{C}$, $S_f \equiv 0.00 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$
- R-134a Tables : @ $T = -40^\circ\text{C}$, $S_f \equiv 0.00 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

- You obtain Entropy from these tables in the same way you have been obtaining values for v , u , and h :



Lecture 27 The Entropy Balance Equation for Closed and Open Systems ①

(I) Entropy "Flow" or "Transfer" Due to $\frac{dT}{dx} \neq 0$

- Recall that the net Entropy flow into a system is, in general, irreversible whenever $\frac{dT}{dx} \neq 0$ and is quantified by

$$S_{\text{Net, in}}^{\text{flow}} = S_{\text{in}}^{\text{flow}} - S_{\text{out}}^{\text{flow}} = \int_1^2 \frac{\delta Q_{\text{Net, in}}}{T} \quad (1)$$

Where T is evaluated at the system boundary.

- If $T = \text{constant}$, Then the process is reversible so

$$S_{\text{Net, in}}^{\text{flow}} = \frac{1}{T} \int_1^2 \delta Q_{\text{Net, in}}$$

$$S_{\text{Net, in}}^{\text{flow}} = \left(\frac{Q_{\text{Net, in}}}{T} \right)_{\text{rev}} \quad (2)$$

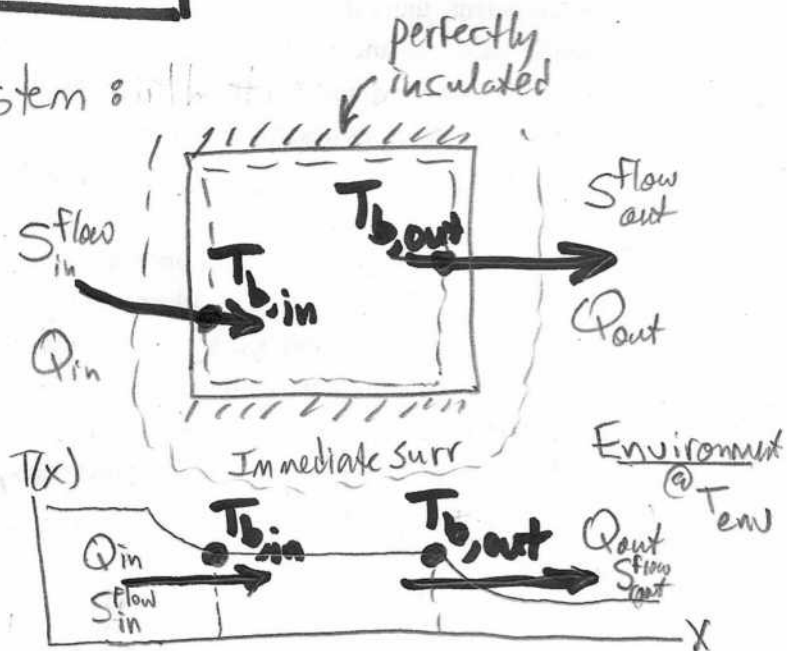
- Consider the Closed System:

- At the System Boundary points

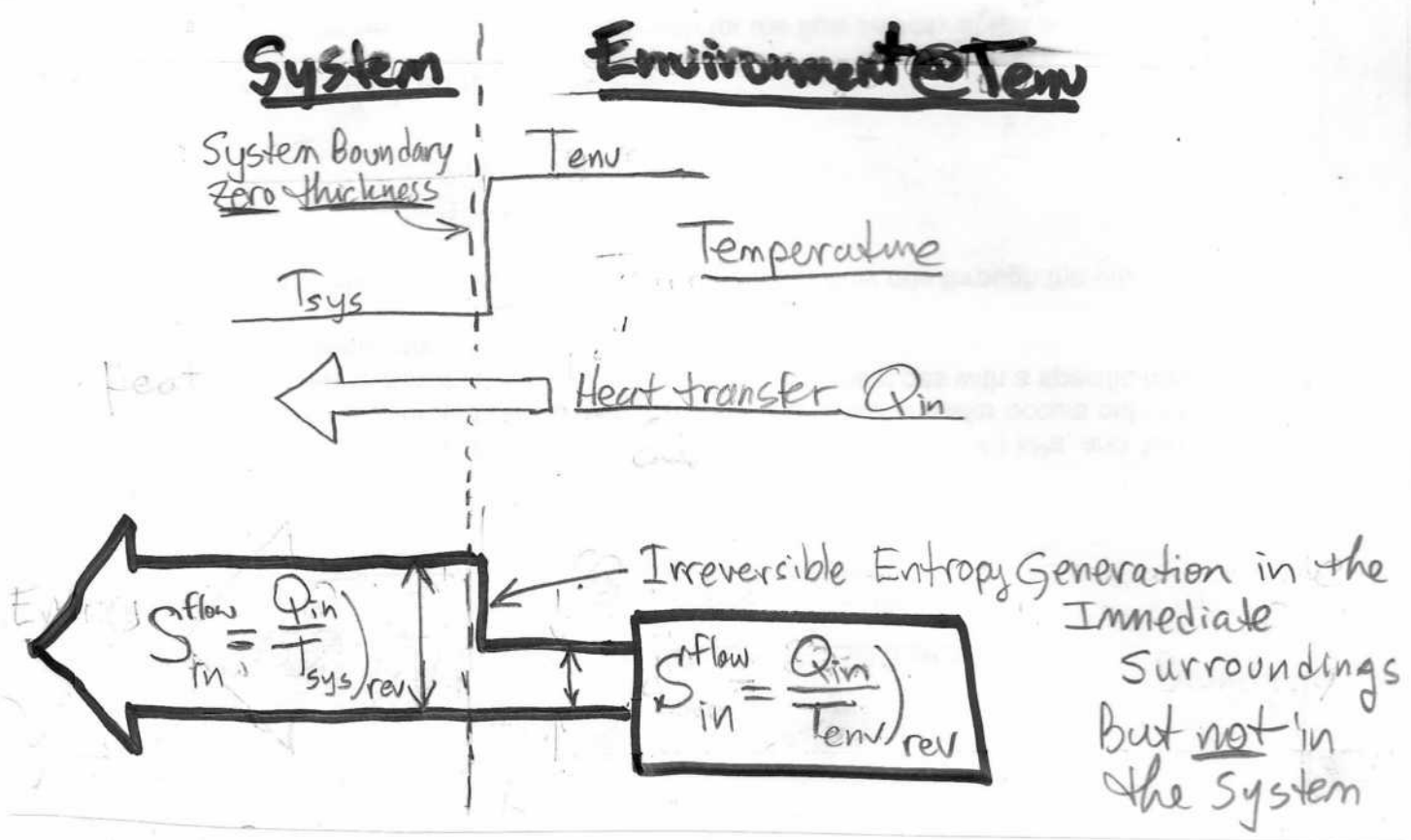
$T_{b, \text{in}}$ and $T_{b, \text{out}}$, $\frac{dT}{dx} = 0$

So

$$\begin{aligned} S_{\text{out}}^{\text{flow}} &= \left(\frac{Q_{\text{out}}}{T_{b, \text{out}}} \right)_{\text{rev}} \\ S_{\text{in}}^{\text{flow}} &= \left(\frac{Q_{\text{in}}}{T_{b, \text{in}}} \right)_{\text{rev}} \end{aligned}$$



- No Entropy can be created or destroyed at a boundary having no thickness or volume



- We assume that there is $\frac{dT}{dx} \neq 0$ in the immediate surroundings only
- Note above that $S_{flow, in}$ inside the System compared to Outside the System because $T_{sys} < T_{env}$ with $Q_{in} = \text{constant across the boundary!}$

For the given System:

$S_{flow, net, in} = S_{flow, in} - S_{flow, out}$

$S_{flow, net, in} = \frac{Q_{in}}{T_{b, in}} - \frac{Q_{out}}{T_{b, out}} = \sum_k \frac{Q_{k, net, in}}{T_k}$ $k = \text{Boundary \#}$

Boundary #1

Boundary #2

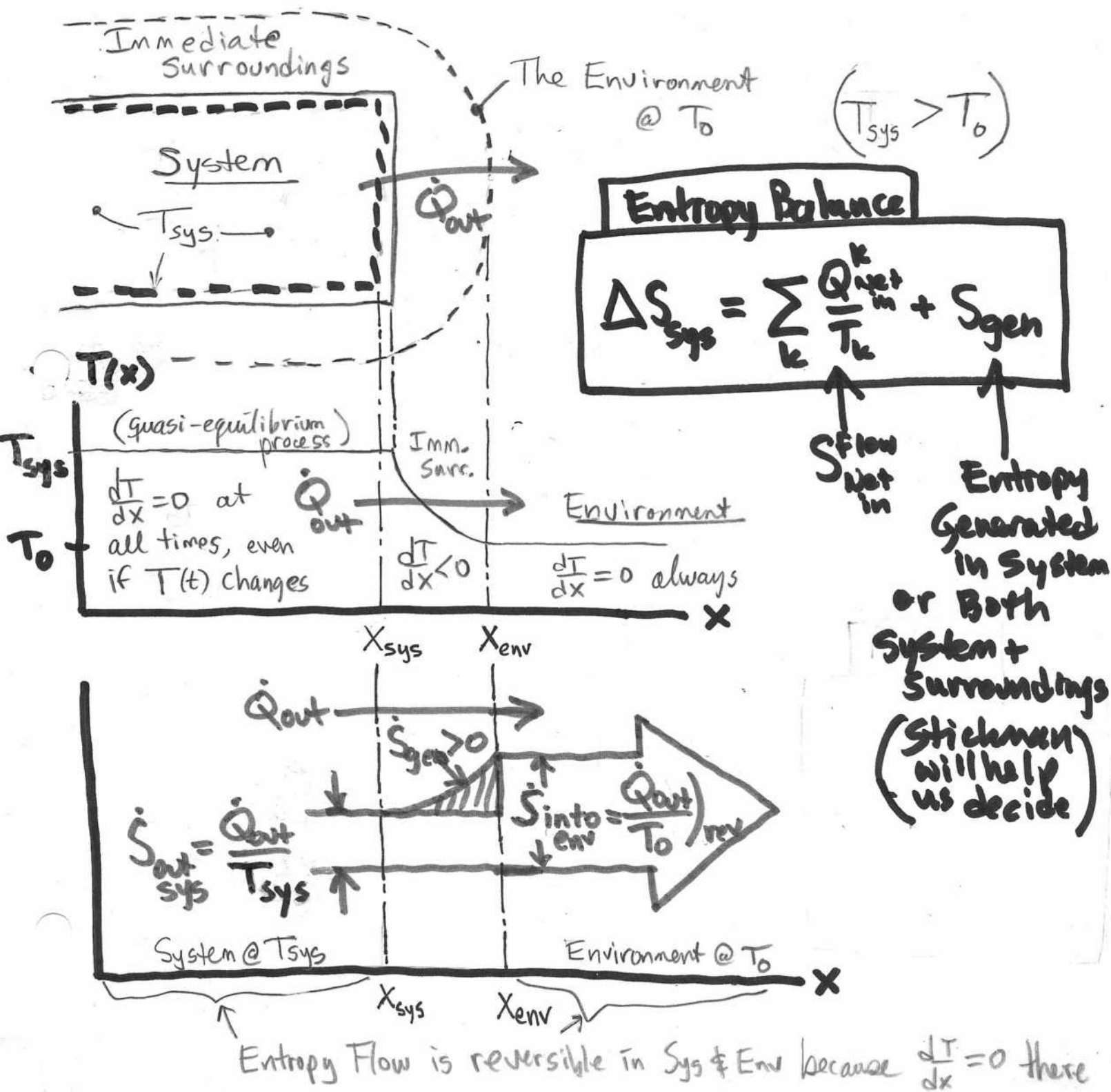
Q_{in}

$T_{b, in}$

Q_{out}

$T_{b, out}$

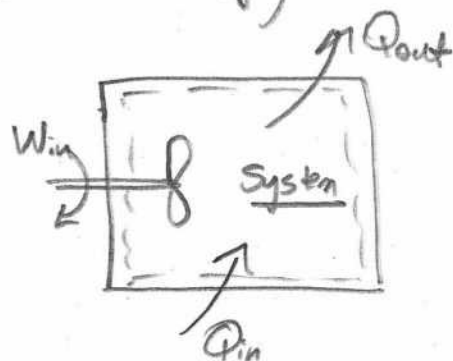
- Consider a quasi-equilibrium process in which heat transfer occurs out of the System and into the Environment.



(II) Entropy Change for System only or for Both System + Surroundings

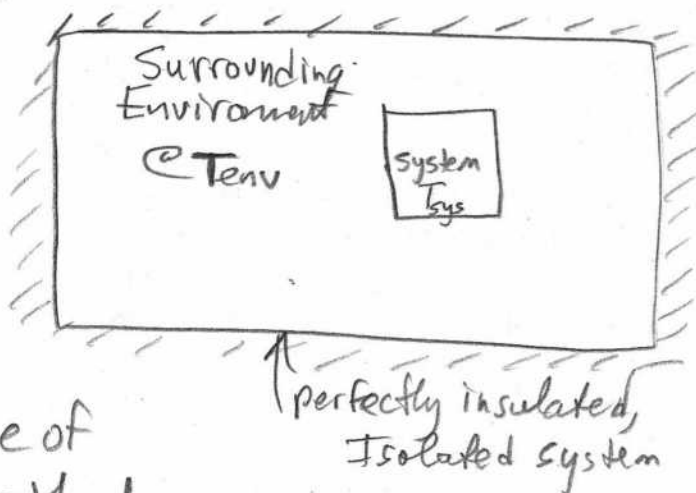
- For the System only, the Increase of Entropy Principle states that

$$S_{gen, sys} = \Delta S_{sys} \geq 0$$



- The Surrounding Environment is also a System, and many times we want to combine the System and Surroundings into one big system.

- Now we consider the System and Surroundings as an isolated system →



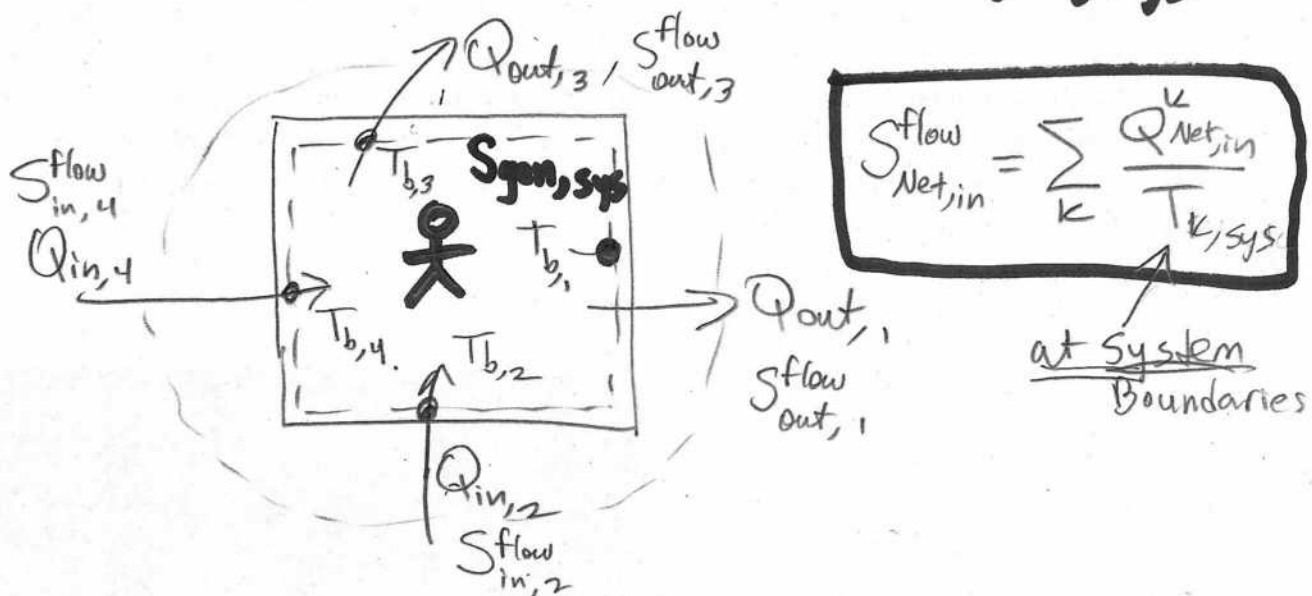
- In this case, the Increase of Entropy principle states that

$$S_{gen, sys + surr} = \Delta S_{surr} + \Delta S_{sys} \geq 0$$

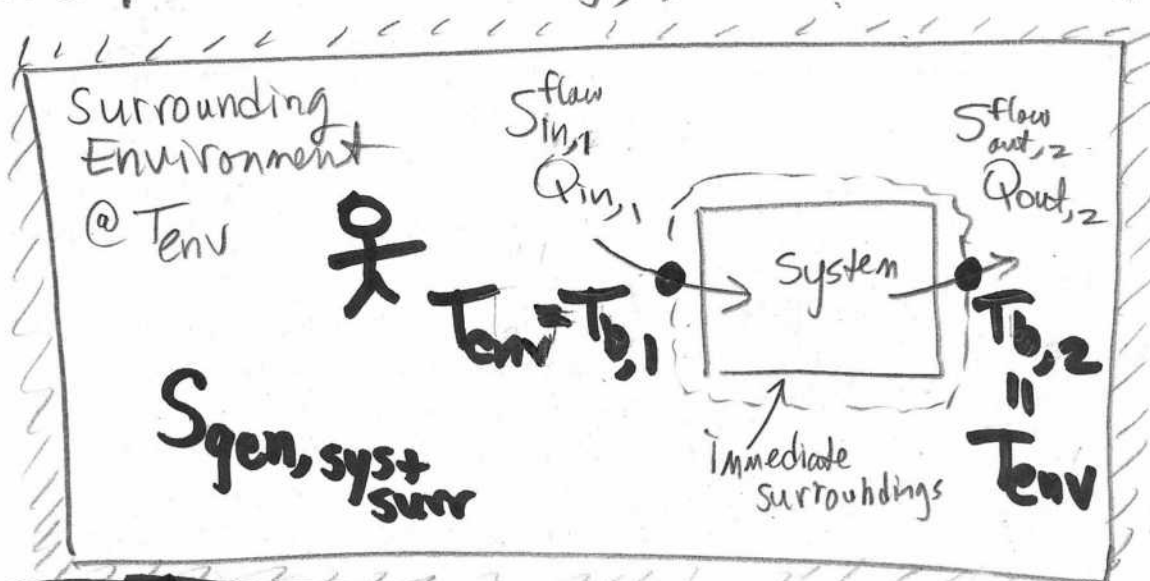
Stickman ♀

(5)

- If we are asked to find S_{gen} for the System, then we place Stickman inside the System where he sees the boundary Temperatures inside the System, and we write $S_{gen, sys}$



- If we are asked to find S_{gen} for the System + Surroundings (or for the Universe due to this process under study), then Stickman goes here:



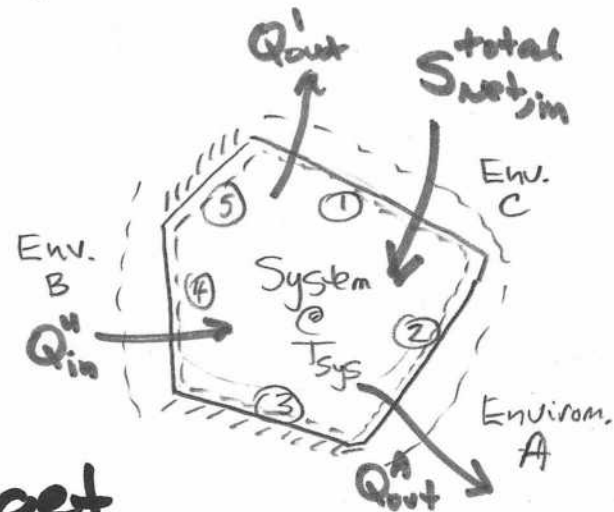
$$S_{flow_{Net,in}} = \sum_k \frac{Q_{Net,in}^k}{T_{k,env}}$$

← T_k evaluated at Environment Temperature on Outer boundary

- The total Net reversible Entropy transfer accounting for ALL boundaries of the system is

$$S_{\text{Net},in} = \sum_{k=1}^m S_{\text{Net},in}^k$$

$$S_{\text{Net},in}^{\text{total}} = \sum_{k=1}^m \frac{Q_{\text{Net},in}^k}{T_k}$$



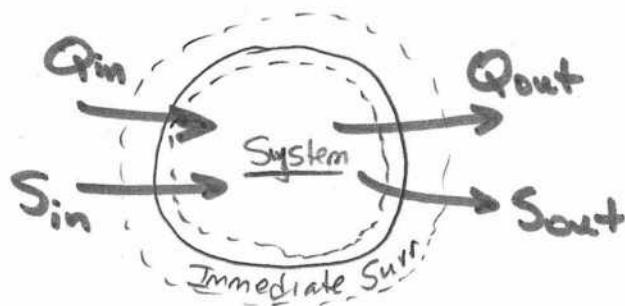
- Here is where students get into problems: For a given Boundary #, which temperature do you use? Should you use T_{sys} or the Environment Temperature in the denominator?!
- Answer: It depends on the question you are asked to answer!

• "Stickman" ☿

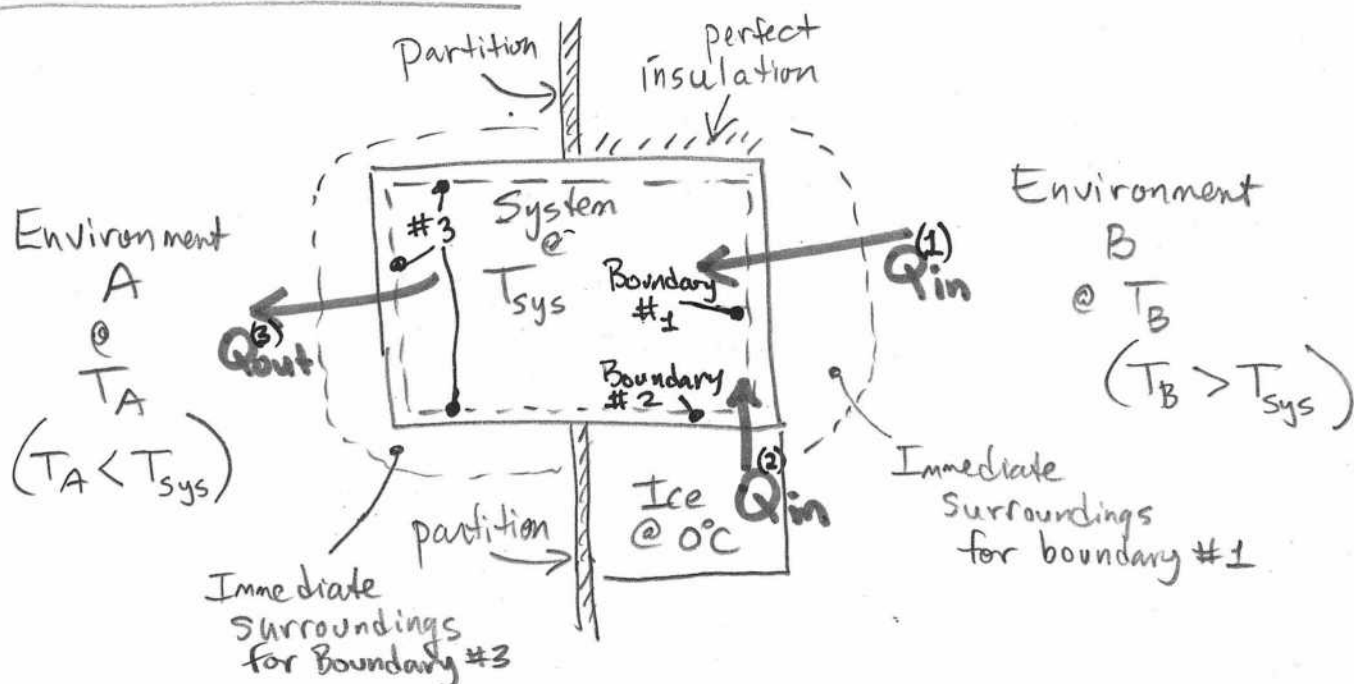
⇒ Until you become comfortable determining which temperature to use in $\frac{Q_{\text{Net},in}^k}{T_k}$, I have invented "Stickman" to help.

- Consider a generic Closed System. The net reversible Entropy transfer across the System boundary due to heat transfer is

$$S_{in} - S_{out} \equiv S_{Net,in} = \frac{Q_{Net,in}}{T}_{rev}$$



- Suppose the System has Multiple Boundaries Exposed to Different Environments:



⇒ Each Boundary is labeled $k = 1, 2, 3, \dots$ etc.

⇒ The reversible Entropy transfer at each boundary is

$$S_{Net,in}^k = \frac{Q_{Net,in}^k}{T_k}$$

⇒ Example for Boundary #2: $S_{Net,in}^2 = S_{in}^2 - \cancel{S_{out}^2} = \frac{(Q_{in}^2 - \cancel{Q_{out}^2})}{T_2}_{rev}$

So $S_{in}^2 = \frac{Q_{in}^2}{T_2}_{rev}$

(III) Closed System Entropy Balance

$$\Delta S_{\text{sys}} = (S_2 - S_1)_{\text{sys}} = S_{\text{in}} - S_{\text{out}} + S_{\text{gen}}$$

"change form"
(kJ/K)

Net change in System Entropy.
 S_1 & S_2 are State properties from Tables or formulas

$\sum_{k=1}^m \frac{\dot{Q}_{\text{net},in}^k}{T_k}$
Net Entropy transfer due to heat transfer across system boundaries

Entropy generated either within the System, the Environment, or both
($S_{\text{gen}} \geq 0$)

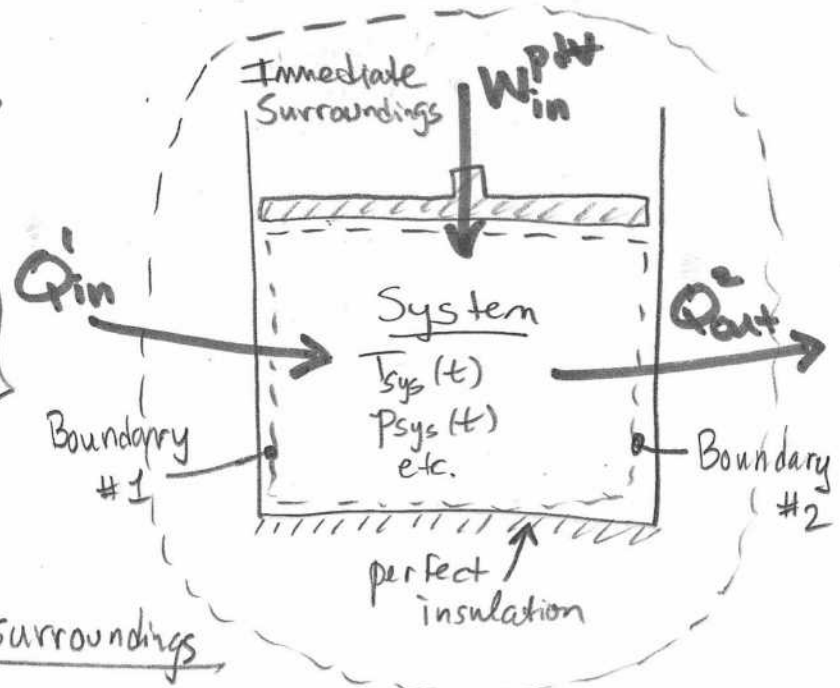
$$\left(\frac{dS}{dt}\right)_{\text{sys}} = \sum_{k=1}^m \frac{\dot{Q}_{\text{net},in}^k}{T_k} + \dot{S}_{\text{gen}}$$

"rate form"
(kW/K)

Example 1

Find Entropy generated within both the System and the Surroundings (i.e., the Entire Universe) due to the given process

- Given: Piston-cylinder where system is compressed quasistatically but changes with time
(i.e., the temperature, pressure, etc. are uniform throughout the system but does change with time)



- Find: S_{gen} for system + surroundings

• Analysis

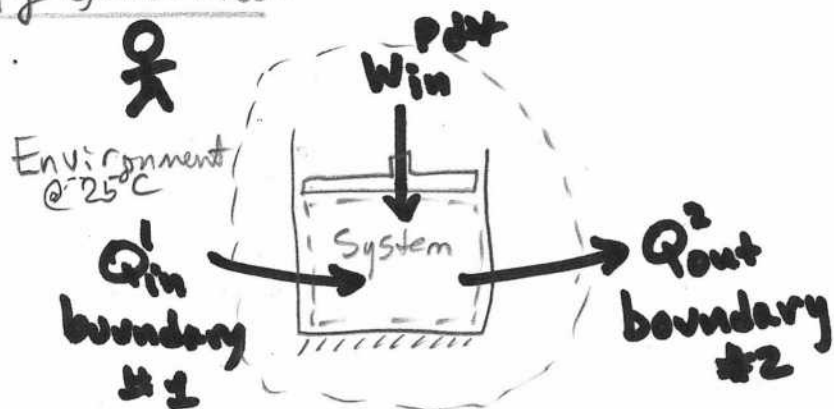
⇒ Use the "change form" ($\frac{kJ}{K}$)

Environment at
 $T_{\text{env}} = 25^\circ\text{C}$

- ⑨
- The question asks for the rate of Entropy generated in both System + Surroundings

➔ This means you have to include not only the Entropy generated in the System, but also the Immediate Surroundings where there is a temperature gradient up to the Environment

➔ Place Stickman outside, within the Environment where he feels $T_{env} = 25^\circ\text{C}$ and can "see" all the Entropy Generated:



- The Closed System Entropy Balance, written for both System + Surrounding is:

$$\Delta S)_{sys} = \sum_{k=1}^M \frac{Q_{net,in}^k}{T_k} + S_{gen,sys+surr}$$

For two boundaries,

$$\Delta S)_{sys} = \frac{Q_{net,in}^{(1)}}{T_{env}} + \frac{Q_{net,in}^{(2)}}{T_{env}} + S_{gen,sys+surr}$$

Stickman feels this environment Temperature

- Simplify $Q_{\text{net},\text{in}}^1$ and $Q_{\text{net},\text{in}}^2$: (for #1) (for boundary #2)

$$\Delta S)_{\text{sys}} = \left(\frac{Q_{\text{in}}^{(1)} - Q_{\text{out}}^{(1)}}{T_{\text{env}}} \right) + \left(\frac{Q_{\text{in}}^{(2)} - Q_{\text{out}}^{(2)}}{T_{\text{env}}} \right) + S_{\text{gen},\text{sys}+\text{surr}}$$

$$\Delta S)_{\text{sys}} = \frac{Q_{\text{in}}^{(1)}}{T_{\text{env}}} - \frac{Q_{\text{out}}^{(2)}}{T_{\text{env}}} + S_{\text{gen},\text{sys}+\text{surr}}$$

rearranging,

$$S_{\text{gen},\text{sys}+\text{surr}} = \left[\frac{Q_{\text{out}}^{(2)} - Q_{\text{in}}^{(1)}}{T_{\text{env}}} \right] + \Delta S)_{\text{sys}}$$

This term is the system's Property S from State ① → State ②, so

$$\Delta S)_{\text{sys}} = (S_2 - S_1)_{\text{sys}}$$

- To make any further progress, you need more information about the process itself, such as isothermal compression, etc.

- If the system is an Ideal gas with constant specific heats, then

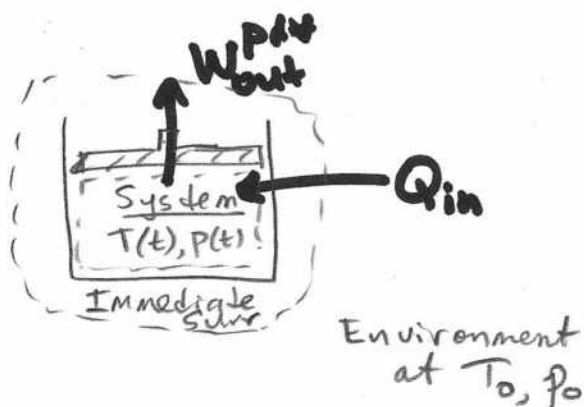
$$(S_2 - S_1)_{\text{sys}} = C_{p,\text{avg}} \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)$$

Example 2

Find the Entropy generated only within the System, which has only one boundary

Given: • piston-cylinder system with heat transfer across a single boundary:

- The Environment is at temperature T_0 and pressure P_0



Find: S_{gen} for System only

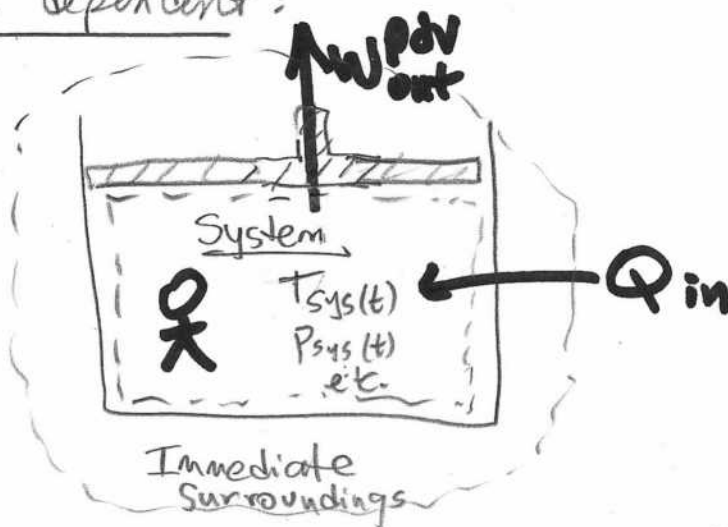
Analysis

- We again use the "Change form" because the interactions are given in terms of energy, not power:

$$\Delta S)_{sys} = \sum_{k=1}^m \frac{Q_{net,in}^k}{T_k} + S_{gen,sys}$$

- Stickman must be placed inside the system and he feels the system temperature $T_{sys}(t)$, which could be time dependent:

- Now simplify the Entropy Balance Equation:



$$\Delta S)_{sys} = \frac{(Q_{in} - Q_{out})}{T_{sys}(t)} + S_{gen,sys}$$

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$$\Delta S)_{sys} = \frac{Q_{in}}{T_{sys}(t)} + S_{gen,sys}$$

$$S_{gen,sys} = \Delta S)_{sys} - \frac{Q_{in}}{T_{sys}(t)}$$

• Without further information regarding

(a) $\Delta S)_{sys} = (S_2 - S_1)_{sys}$

(b) $T_{sys}(t)$

You cannot proceed further. If $T_{sys}(t)$ changes with time, then all you can do is estimate

$S_{gen,sys}$ by taking the average temperature between States ① & ②: $T_{sys}(t) \approx \frac{1}{2}[T_1 + T_2]$

Of course, this means you need to know the quantities T_1 and T_2 .

IV) Open System Entropy Balance

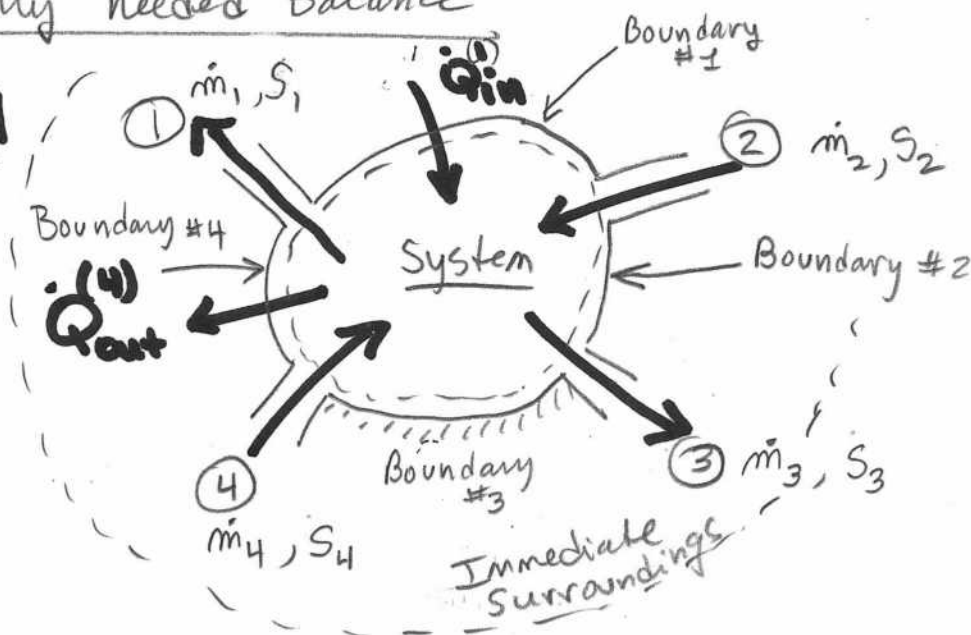
13

$$\left(\frac{dS}{dt}\right)_{sys} = \sum_{k=1}^m \frac{\dot{Q}_{Net,in}^k}{T_k} + \sum_{\text{into system}} \dot{m} S - \sum_{\text{out of system}} \dot{m} S + \dot{S}_{gen}$$

"rate form"
(kW/K)

- For an open System, the "rate form" is the most commonly needed Balance

General



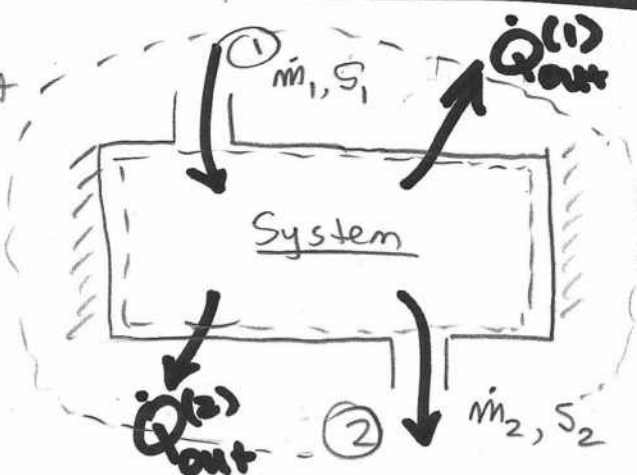
Example 3

Find Entropy generation rate for the Universe due to the given Steady-State Conditions

Given:

- Two boundaries
- $\dot{m}_1 \equiv \dot{m}_2 \equiv \dot{m}$ (steady mass flow rate)
- The Entropy Balance Eqn is

Environment @ T_0, p_0



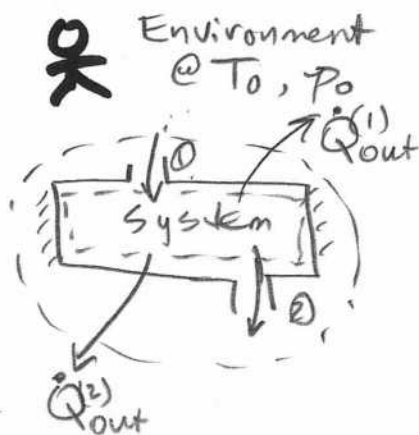
$$\left(\frac{dS}{dt}\right)_{sys} = \sum_{k=1}^m \frac{\dot{Q}_{Net,in}^k}{T_k} + \sum_{in} \dot{m} S - \sum_{out} \dot{m} S + \dot{S}_{gen, sys+surr}$$

- Stickman is placed in the Environment so that he "sees" all the Entropy Generated by both System and Immediate Surroundings and, doing so, feels temperature T_0 .

- Now Simplify the Eqn.

$$\frac{ds}{dt}_{\text{sys}} = \frac{\dot{Q}_{\text{net},in}^{(1)}}{T_0} + \frac{\dot{Q}_{\text{net},in}^{(2)}}{T_0} + \dot{m}_1 s_1 - \dot{m}_2 s_2 + \dot{S}_{\text{gen, sys+surr}}$$

Steady State Conditions



$$0 = \frac{(\dot{Q}_{\text{in}}^{(1)} - \dot{Q}_{\text{out}}^{(1)})}{T_0} + \frac{(\dot{Q}_{\text{in}}^{(2)} - \dot{Q}_{\text{out}}^{(2)})}{T_0} + \dot{m}_1 s_1 - \dot{m}_2 s_2 + \dot{S}_{\text{gen, sys+surr}}$$

$$0 = -\frac{\dot{Q}_{\text{out}}^{(1)}}{T_0} - \frac{\dot{Q}_{\text{out}}^{(2)}}{T_0} - \dot{m}(s_2 - s_1)_{\text{sys}} + \dot{S}_{\text{gen, sys+surr}}$$

$$\dot{S}_{\text{gen, sys+surr}} = \frac{1}{T_0} [\dot{Q}_{\text{out}}^{(1)} + \dot{Q}_{\text{out}}^{(2)}] + \dot{m}(s_2 - s_1)_{\text{sys}}$$

This term is the change in the System Entropy (Property) Between Inlet State ① and Outlet State ②

- If we assume an Ideal gas with constant specific heats, $P_1 = P_2$, and T_1 & T_2 known, then

$$(s_2 - s_1)_{\text{sys}} = C_{p,\text{avg}} \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)$$

$$(s_2 - s_1)_{\text{sys}} = C_{p,\text{avg}} \ln(T_2/T_1)$$

(1) Zero Wall thickness

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Look at a Cross-section of the System and Surroundings as we have been modelling it using a wall of zero thickness, assuming a Quasiequilibrium Process

System

@ $T_{sys}(t)$

Internally Reversible

System Control Volume Boundary →

T_{sys} is either spatially uniform, or, at most $T_{sys}(t)$, since quasiequilibrium

Immediate Surroundings

Environment

@ T_{env}

Externally Irreversible Due to $T(x,t)$ in the Immediate Surroundings

$T_{env} = \text{Constant in space \& time}$

$T(x,t)_{Imm\ surr}$

Q_{in}

S_{gen}

The Immediate Surroundings is where the Heat transfer Irreversibility is occurring
⇒ Here, NEW Entropy is generated!

$$S_{sys\ rev} = \frac{Q_{in}}{T_{sys, rev}}$$

$$S_{env\ rev} = \frac{Q_{in}}{T_{env, rev}}$$

Entropy "flows" into the System reversibly (since T is uniform), but

$S_{sys\ rev} > S_{env\ rev}$ due to the new Entropy generated

Entropy "Flows" reversibly out of the Environment (because T_{env} is constant), until it reaches the Immediate Surroundings

- Quasiequilibrium process
- No friction
- No chemical reactions
- No Mixing etc.

(2) Wall having non-zero thickness

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System
@ $T_{sys}(t)$

Internally Reversible

Control Volume Boundary →

- Quasiequilibrium Process
- No Friction
- No Mixing, etc.

$T_{sys} = \text{Const or } T_{sys}(t)$

Environment
@ T_{env}

Externally Irreversible

$T_{env} = \text{Const in space and time}$

$\frac{dT}{dx} \neq 0$ in Imm. Surr.

Temperature gradient outside System C.V. But within the wall

Q_{in}

Heat Transfer inside wall is Irreversible, so New Entropy is Created!

$S_{gen, wall} > 0$

$S_{gen, Imm. Surr} > 0$, New Entropy is created

$$S_{sys, rev} = \left(\frac{Q_{in}}{T_{sys}} \right)_{rev}$$

$$S_{env, rev} = \left(\frac{Q_{in}}{T_{env}} \right)_{rev}$$

Entropy Flows reversibly out of the Environment (because $T_{env} = \text{Const}$), until it reaches the Immediate Surroundings